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An Improved Construction Method for MIHCs on Cycle Composition Networks

Hsun Su, Yuan-Kang Shih, and Shin-Shin Kao

Abstract—Many well-known interconnection networks, such as k-ary n-cubes, recursive circulant graphs, generalized recursive circulant graphs, circulant graphs and so on, are shown to belong to the family of cycle composition networks. Recently, various studies about mutually independent hamiltonian cycles, abbreviated as MIHC's, on interconnection networks are published. In this paper, using an improved construction method, we obtain MIHC's on cycle composition networks with a much weaker condition than the known result. In fact, we established the existence of MIHC's in the cycle composition networks and the result is optimal in the sense that the number of MIHC's we constructed is maximal.

Keywords—Hamiltonian cycle, k-ary n-cube, cycle composition networks, mutually independent.

I. Introduction and Preliminaries

The architecture of an interconnection network is usually represented by a graph, in which vertices and edges correspond to processors and communication links, respectively. Thus, we use the terms graph and network interchangeably.

For the graph definitions and notations, we follow [1]. A graph G consists of a nonempty set $V(G)$ and a subset $E(G)$ of $\{(u, v) \mid (u, v)$ is an unordered pair of $V(G)\}$. The set $V(G)$ is called the *vertex set* of G and $E(G)$ is called the *edge set*. Two vertices u and v are *adjacent* if $(u, v) \in E(G)$. For a vertex u of G, we denote the *degree* of u by $deg(u) = |\{v \mid (u, v) \in E(G)\}|$. A graph G is *r-regular* if for every vertex $u \in G$, $deg(u) = r$.

A *matching* of size n in a graph G is a set of n edges with no shared endpoints. The vertices belonging to the edges of a matching are *saturated* by the matching; the others are unsaturated. A *perfect matching* is a matching that saturates every vertex of G.

A *path* is represented by a finite sequence of vertices $\langle v_0, v_1, v_2, \ldots, v_n\rangle$, where every two consecutive vertices are adjacent. The *length* of a path P is the number of edges in P. We write the path $\langle v_0, v_1, v_2, \ldots, v_n\rangle$ as $\langle v_0, v_1, \ldots, v_s, P_1, v_i, \ldots, v_j, P_2, v_t, \ldots, v_n\rangle$, where $P_1 = \langle v_s, v_{s+1}, \ldots, v_i\rangle$ and $P_2 = \langle v_j, v_{j+1}, \ldots, v_t\rangle$. A *hamiltonian path* between u and v, where u and v are two distinct vertices of G, is a path joining u to v that visits every vertex of G exactly once. Two paths $P_1 = \langle u_0, u_1, \ldots, u_m\rangle$ and $P_2 = \langle v_0, v_1, \ldots, v_m\rangle$ from a to b are *independent* if $u_0 = v_0 = a$, $u_m = v_m = b$, and $u_i \neq v_i$ for $1 \leq i \leq m-1$ [9]. In [6], two paths $P'_1 = \langle u_0, u_1, \ldots, u_m\rangle$ and $P'_2 = \langle v_0, v_1, \ldots, v_m\rangle$ are *full-independent* if $u_i \neq v_i$ for all $0 \leq i \leq m$. Paths with the same number of vertices are *mutually independent*(resp. *mutually full-independent*) if every two different paths are independent(resp. full-independent). A graph G is *hamiltonian connected* if there is a hamiltonian path joining any two distinct vertices of G. A graph G is called *1-vertex-fault-tolerant hamiltonian connected* if it remains hamiltonian connected after removing any vertex in G.

A *cycle* is a path of at least three vertices such that the first vertex is the same as the last vertex. A *hamiltonian cycle* of G is a cycle that traverses every vertex of G exactly once. A *hamiltonian graph* is a graph with a hamiltonian cycle. The *length* of a cycle C is the number of edges/vertices in C. Two cycles $C_1 = \langle u_0, u_1, \ldots, u_k, u_0\rangle$ and $C_2 = \langle v_0, v_1, \ldots, v_k, v_0\rangle$ beginning at s are *independent* if $u_0 = v_0 = s$ and $u_i \neq v_i$ for $1 \leq i \leq k$ [11]. Cycles beginning at s with the same length are *mutually independent* if every two different cycles are independent. A graph G is said to *contain n MIHCs* if there exist n hamiltonian cycles in G beginning at any vertex s such that the n cycles are mutually independent. There are numerous studies in MIHCs. Readers can refer to [7]–[15].

In 2008, Kueng et al. introduced the cycle composition networks [4], abbreviated as CCN's. Let $k \geq 4$, $n \geq 6$, $r \geq 2$ be integers, and G_i be a r-regular graph with n vertices for $0 \leq i \leq k-1$. From now on, all additions and subtractions are considered modulo k. Let $M_{i,j}$ be an arbitrary perfect matching between the vertices of G_i and those of G_j and $\mathcal{M} = \bigcup_{i=0}^{k-1} M_{i,i+1}$. The *cycle composition network* $\tilde{G} = \text{CCN}(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$ is defined to be the graph with the vertex set $V(\tilde{G}) = \bigcup_{i=0}^{k-1} V(G_i)$ and the edge set $E(\tilde{G}) = \bigcup_{i=0}^{k-1}(E(G_i)) \cup \mathcal{M}$. We abbreviate $\text{CCN}(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$ as CCN_k. See Figure 1 for an illustration. Many well-known interconnection networks, such as k-ary n-cubes, recursive circulant graphs, generalized recursive circulant graphs, circulant graphs and so on, are shown to belong to the family of CCN's. Hence, CCN's have attracted many studies and research interests [2]–[4].

Suppose that each G_i contains r MIHCs. More precisely, for any vertex s_i of G_i, where $0 \leq i \leq k-1$, there exist r hamiltonian cycles in G_i beginning at the vertex s_i such that the r cycles are mutually independent in G_i. Is it true that $\tilde{G} = \text{CCN}(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$ contains $(r+2)$ MIHCs? In [5], M.-F. Hsieh et. al. derived the following result.

Theorem 1: For $k \geq 6$, let $\{G_i\}_{i=0}^{k-1}$ be k r-regular hamil-

H. Su and S.-S. Kao are with the Department of Applied Mathematics, Chung-Yuan Christian University, Chungli, Taiwan 32023, R.O.C.
H. Su is with Department of Public Finance and Taxation, Takming University of Science and Technology, Taipei City, Taiwan 11451, R.O.C..
Y.-K. Shih is with the Department of Computer Science, National Chiao Tung University, Hsinchu, Taiwan 30010, R.O.C.

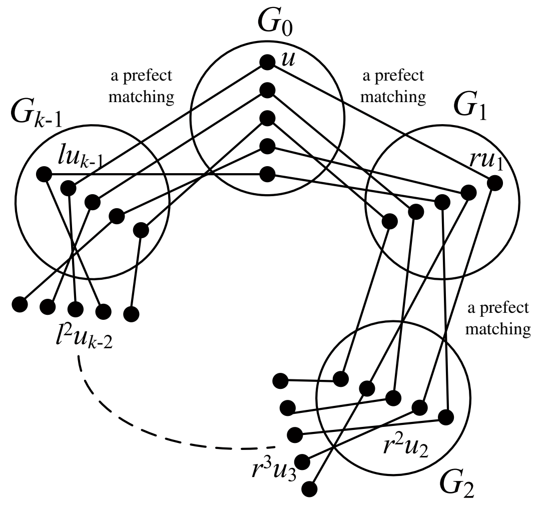

Fig. 1. An illustration for $\text{CCN}_k = \text{CCN}(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$.

tonian graphs with n vertices. Suppose that each G_i contains r MIHCs and r mutually full-independent hamiltonian paths between any r pairs of distinct vertices of G_i, and is 1-vertex-fault-tolerant hamiltonian connected. Then there exist $r+2$ MIHCs in CCN_k.

Obviously, each vertex of $\tilde{G}$ has exactly $r+2$ neighbors. However, the requirement that each G_i contains r mutually full-independent hamiltonian paths between any r pairs of vertices of G_i seems to be unnecessarily strict. Besides, to check whether this requirement is satisfied on each G_i is a difficult task. Consequently, Theorem 1 is of little practical use. In this paper, using a different construction scheme, we are able to achieve the same result with a much weaker condition on G_i. (See Theorem 4).

The following notations are defined for the rest of the paper. Let u_i be a vertex of G_i for some i. We use $l^j u_{i-j}$ to denote the vertex of G_{i-j} such that there exists a path in $\mathcal{M}$ of the form $\langle u_i, lu_{i-1}, l^2u_{i-2}, \ldots, l^j u_{i-j}\rangle$. Similarly, we use $r^j u_{i+j}$ to denote the vertex of G_{i+j} such that there exists a path in $\mathcal{M}$ of the form $\langle u_i, ru_{i+1}, r^2u_{i+2}, \ldots, r^j u_{i+j}\rangle$. W.L.O.G., let $u \in G_0$ and $u = u_0$. See Figure 1 for an illustration. It is possible that there exists a cycle beginning at u with the length k of the form $\langle u = u_0, ru_1, r^2u_2, \ldots, r^{k-2}u_{k-2}, r^{k-1}u_{k-1}, u_0\rangle$. More specifically, $r^i u_i = l^{k-i}u_{i-k}$ for any $1 \le i \le k-1$.

II. Main Results

Let $k \ge 4$ and $n \ge 6$. Throughout this section, we use the symbol CCN_k for $CCN(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$, which is a cycle composition network composed of k graphs $\{G_i \mid G_i$ is a r-regular graph with $|G_i| = n$ for $0 \le i \le k-1\}$ and k perfect matchings $\mathcal{M} = \bigcup_{i=0}^{k-1} M_{i,i+1}$, for simplicity.

Lemma 1: Consider any CCN_k. Suppose that G_0 contains r MIHCs beginning at any given vertex, denoted by $\{\overline{C_0^i} \mid 0 \le i \le r-1\}$, and there exists some edge (a_0, b_0) such that $(a_0, b_0) \in \overline{C_0^i}$ for all $0 \le i \le r-1$. Let $a_1 \in V(G_1)$ and $b_{r-1} \in V(G_{r-1})$ be arbitrary. If there is a path P between a_1 and b_{r-1} such that P visits each vertex of $\bigcup_{i=1}^{k-1} G_i$ in CCN_k exactly once, then CCN_k contins r MIHCs starting with any vertex in G_0 and passing through a common edge.

Proof: W.L.O.G., let $s_0 \in V(G_0)$ be the beginning vertex. Obviously, for $1 \le i \le r-1$, $\overline{C}_0^i$ is of the form $\langle s_0, A^i, a_0, b_0, B^i, s_0\rangle$, where A^i and B^i are two disjoint paths in G_0 such that A^i is between s_0 and a_0, B^i is between b_0 and s_0, and $A_i \bigcup B_i = V(G_0)$. Since $\{\overline{C_0^i} \mid 0 \le i \le r-1\}$ are MIHCs in G_0, it must be $|A_i| \ne |A_j|$ and $|B_i| \ne |B_j|$ for $i \ne j$. Otherwise, a_0 or b_0 might appear at the same timestep on different $\overline{C}_0^i$'s.

Note that $ra_1 \in V(G_1)$ and $lb_{r-1} \in V(G_{r-1})$. It is known that there is a path P between ra_1 and lb_{r-1} such that P visits every vertex of $\bigcup_{i=1}^{k-1} G_i$ in CCN_k exactly once. Let $C_i = \langle s_0, A^i, a_0, ra_1, P, lb_{r-1}, b_0, B^i, s_0\rangle$. It is easy to see that $\{C_i \mid 1 \le i \le r-1\}$ forms a set of r MIHCs of CCN_k, and each C_i contains the edge (a_0, ra_1), which is the common edge. ∎

Theorem 2: Consider CCN_4. For $0 \le i \le 3$, suppose that G_i satisfies the following two requirements – (1) G_i is 1-vertex-fault-tolerant hamiltonian connected. (2) Starting from any vertex of G_i, there exist r MIHCs passing through a common edge of G_i. Then CCN_4 contains $r+2$ MIHCs passing through a common edge.

Proof: W.L.O.G., let s_0 be an arbitrary vertex of G_0. We want to construct $r+2$ MIHCs starting at s_0 in CCN_4. It is known that G_0 contains r MIHCs beginning at s_0 and passing through a common edge of G_0. Let u_1 and v_2 be any two vertices in G_1 and G_2, respectively. Since G_i is hamiltonian connected, there exist three hamiltonian paths P_1, P_2 and P_3, such that P_1 connects ra_1 and u_1 in G_1, P_2 connects ru_2 and v_2 in G_2, and P_3 connects rv_3 and lb_3 in G_3. Then $P = \langle ra_1, P_1, u_1, ru_2, P_2, v_2, rv_3, P_3, lb_3\rangle$ is a path between ra_1 and lb_3 that visits each vertex of $\{G_i \mid 1 \le i \le 3\}$ exactly once. By Lemma 1, CCN_4 contains r MIHCs, denoted by $\{C_i \mid 0 \le i \le r-1\}$, and each C_i contains the common edge (a_0, ra_1).

Now, we construct the $(r+1)$-th MIHC of CCN_4 beginning at s_0. In G_3, choose a vertex x_3 which is adjacent to ls_3 and $x_3 \ne lb_3$. Since G_3 is 1-vertex-fault-tolerant hamiltonian connected, there is a hamiltonian path T_3 of $G_3 - \{ls_3\}$ between la_3 and x_3. We can write T_3 as $\langle la_3, Q_3, y_3, x_3\rangle$. Since G_i is 1-vertex-fault-tolerant hamiltonian connected, $G_0 - \{s_0\}$ contains a hamiltonian path Q_0 that connects ry_0 and a_0, $G_1 - \{rs_1\}$ contains a hamiltonian path Q_1 that connects ra_1 and l^2x_1, and $G_2 - \{lx_2\}$ contains a hamiltonian path Q_2 that connects r^2s_2 and l^2a_2. Let $C_r = \langle s_0, rs_1, r^2s_2, Q_2, l^2a_2, la_3, Q_3, y_3, ry_0, Q_0, \underline{a_0, ra_1}, Q_1, l^2x_1, lx_2, x_3, ls_3,\ s_0\rangle$. It is easy to see that C_r is mutually independent of the r MIHCs $\{C_i \mid 0 \le i \le r-1\}$ constructed by Lemma 1, and C_r passes through the edge (a_0, ra_1), which is underlined in C_r.

Finally, we construct the $(r+2)$-th MIHC of CCN_4 beginning at s_0. In G_2, choose a vertex w_2 such that w_2 is adjacent to r^2s_2 and $w_2 \ne lx_2$. Choose another vertex z_2 in G_2 such that $z_2 \ne r^2a_2$ and $z_2 \ne r^2s_2$. Since G_3 is hamiltonian connected, there exists a hamiltonian path R_3 of G_3 that connects ls_3 and rz_3. Since G_i is 1-vertex-fault-tolerant hamiltonian connected, $G_0 - \{s_0\}$ contains a hamiltonian path R_0 between r^2z_0 and a_0, $G_1 - \{rs_1\}$ contains a hamiltonian path R_1 between ra_1 and lz_1, and $G_2 - \{r^2s_2\}$ contains a hamiltonian path R_2 between z_2 and w_2. Let $C_{r+1} = \langle s_0, ls_3, R_3, rz_3, r^2z_0, R_0, \underline{a_0, ra_1}, R_1, lz_1, z_2, R_2, w_2, r^2s_2, rs_1, s_0\rangle$. Consequently, C_{r+1} is mutually independent of $\{C_i \mid 0 \le i \le r\}$ constructed above and passes through the common edge (a_0, ra_1), which is underlined. See Figure 2 for an illustration. ■

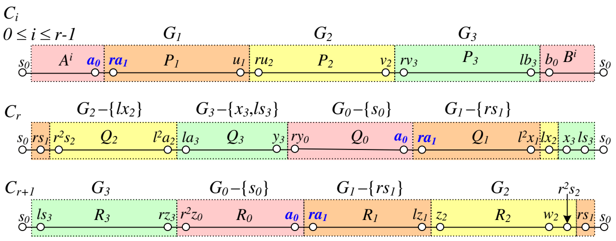

Fig. 2. An illustration of Theorem 2 .

Theorem 3: Suppose that CCN_5 is constructed by five r-regular graphs G_i with n vertices for $0 \le i \le 4$, and each G_i is 1-vertex-fault-tolerant hamiltonian connected and contains r MIHCs passing through a fixed edge from any vertex of G_i. Then CCN_5 contains $r+2$ MIHCs passing through a fixed edge.

Proof: W.L.O.G., we let s_0 be the beginning vertex and $\overline{C}_0^0, \overline{C}_0^1, \cdots, \overline{C}_0^{r-1}$ be the r MIHCs beginning at s_0 and passing through the fixed edge (a_0, b_0) in G_0. Hence we can write $\overline{C}_0^i$ as $\langle s_0, A^i, a_0, b_0, B^i, s_0\rangle$ for $0 \le i \le r-1$. We will construct the $r+2$ MIHCs beginning at s_0 and passing through a fixed edge in CCN_5.

Consider the first r MIHCs of CCN_5 beginning at s_0. Let u_1, v_2 and u_3 be any three vertices in G_1, G_2 and G_3, respectively. There exist four hamiltonian paths P_1, P_2, P_3 and P_4 joining from ra_1 to u_1, ru_2 to v_2, rv_3 to u_3 and ru_4 to lb_4 in G_1, G_2, G_3 and G_4, respectively. Set $C_i = \langle s_0, A^i, \underline{a_0, ra_1}, P_1, u_1, ru_2, P_2, v_2, rv_3, P_3, u_3, ru_4, P_4, lb_4, b_0, B^i, s_0\rangle$ for $0 \le i \le r-1$. Then, the r MIHCs are C_0, C_1, $\cdots$, C_{r-1}, which pass through the fixed edge (a_0, ra_1).

Now, we consider the $(r+1)$-th MIHC of CCN_5 beginning at s_0. In G_4, choose a vertex x_4 which is adjacent to ls_4 and $x_4 \ne lb_4$. Since G_4 is 1-vertex-fault-tolerant hamiltonian connected, there is a hamiltonian path T_4 of $G_4 - \{ls_4\}$ between la_4 and x_4. W.L.O.G., T_4 can be written as $\langle la_4, Q_4, y_4, x_4\rangle$. In G_3, choose a vertex $lx_3 \ne r^3s_3$. If $lx_3 = r^3s_3$, we have to choose another vertex x_4, which is adjacent to ls_4, for $lx_3 \ne r^3s_3$. Since G_3 is 1-vertex-fault-tolerant hamiltonian connected, there is a hamiltonian path Q_3 of $G_3 - \{lx_3\}$ between rd_3 and l^2a_3, it can be written as $\langle rd_3, Q_3, l^2a_3\rangle$. Let d_2 be any vertex in G_2 not adjacent to l^2a_3. Using the 1-vertex-fault-tolerant hamiltonian connected property of G_0, G_1 and G_2, there exist three hamiltonian paths Q_0, Q_1 and Q_2 of $G_0 - \{s_0\}$, $G_1 - \{rs_1\}$ and $G_2 - \{l^2x_2\}$ from ry_0 to a_0, ra_1 to l^3x_1 and r^2s_2 to d_2, respectively. Let $C_r = \langle s_0, rs_1, r^2s_2, Q_2, d_2, rd_3, Q_3, l^2a_3, la_4, Q_4, y_4, ry_0, Q_0, \underline{a_0, ra_1}, Q_1, l^3x_1, l^2x_2, lx_3, x_4, ls_4, s_0\rangle$. Therefore, C_r is mutually independent of the first r MIHCs $C_0, C_1, \cdots, C_{r-1}$ and passes the fixed edge (a_0, ra_1).

Finally, we consider the last MIHC of CCN_5 beginning at s_0. Let w_1, z_2 and w_4 be any three vertices in G_1, G_2 and G_4, where w_4 is not adjacent to a_0 and z_2 is not adjacent to r^3s_3. There exist two hamiltonian paths R_3 and R_4 joining from rz_3 to r^3s_3 and ls_4 to w_4 in G_3 and G_4. And using the 1-vertex-fault-tolerant hamiltonian connected property of G_0, G_1 and G_2, there exist three hamiltonian paths R_0, R_1 and R_2 of $G_0 - \{s_0\}$, $G_1 - \{rs_1\}$ and $G_2 - \{r^2s_2\}$ from rw_0 to a_0, ra_1 to w_1 and rw_2 to z_2, respectively. We let $C_{r+1} = \langle s_0, ls_4, R_4, w_4, rw_0, R_0, \underline{a_0, ra_1}, R_1, w_1, rw_2, R_2, z_2, rz_3, R_3, r^3s_3, r^2s_2, rs_1, s_0\rangle$. So, C_{r+1} is mutually independent of the first $r+1$ MIHCs C_0, C_1, $\cdots$, C_r and passes the fixed edge (a_0, ra_1). See Figure 3 for an illustration. ■

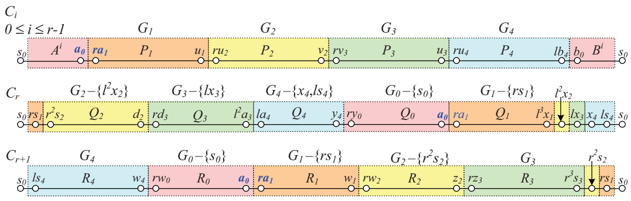

Fig. 3. An illustration of Theorem 3 .

Theorem 4: Suppose that CCN_k is constructed by k r-regular graphs G_i with n vertices for $0 \le i \le k-1$. If each G_i is 1-vertex-fault-tolerant hamiltonian connected and contains r MIHCs passing through a fixed edge from any vertex of G_i, then CCN_k contains $r+2$ MIHCs passing through a fixed edge.

Proof: W.L.O.G., we let s_0 be the beginning vertex and $\overline{C}_0^0, \overline{C}_0^1, \cdots, \overline{C}_0^{r-1}$ be the r MIHCs beginning at s_0 and passing through the fixed edge (a_0, b_0) in G_0. Hence we can write $\overline{C}_0^i$ as $\langle s_0, A^i, a_0, b_0, B^i, s_0\rangle$ for $0 \le i \le r-1$. We will construct the $r+2$ MIHCs beginning at s_0 passing through a fixed edge in CCN_k.

Consider the first r MIHCs of CCN_k beginning at s_0. We choose distinct vertices $u_i, v_i \in G_i$ for $2 \le i \le k-3$ such that $(v_{i-1}, u_i) \in E(CCN_k)$ for $3 \le i \le k-3$. Let $U_1(u_2, v_{k-3}) = \langle u_2, P_2, v_2, u_3, P_3, v_3, \cdots, u_{k-3}, P_{k-3}, v_{k-3}\rangle$, where P_i is a hamiltonian path of C_i between u_i and v_i for $2 \le i \le k-3$. Let u_1 and u_{k-2} be any two vertices in G_1 and G_{k-2}, respectively. There exist three hamiltonian paths P_1, P_{k-2} and P_{k-1} joining from ra_1 to u_1, rv_{k-2} to u_{k-2} and ru_{k-1} to lb_{k-1} in G_1, G_{k-2} and G_{k-1}, respectively. Set $C_i = \langle s_0, A^i, \underline{a_0, ra_1}, P_1, u_1, ru_2, U_1(ru_2, v_{k-3}), v_{k-3}, rv_{k-2}, P_{k-2}, u_{k-2}, ru_{k-1}, P_{k-1}, lb_{k-1}, b_0, B^i, s_0\rangle$ for $0 \le i \le r-1$. Then, the r MIHCs are C_0, C_1, $\cdots$, C_{r-1} which pass through the fixed edge (a_0, ra_1).

Now, we consider the $(r+1)$-th MIHC of CCN_k beginning at s_0. We choose distinct vertices $c_i, d_i \in G_i$ for $2 \le i \le k-3$ such that $(c_{i-1}, d_i) \in E(CCN_k)$ for

$3 \le i \le k-3$. Since G_i is 1-vertex-fault-tolerant hamiltonian connected for $2 \le i \le k-3$, let $U_x(c_2, d_{k-3}) = \langle c_2, Q_2, d_2, c_3, Q_3, d_3, \cdots, c_{k-3}, Q_{k-3}, d_{k-3}\rangle$, where Q_i is a hamiltonian path of $C_i - \{l^{k-i-1}x_i\}$ between c_i and d_i for $2 \le i \le k-3$. In G_{k-1}, choose a vertex x_{k-1} which is adjacent to ls_{k-1} and $x_{k-1} \ne lb_{k-1}$. Since G_{k-1} is 1-vertex-fault-tolerant hamiltonian connected, there is a hamiltonian path T_{k-1} of $G_{k-1} - \{ls_{k-1}\}$ between la_{k-1} and x_{k-1}. W.L.O.G., T_{k-1} can be written as $\langle la_{k-1}, Q_{k-1}, y_{k-1}, x_{k-1}\rangle$. Let d_{k-2} be any vertex in $G_{k-2} - \{lx_{k-2}, l^2a_{k-2}\}$. Using the 1-vertex-fault-tolerant hamiltonian connected property of G_0, G_1 and G_{k-2}, there exist three hamiltonian paths Q_0, Q_1 and Q_{k-2} of $G_0 - \{s_0\}$, $G_1 - \{rs_1\}$ and $G_2 - \{lx_2\}$ from ry_0 to a_0, ra_1 to $l^{k-2}x_1$ and d_{k-2} to l^2a_{k-2}, respectively. And let $X_Q = \langle l^{k-3}x_2, l^{k-4}x_3, \cdots, lx_{k-2}\rangle$. Let $C_r = \langle s_0, rs_1, r^2s_2, U_x(r^2s_2, ld_{k-2}), ld_{k-2}), d_{k-2}, Q_{k-2}, l^2a_{k-2}, la_{k-1}, Q_{k-1}, y_{k-1}, ry_0, Q_0, \underline{a_0, ra_1}, Q_1, l^{k-2}x_1, l^{k-3}x_2, X_Q, lx_{k-2}, x_{k-1}, ls_{k-1}, s_0\rangle$. Therefore, C_r is mutually independent of the first r MIHCs C_0, C_1, $\cdots$, C_{r-1} and passes the fixed edge (a_0, ra_1).

Finally, we construct the last MIHC of CCN_k beginning at s_0. We choose distinct vertices $w_i, z_i \in G_i$ for $2 \le i \le k-3$ such that $(w_{i-1}, z_i) \in E(CCN_k)$ for $3 \le i \le k-3$. Since G_i is 1-vertex-fault-tolerant hamiltonian connected for $2 \le i \le k-3$, let $U_x(w_2, z_{k-3}) = \langle w_2, R_2, z_2, w_3, R_3, z_3, \cdots, w_{k-3}, R_{k-3}, z_{k-3}\rangle$, where R_i is a hamiltonian path of $C_i - \{r^is_i\}$ between w_i and z_i for $2 \le i \le k-3$. Let w_1, w_{k-2} and z_{k-1} be any three vertices in G_1, G_{k-2} and G_{k-1}, where z_{k-1} is not adjacent to a_0. There exist a hamiltonian path R_{k-2} joining from l^2s_{k-2} to w_{k-2} in G_{k-2}. Using the 1-vertex-fault-tolerant hamiltonian connected property of G_0, G_1 and G_{k-1}, there exist three hamiltonian paths R_0, R_1 and R_{k-1} of $G_0 - \{s_0\}$, $G_1 - \{rs_1\}$ and $G_{k-1} - \{ls_{k-1}\}$ from rz_0 to a_0, ra_1 to w_1 and rw_{k-1} to z_{k-1}, respectively. Let $S_R = \langle r^{k-3}s_{k-3}, r^{k-2}s_{k-2}, \cdots, rs_1\rangle$, and z_{k-3} be adjacent to $r^{k-3}s_{k-3}$. We let $C_{r+1} = \langle s_0, ls_{k-1}, l^2s_{k-2}, R_{k-2}, w_{k-2}, rw_{k-1}, R_{k-1}, z_{k-1}, rz_0, R_0, \underline{a_0, ra_1}, R_1, w_1, rw_2, U_s(rw_2, z_{k-3}), z_{k-3}, r^{k-3}s_{k-3}, S_R, rs_1, s_0\rangle$. To avoid the collision of $U_1(ru_2, v_{k-3})$ and $U_s(rw_2, z_{k-3})$, which means $|S_R| + |R_{k-3}| \le |B^i| + |P_{k-1}| + |P_{k-1}|$, we have $(k-3) + (n-1) \le 1 + n + n \Rightarrow k \le n+5$. So, C_{r+1} is mutually independent of the first $r+1$ MIHCs C_0, C_1, $\cdots$, C_r and passes the fixed edge (a_0, ra_1). See Figure 4 for an illustration. ■

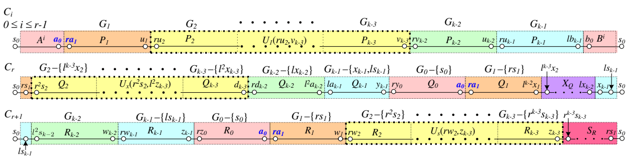

Fig. 4. An illustration of Theorem 4 .

III. Conclusion

Let $k \ge 4$, $n \ge 6$, $r \ge 2$ be integers and G_i be a r-regular graph with n vertices for $0 \le i \le k-1$. In this paper, we prove that under a much weaker condition than [5], given any vertex u of the cycle composition network $\mathrm{CCN}_k = \mathrm{CCN}(G_0, G_1, \ldots, G_{k-1}; \mathcal{M})$, there exist $(r+2)$ hamiltonian cycles in CCN_k beginning at u such that the $(r+2)$ cycles are mutually independent. The result is optimal since each vertex of the cycle composition network has exactly $(r+2)$ neighbors. It is known that many well-known interconnection networks, such as k-ary n-cubes, recursive circulant graphs $G(cd^m, d)$, generalized recursive circulant graphs $G(h_k, h_{k-1}, \ldots, h_1)$, circulant graphs $C(n : c_1, c_2, \ldots, c_k)$ and so on, belong to the family of the cycle composition networks. To our knowledge, the above results of $G(cd^m, d)$, $G(h_k, h_{k-1}, \ldots, h_1)$ and $C(n : c_1, c_2, \ldots, c_k)$ have not been published yet. Our study has established the existence of MIHCs in these three families as long as the conditions of Theorem 4 are verified.

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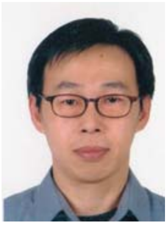

Hsun Su received his Bachelor and Master Degree from Feng Chia University, Taiwan in 1987 and 1989, respectively. He has been working in the Department of Public Finance and Taxation, Takming University of Science and Technology since 1989 until now. His research interests include combinatorial optimization and graph theory in interconnection networks.

Yuan-Kang Shih received the B.S. degree in the Department of Mathematics from Fu Jen Catholic University, Hsinchuang, Taipei County, Taiwan, R.O.C. in 2004. He received his M.S. degree from the Department of Applied Mathematics from Chung Yuan Christian University, Chungli, Taiwan, R.O.C. in 2006. He is now a student in the Ph.D. program in the College of Computer Science in the National Chiao Tung University, Hsinchu, Taiwan, R.O.C. His research interests include interconnection networks, fault-tolerant problems, and graph theory.

Shin-Shin Kao received her Bachelor from National Tzing Hua University in Taiwan in 1990, and her Ph.D. from University of California, Los Angeles, USA in 1995. She has joined the Department of Applied Mathematics in Chung-Yuan Christian University in Taiwan since 1995 and is a full professor now. Her research interests include combinatorial optimization, interconnection networks, and graph theory.