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An Improved GA to Address Integrated Formulation of Project Scheduling and Material Ordering with Discount Options

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Abstract**—**Concurrent planning of the resource constraint project scheduling and material ordering problems have received significant attention within the last decades. Hence, the issue has been investigated here with the aim to minimize total project costs. Furthermore, the presented model considers different discount options in order to approach the real world conditions. The incorporated alternatives consist of all-unit and incremental discount strategies. On the other hand, a modified version of the genetic algorithm is applied in order to solve the model for larger sizes, in particular. Finally, the applicability and efficiency of the given model is tested by different numerical instances.

Keywords**—**Genetic algorithm, material ordering, project management, project scheduling.

I. Introduction

On a traditional planning base, project scheduling and material ordering issues were treated as distinctive problems. This approach resulted in neglecting the trade-off between the project corresponding costs, mainly consisting of the ordering, holding, and penalty (reward payments) costs for late (early) project completion. On the contrary, simultaneous consideration of the aforementioned problems can improve the plans with respect to accommodation of opportunity to order materials at the time of activities scheduling. Concurrent formulation of these two problems, to the best of our knowledge, goes back initially to the work of Aquilano and Smith [1], who introduced the integrated problem by developing a hybrid model of the critical path method with material requirement planning. Later, Smith-Daniels and Aquilano [2] regarded an enhancement to the problem by a heuristic scheduling for large-sized projects based on the least slack rule. Smith-Daniels and Smith-Daniels [3] addressed fixed duration for the activities and showed that the latest starting time schedule could yield to an optimal solution. Their proposed objective function (OF) consisted of minimization of total costs corresponding to the inventory holding, material ordering, completed activities holding, and project delay.

One of the research studies to which the aforementioned subject is nicely associated pertains to the one of Dodin and Elimam [4] who developed the problem by total costs minimization under activity crashing possibility, rewards for early completion, and materials quantity discounts. Schmitt and Faaland [5] proposed a heuristic algorithm for scheduling a recurrent construction to the net present value maximization of cash flows, where an initial schedule is developed and worker teams are dispatched to the tasks related to the backlogged products. In another research, Sheikh Sajadieh et al. [6] applied a genetic algorithm (GA) to solve an extended version of [4]. However, the crashing cost had been assumed to follow a constant slope for every activity.

The notes that can differentiate the proposed model from the existing works are that discount options are taken into consideration here, in addition to development of an efficient heuristic to solve the problem for larger sizes, in particular.

The rest of the paper is organized as follows. The mathematical model is explained in Section II. The next Section discusses the solution methodology and numerical results are presented in Section IV. Finally, the conclusions and future research directions are incorporated in the last section.

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II. Problem Definition

This section deals with the problem definition, as well as the mathematical formulation. The proposed model for simultaneous project scheduling and material ordering aims to minimize total costs of the project including penalty/reward, purchasing, ordering, and holding.

A. Mathematical Model Formulation

The mathematical model, the indices, parameters, and decision variables are introduced, as follows.

Indices

$j=1,2,\dots,N$	Index of project activities
$m=1,2,\dots,M$	Index of required materials
$t=0,1,\dots,l_N$	Index of time
$s=1,2,\dots,S$	Index of suppliers
$k=1,2,\dots,K_{ms}$	Index of price ranges for all-unit discount
$k=1,2,\dots,K'_{ms}$	Index of price ranges for incremental discount

Parameters

P_{rj}	Set of activities preceding *j*.
e_j	Earliest finish time of activity *j*.
l_j	Latest finish time of activity *j*.
C_j	Cost of activity *j*.
G_{ms}	Ordering cost of material *m* to supplier *s*.
G'_{ms}	Ordering cost of material *m* to supplier s' .

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δ_{mks}	Unit cost of material m in quantity range k purchased from supplier s.
δ'_{mks}	Unit cost of material m in quantity range k purchased from supplier s'.
R_{jm}	Requirement amount of material m to perform activity j.
h_m	Holding cost of material m.
L_{ms}	Lead time of material m ordered to supplier s.
L'_{ms}	Lead time of material m ordered to supplier s'.
d_j	Duration time of activity j.
α_{mks}	Limit on quantity range k of material m for supplier s.
α'_{mks}	Limit on quantity range k of material m for supplier s'.
K_{ms}	Number of quantity discount ranges for material m proposed by supplier s.
K'_{ms}	Number of quantity discount ranges for material m proposed by supplier s'.

Decision Variables

x_{jt}	1 if activity j is completed at time t and 0, otherwise.
λ_{mkst}	1 if material m is ordered within quantity range k to supplier s in period t and 0, otherwise.
ρ_{mkstj}	1 if material m is ordered within quantity range k to supplier s for activity j in period t and 0, otherwise.
I_{mt}	Inventory amount of material m in period t.

Now, the proposed model can be formulated according to the following mixed-integer programming model.

$$Min \quad Z=\sum_{j=1}^{N} C_j x_j+\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{t=1}^{l_N-L_{ms}} G_{ms}\sum_{k=1}^{K_{ms}}\lambda_{mkst}$$
$$+\sum_{m=1}^{M}\sum_{s=1}^{S'}\sum_{t=1}^{l_N-L'_{ms}} G'_{ms}\sum_{k=1}^{K'_{ms}}\lambda_{mks't}+\sum_{m=1}^{M}\sum_{k=1}^{K_{ms}}\sum_{s=1}^{S}\sum_{t=1}^{l_N-L_{ms}}\sum_{j=1}^{N}\delta_{mks}R_{jm}\rho_{mkstj} \quad (1)$$
$$+\sum_{m=1}^{M}\sum_{k=1}^{K'_{ms}}\sum_{s=1}^{S'}\sum_{t=1}^{l_N-L'_{ms}}\sum_{j=1}^{N}\delta'_{mks'}R_{jm}\rho_{mk'stj}+\sum_{m=1}^{M}\sum_{t=1}^{l_N-1}h_m I_{mt}$$

S.t:

$$\sum_{t=e_i}^{l_i} t x_{it}+d_j\le\sum_{t=e_j}^{l_j} t x_{jt} \quad ;\forall i\in \mathrm{Pr}_j \quad (2)$$

$$\sum_{t=e_j}^{l_j} x_{jt}=1 \quad ;\forall j\in 1,2,...,N, \quad x_{10}=1 \quad (3)$$

$$I_{mt}=I_{m(t-1)}+\sum_{k=1}^{K_{ms}}\sum_{s=1}^{S}\sum_{j=1}^{N}R_{jm}\rho_{mks(t-L_{ms})j} \quad (4)$$
$$+\sum_{k=1}^{K'_{ms}}\sum_{s=1}^{S'}\sum_{j=1}^{N}R_{jm}\rho_{mks'(t-L_{ms'})j}-\sum_{j=1}^{N}\sum_{t'=Max(t,e_j)}^{Min(t+d_j-1,l_j)}\frac{R_{jm}}{d_j}x_{jt'}$$
$$;\forall m=1,2,...,M,\forall t=1,2,...,l_N$$

$$\alpha_{m(k-1)s}\lambda_{mkst}\le\sum_{j=1}^{N}R_{jm}\rho_{mkstj}\le\alpha_{mks}\lambda_{mkst} \quad (5)$$
$$;\forall m=1,2,...,M,\forall s=1,2,...,S,$$
$$\forall t=1,2,...,l_N,\forall$$
$$k\in 1,2,...,K_{ms}$$

$$\alpha_{m(k-1)s'}\lambda_{mks't}\le\sum_{j=1}^{N}R_{jm}\rho_{mks'tj}\le\alpha_{mks'}\lambda_{mks't} \quad (6)$$
$$;\forall m=1,2,...,M,\forall s=1,2,...,S',$$
$$\forall t=1,2,...,l_N,\forall$$
$$k\in 1,2,...,K'_{ms}$$

$$\sum_{k=1}^{K_{ms}}\sum_{s=1}^{S}\lambda_{mkst}\le 1 \quad ;\forall m=1,2,...,M, \quad (7)$$
$$\forall t\in 1,2,...,l_N$$

$$\sum_{k=1}^{K'_{ms}}\sum_{s=1}^{S'}\lambda_{mks't}\le 1 \quad ;\forall m=1,2,...,M, \quad (8)$$
$$\forall t\in 1,2,...,l_N$$

$$\sum_{k=1}^{K_{ms}}\sum_{s=1}^{S}\sum_{t=1}^{l_N-1}\rho_{mkstj}+\sum_{k=1}^{K'_{ms}}\sum_{s=1}^{S'}\sum_{t=1}^{l_N-1}\rho_{mks'tj}=1 \quad (9)$$
$$;\forall j=1,2,...,N, \quad \forall m\in 1,2,...,M$$

$$\sum_{k=1}^{K_{ms}}\sum_{s=1}^{S}\sum_{t=1}^{l_N-1}t\rho_{mkstj}+L_{ms}\le\sum_{t=e_j}^{l_j}t x_{jt}-d_j+1 \quad (10)$$
$$;\forall j=1,2,...,N,\forall m\in 1,2,...,M$$

$$\sum_{k=1}^{K'_{ms}}\sum_{s=1}^{S'}\sum_{t=1}^{l_N-1}t\rho_{mks'tj}+L'_{ms}\le\sum_{t=e_j}^{l_j}t x_{jt}-d_j+1 \quad (11)$$
$$;\forall j=1,2,...,N,\forall m\in 1,2,...,M$$

$$x_{jt},\lambda_{mkst},\rho_{mkstj}\in[0,1], \quad I_{mt}\ge 0 \quad (12)$$

Equation (1) shows the OF which aims to minimize overall costs of the project execution, including activities execution, ordering materials, purchasing materials, and holding costs, respectively. Afterwards, the model constraints are written by (2)-(12). In this regard, (2) addresses the precedence constraints where the successor activity cannot be completed before its predecessors. It means that the completion time of an activity must be equal to or larger than all its predecessors and the associated duration time. Equation (3) guarantees that each activity is completed within its relevant earliest and latest finish bounds for sure. Equation (4), calculates the inventory level of materials according to the delivered and consumed quantities, respectively. Equations (5) and (6) point to the embedment of purchased materials' quantities within the lower and upper price ranges for all-unit and incremental discounts, respectively. Equations (7) and (8) reiterate that only one price interval and supplier is selected for each order at a given time period. Likewise, (9) reveals that a given requirement is satisfied by one of the discount options. The orders lead time is taken into consideration by (10) and (11) for suppliers offering all-unit or incremental discounts. Finally, the domain of decision variables is shown by (12).

III. Solution Methodology

This section deals with the solution methodology for the proposed model, in which an enhanced version of GA is developed. First, a brief description of GA is presented and the memetic algorithm is addressed then.

A. Genetic Algorithm

GA has proved quite efficient in dealing with discrete optimization problems, in particular, as it can even search in complicated spaces [7]. A preprocessing is instrumental to develop appropriate genotypes, i.e., the schedule representation, and put aside infeasible schedules in the GA initialization. The proposed chromosome representation is as follows.

The proposed chromosome, shown by ∇, accommodates a $(2M+1)\times N$ matrix, as depicted in Fig. 1. F_N, S_{MN}, and OT_{MN} represent for activities' finish times, selected suppliers, and materials' ordering times, respectively.

$$\nabla = \begin{bmatrix} F_1^{\nabla} & F_2^{\nabla} & \cdots & F_N^{\nabla} \\ S_{11}^{\nabla} & S_{12}^{\nabla} & \cdots & S_{1N}^{\nabla} \\ \vdots & \vdots & & \vdots \\ S_{M1}^{\nabla} & S_{M2}^{\nabla} & \cdots & S_{MN}^{\nabla} \\ OT_{11}^{\nabla} & OT_{12}^{\nabla} & \cdots & OT_{1N}^{\nabla} \\ \vdots & \vdots & & \vdots \\ OT_{M1}^{\nabla} & OT_{M2}^{\nabla} & \cdots & OT_{MN}^{\nabla} \end{bmatrix}$$

Fig. 1 The proposed chromosome representation

To abide by the individuals feasibility, activities' completion times are randomly assigned between their corresponding earliest and latest finish times. Likewise, the ordering times are considered to be between $[0, f_j\text{-}d_j+1\text{-}LT_{ms}]$.

B. Enhanced GA

In the enhanced version of the GA, it has been tried to improve the GA performance with respect to the local search strength, since it can exploit the solution space intensively. The proposed approach consists of a variable neighborhood search (VNS) which is based on the GA as the main framework. VNS was first introduced by Mladenovic and Hansen [8] as a method to systematically exploit the concept of neighborhood change in escaping from a local optimal solution.

The integrated method can improve the solution process by maintaining the diversity, within the convergence. It functions according to development of more qualified solution neighbors within the individuals' evolution [9]. In the course of the VNS application, it is also required to maintain the feasibility.

One of the intrinsic advantages of VNS is the account for neighborhood structures, which has been utilized here with respect to two distinctive cases, as follows.

- Neighborhood structure N_1. This structure aims to improve the OF with regard to the holding costs. Therefore, those neighbors are preferred that result in the ordering times closer to the start time of associated activity.
- Neighborhood structure N_2. This structure aims to improve the OF with regard to the procurement costs. Hence, those moves are preferred that result in orders with the same supplier at the same time.

However, only those moves are welcomed that lead to the OF value improvement, i.e., $OF(\nabla_{New}) \le OF(\nabla_{Old})$.

IV. Computational Experiments

This section pertains to test of the model performance and applicability in practice. To solve the model, GAMS 22.1 solver is used and the results are compared with those obtained from the heuristic method. Likewise, the heuristic, whose underlying parameters have been tuned by the Taguchi method, is solved by C++ Programming Language on a Core i3 Pentium 4 PC with 2.0 GHz CPU speed and 4 GB of RAM.

Since the problem is NP-Hard, it takes an exponential increase in the solution time and thus the ordinary branch and bound method cannot function efficiently. On the other hand, a two-hour time has been considered for the solution process and it has been intentionally interrupted for the cases that elapsed time has reached the limit.

We have randomly generated the parameters of the planning model in a rational basis, shown in Table I. The results are compared according to both the OF-based and the solution elapsed time measures, respectively. The status of the OF is written according to the central and dispersion measures, in which the best and mean values are calculated for a ten-time repetition of the instances. The results are accumulated in Tables II, III, as follows.

As can be seen, the heuristics have shown efficient performance in Table II, compared with the ordinary branch and bound method. The results prove that local search-based GA has led to improved schedules, with regard to the mean and standard deviation measures. On the other hand, the solution elapsed time increases for the enhanced-version of the GA, which originates from the further processing of the individuals.

On the contrary to Table II, which belongs to the small instances, Table III accommodates the results comparison for large projects. It shows that GAMS could not perform efficiently within the set elapsed time. In fact, we could only obtain a solution interval by the GAMS. However, the heuristics showed appropriate substitutions, such that they could end in the near-optimal solutions in a rather short period of time. The enhanced-version proved quite efficient for this case in which the improvement is more noticeable than the small-sized instances.

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TABLE I
DATA GENERATION METHOD

Parameters	Random distribution function
C_j	~ U [60, 100]
δ_{mks}	~ U [3, 8]
G_{ms}	~ U [5, 10]
h_m	~ U [1, 5]
α_{mks}	~ U [5, 15]
R_{jm}	~ U [1, 4]
d_j	~ U [1, 10]
L_{ms}	~ U [1, 15]
K_{ms}	~ U [1, 3]

V. CONCLUSIONS

The simultaneous planning of the project scheduling problem and material procurement was considered in this paper. The proposed mathematical model could deal with the presence of multiple suppliers offering either all-unit or incremental discounts. To solve the model, a GA was developed based on the VNS as the neighborhood search technique. The key factors of the solution method was calibrated by the Taguchi method to provide robust solutions. The instances were separated with respect to the size, i.e., number of the activities. This was taken into consideration for problems with different sizes, in which the results proved the efficiency of the proposed solution method.

TABLE II
OBTAINED RESULTS FOR SMALL PROJECTS

Instance No.	GAMS		GA				VNS-based GA			
			Elapsed time (Sec.)	OF value		StD of the OF value	Elapsed time (Sec.)	OF value		StD of the OF value
	Elapsed time (Sec.)	OF value		Mean of the OF value	Best of the OF value			Mean of the OF value	Best of the OF value	
1	28	5904.1	51	5856.3	5904.1	41.6	63	5894.1	5904.0	40.6
2	39	3911.3	74	3837.2	3911.3	40.2	86	3903.0	3911.2	29.3
3	46	5766.0	98	5663.1	5766.4	83.7	112	5752.8	5765.9	88.4
4	75	4017.4	150	3953.8	4013.5	32.9	150	4002.7	4011.3	30.2
5	361	4934.3	227	4835.4	4926.6	60.5	282	4905.9	4927.3	68.7
6	449	3973.2	293	3912.5	3962.3	44.2	293	3933.5	3968.7	33.6
7	3066	4021.3	324	3938.0	4022.4	68.0	389	3971.6	4023.8	61.4
8	7514	5019.4	381	4858.2	5011.5	41.1	474	4955.9	5017.0	67.9

TABLE III
OBTAINED RESULTS FOR LARGE PROJECTS

Instance No	GAMS		GA				MA			
				OF value				OF value		
	Elapsed time (Sec.)	OF value	Elapsed time (Sec.)	Mean of the OF value	Best of the OF value	StD of the OF value	Elapsed time (Sec.)	Mean of the OF value	Best of the OF value	StD of the OF value
1	7200	[19000-21500]	5589	20186.4	20767.4	559.3	5904	20448.6	20844.6	358.2
2	7200	[17000-18200]	5925	17104.3	17725.5	596.2	6573	17948.4	18371.3	386.3
3	7200	[20100-21700]	5863	19865.2	20628.6	752.6	6742	20391.3	20913.6	480.4
4	7200	[17500-18400]	6314	17678.3	18453.7	748.6	7223	18161.2	18664.1	461.5
5	7200	[14000-14800]	6782	13576.4	14216.3	601.89	7934	14302.6	14775.2	424.2
6	7200	[19000-20700]	6750	19844.0	20868.4	984.0	8507	20593.2	21208.3	582.0
7	7200	[17500-18900]	7174	17521.1	18579.5	1019.3	8828	18217.6	18917.4	616.3
8	7200	[14300-15200	7636	13054.7	14083.7	1006.2	8669	14024.9	14593.0	538.5

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