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An Improved Adaptive Dot-Shape Beamforming Algorithm Research on Frequency Diverse Array

Yanping Liao, Zenan Wu, Ruigang Zhao

Abstract**—**Frequency diverse array (FDA) beamforming is a technology developed in recent years, and its antenna pattern has a unique angle-distance-dependent characteristic. However, the beam is always required to have strong concentration, high resolution and low sidelobe level to form the point-to-point interference in the concentrated set. In order to eliminate the angle-distance coupling of the traditional FDA and to make the beam energy more concentrated, this paper adopts a multi-carrier FDA structure based on proposed power exponential frequency offset to improve the array structure and frequency offset of the traditional FDA. The simulation results show that the beam pattern of the array can form a dot-shape beam with more concentrated energy, and its resolution and sidelobe level performance are improved. However, the covariance matrix of the signal in the traditional adaptive beamforming algorithm is estimated by the finite-time snapshot data. When the number of snapshots is limited, the algorithm has an underestimation problem, which leads to the estimation error of the covariance matrix to cause beam distortion, so that the output pattern cannot form a dot-shape beam. And it also has main lobe deviation and high sidelobe level problems in the case of limited snapshot. Aiming at these problems, an adaptive beamforming technique based on exponential correction for multi-carrier FDA is proposed to improve beamforming robustness. The steps are as follows: first, the beamforming of the multi-carrier FDA is formed under linear constrained minimum variance (LCMV) criteria. Then the eigenvalue decomposition of the covariance matrix is performed to obtain the diagonal matrix composed of the interference subspace, the noise subspace and the corresponding eigenvalues. Finally, the correction index is introduced to exponentially correct the small eigenvalues of the noise subspace, improve the divergence of small eigenvalues in the noise subspace, and improve the performance of beamforming. The theoretical analysis and simulation results show that the proposed algorithm can make the multi-carrier FDA form a dot-shape beam at limited snapshots, reduce the sidelobe level, improve the robustness of beamforming, and have better performance.

Keywords**—**Multi-carrier frequency diverse array, adaptive beamforming, correction index, limited snapshot, robust.

I. Introduction

ELECTRONIC warfare is an indispensable part of modern high-tech military warfare. As an important branch of electronic warfare [1], radar electronic warfare aims to using professional interference equipment to detect, interfering to enemy's radar, and defending against the enemy radar's electronic attack on our radar. Radar interference is an important part of modern radar electronic warfare. The research on radar interference has its important and special status in the field of national defense and military [2]. The effectiveness of modern radar interference will directly affect the success or failure of war.However, when transmitting interference to an enemy target, it must be ensured that the energy of the interference signal is large enough for the interference signal to enter the radar receiver. Beamforming technology [3] can fulfill this need and it is a key technology for transmitting interference. However, traditional beamforming, whether classical beamforming or adaptive beamforming, is based on phased array [4]. Since the frequency of the transmitted signal on all of the array elements are the same, the beam points in one direction at all distances which has the same energy. It cannot change the beam pointing as the distance changes, and also cannot accurately point to a specific position. As a result, when the array transmitting interference, the beam cannot be flexibly controlled, the beam cannot be formed at the desired position, and a null can be formed at other positions. The proposal of the FDA provides the possibility to solve this problem.

In 2006, the FDA [5], [6] was first proposed by Antonik and Wicks as a new concept. This new array imposes a small frequency offset between adjacent antenna elements. The FDA has a unique angle-distance dependence characteristic under its small frequency offset [7]. Moreover, the beam from the FDA has a certain periodicity [8], and the periodically generated factors are related to the frequency offset [9]. After analyzing the frequency offset, the results show that by adjusting the frequency offset, not only the periodicity but also the beam steering of the FDA can be controlled [10]. However, the angle-distance dependent characteristic of the FDA results that its beam cannot be controlled accurately. By varying the structure of the array, the FDA can achieve angle-distance decoupling, which can form a relatively concentrated single-point beam in space [11]-[14]. Yet, in order to achieve accurate interference to the enemy's specific targets, it is still necessary to improve the concentration and resolution of the beam energy so that a dot-shape beam can be formed in the space.

In order to solve this problem and improve the robustness of the beam, the array weighting vector is optimized by adaptive beamforming algorithm [15] while being changed the array structure and frequency offset, which aims to obtaining a dot-shape beam that can adaptively point to the enemy's target position with flexibility and robustness. However, when the number of snapshots is limited, the beam will appear the deviation of main lobe, its sidelobe level will rise significantly with other robustness problems. Therefore, in recent years, a large number of studies on beamforming robustness in the case the number of snapshots is limited: The traditional

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beam-management beamforming algorithm is a simple and effective method [16], but it needs to correctly estimate the specific value of the diagonal load. The beamforming algorithm is based on Krylov subspace [17], in the case of limited snapshot, the convergence speed is fast, but the specific truncation order needs to be predetermined. The subspace-based beamforming algorithm [18] has good performance in limited snapshot conditions, but it is necessary to correctly estimate the number of spatial signal sources. The above beamforming algorithm is applied in the case of limited snapshot, although the problem of main lobe deviation can be solved, the algorithm has certain limitations. In this paper, an adaptive beamforming scheme based on exponential correction for multi-carrier FDA is proposed. The scheme can effectively solve the problem of main lobe deviation and significant increase of sidelobe level, thereby obtaining a robust, energy-concentrated dot-shape beam.

The paper is structured as follows: Section I is the introduction part of this article, Section II is the signal model and array structure of the FDA. In Section III, a multi-carrier FDA adaptive beamforming scheme based on exponential correction is proposed. Section IV is the simulation results, and Section V is the conclusion.

II. Array Structure and Signal Model

The research of FDA beamforming is based on its new array structure and related signal model. Therefore, this section mainly introduces its array structure and signal model.

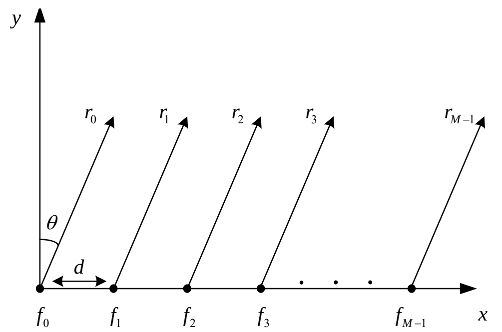

Fig. 1 Uniform linear frequency diverse array (ULA-FDA) structure

In Fig. 1, f_i denotes the transmission frequency of each antenna element, where $i = 0,1,\cdots,M-1$. M is the number of transmitting elements, d is the spacing of adjacent elements, and the leftmost element is used as the reference point. Then, r and θ respectively represent the distance from the far-field observation point to the reference array element and the azimuth angle of the far-field observation point relative to the array normal direction. In order to simplify the signal model, in general, only the radiation characteristics of the plane in which the antenna array is located are discussed, and the characteristics in other orientations of the space are not considered.

The biggest feature of the FDA is its frequency diverse characteristic, that is, a linearly increasing frequency offset between adjacent elements, and $f_m = f_0 + m\cdot\Delta f$, $m = 0,1,\cdots,M-1$ for a ULA-FDA. Assume that the transmitted signal on the mth antenna element is:

$$f_m = f_0 + m\cdot\Delta f,\ m = 0,1,\cdots,M-1 \tag{1}$$

According to the electromagnetic field superposition theory, the signal of any observation point in space is the superposition of all antenna transmission signals, and the signal that reaches the far-field observation point after propagation delay can be expressed as:

$$\begin{aligned} S(t) &= \sum_{m=0}^{M-1} s_m(t-\tau_m) = \sum_{m=0}^{M-1} \frac{1}{r_m}\exp(j2\pi f_m(t-\frac{r_m}{c})) \\ &= \sum_{m=0}^{M-1} \frac{1}{r_m}\exp(j2\pi(f_0 + m\Delta f)(t-\frac{r_m}{c})) \end{aligned} \tag{2}$$

where c is the speed of light, r_m is the distance from the mth antenna element to the far-field observation point, and $1/r_m$ is the distance-distance effect of the amplitude. According to the array synthesis theory, the beam pattern expression corresponding to the FDA can be obtained as:

$$B(\theta,r) = \left|\sum_{m=0}^{M-1}\exp\{j2\pi m(\frac{f_0 d\sin\theta}{c} + \Delta f(t-\frac{r}{c}))\}\right|^2 \tag{3}$$

It can be clearly seen from (3) that the beam pattern of the FDA is a function of distance, angle, time and frequency offset, that is, the beam pattern of the FDA has distance, angle, time and frequency offset dependence. This feature determines that the FDA is a more flexible array, and the beam can be flexibly controlled by controlling the relevant array parameters.

III. Proposed Scheme

As described in the previous section, conventional FDA forms a periodic S-shaped spatial beam with a linearly increasing frequency offset scheme, but such a beam is not always desirable. In electronic warfare, when a precise point-to-point interference is transmitted for the specific location of an enemy target, it is necessary to form a beam only at a specific location of the target and form a null trap in other places. In order to solve this problem, this paper adopts a multi-carrier FDA structure scheme with power exponential frequency offset. The structure can form a dot-shape beam of energy convergence, and furtherly reduce the influence of sidelobes and improve carrier utilization.

Multi-carrier is based on FDA, that is, each array element no longer transmits the traditional single carrier signal, but transmits multi-carrier signals with different frequencies. The structure diagram of the multi-carrier FDA system is shown in Fig. 2.

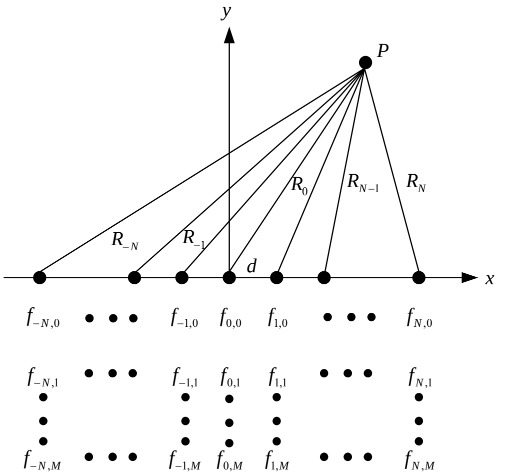

Fig. 2 Multi-carrier FDA model

The intermediate array element of the line array is used as the reference array element, and the two sides of the reference array element are set as symmetric frequency structure, and the symmetry benefits can make the energy more concentrated. Then the mth subcarrier of the nth transmit antenna can be expressed as:

$$f_{n,m} = f_0 + m\Delta f_n \quad -N \le n \le N, 0 \le m \le M \tag{4}$$

where M represents the number of subcarriers and N represents the number of elements, Δf_n is a frequency offset scheme consisting of a power exponential function, which can be expressed as:

$$\Delta f_n = \Delta\delta \cdot |n|^{\frac{1}{2}}, \quad -N \le n \le N \tag{5}$$

where $\Delta\delta$ is a small frequency offset constant. Therefore, the energy transmitted to the target on the nth element can be expressed as:

$$X(t) = e^{j2\pi f_0 (t-\frac{r}{c})} \cdot \sum_{n=-N}^{n=N} \sum_{m=0}^{M} w_{n,m} e^{j2\pi f_0 (\frac{nd\sin\theta}{c})} e^{j2\pi m \Delta f |n|^{\frac{1}{2}} (t-\frac{r}{c})} \tag{6}$$

where (θ, r) is the position of any point in the space, the normalized beam pattern can be expressed as:

$$B(\theta, r) = \left| \sum_{n=-N}^{n=N} \sum_{m=0}^{M} w_{n,m} e^{j2\pi f_0 (\frac{nd\sin\theta}{c})} e^{j2\pi m \Delta f |n|^{\frac{1}{2}} (t-\frac{r}{c})} \right|^2 = \left| \mathbf{W}^H \mathbf{A}(\theta, r) \right|^2 \tag{7}$$

where $\mathbf{A}(\theta, r)$ is the array factor, and $\mathbf{W}$ is the weight vector. In order to adaptively point the main lobe of the beam to the target location to enhance the useful signal and improve the spatial resolution and system performance, the adaptive beamforming of the multi-carrier FDA based on the power exponential frequency offset is implemented by optimizing the $\mathbf{W}$ by the LCMV criterion.

The LCMV beamforming algorithm based on the minimum variance criterion is a classical beamforming algorithm. The output power is minimized while maintaining the desired signal or certain constraints, thereby suppressing the interference noise. This linear constraint optimization problem can be described as:

$$\begin{cases} \min\limits_{w} \mathbf{W}^H \boldsymbol{R}_x \mathbf{W} \\ \boldsymbol{C}^H \mathbf{W} = \mathbf{F} \end{cases} \tag{8}$$

where $\mathbf{W}$ is the weight vector of the spatial domain filter, C is the steering vector constraint matrix, $\mathbf{F}$ is the corresponding constraint response vector, and $\boldsymbol{R}_x$ is the data covariance matrix.

The Lagrange multiplier algorithm is used to solve the above constraint optimization problem, and the LCMV optimal weight vector is obtained:

$$\mathbf{W}_{LCMV} = \boldsymbol{R}_x^{-1} \boldsymbol{C} (\boldsymbol{C}^H \boldsymbol{R}_x^{-1} \boldsymbol{C})^{-1} \mathbf{F} \tag{9}$$

Since $\boldsymbol{R}_x$ is a Hermitian matrix, all eigenvalues are real numbers and the eigenvectors are orthogonal, and eigenvalue decomposition is performed on them:

$$\boldsymbol{R}_x = \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^H + \boldsymbol{U}_n \boldsymbol{\Lambda}_n \boldsymbol{U}_n^H \tag{10}$$

where $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are the matrix representations of $\boldsymbol{R}_x$ interference subspace and noise subspace respectively; $\boldsymbol{\Lambda}_s = diag\{\lambda_1, \lambda_2 \cdots \lambda_m\}$ is a diagonal matrix composed of large eigenvalues corresponding to eigenvectors in $\boldsymbol{U}_s$, and $\boldsymbol{\Lambda}_n = \sigma \boldsymbol{I}$ is a diagonal of small eigenvalues corresponding to eigenvectors in $\boldsymbol{U}_n$ matrix.

In practical applications, the LCMV beamformer is usually implemented by a sampling matrix inversion algorithm (SMI), which uses K times of snapshot data to estimate the convergence speed of $\boldsymbol{R}_x$ with a fast signal-to-interference-and-noise ratio (SINR).

$$\hat{\boldsymbol{R}}_x = \frac{1}{K} \sum_{i=1}^{K} \boldsymbol{x}\boldsymbol{x}^H \tag{11}$$

When the SMI is used, in the case of a limited number of snapshots, the small eigenvalues of the noise subspace will be diverged due to the estimation error, and as the number of snapshots decreases, the divergence will increase, resulting in adaptive beam distortion, which is manifested as: the deviation of main lobe, the sidelobes rise, the energy concentration ability decreases, and the convergence speed becomes slower.

It can be seen from the above analysis that in the case of a limited number of snapshots, the beamforming performance is degraded due to the divergence of small eigenvalues in the noise subspace, which is manifested as follows: the sidelobe level is increased, the beam energy cannot be concentrated, and the dot-shape beam cannot be formed. Therefore, this paper adopts a beamforming algorithm based on exponential correction, which improves the performance of beamforming by improving the degree of small eigenvalue divergence in the noise subspace. Perform an index correction for $\boldsymbol{R}_x$ as follows:

$$\boldsymbol{R}_x^{\alpha} = \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{\alpha} \boldsymbol{U}_s^H + \boldsymbol{U}_n \boldsymbol{\Lambda}_n^{\alpha} \boldsymbol{U}_n^H \tag{12}$$

where α is the correction index and $\alpha \in [0,1]$.

The modified diagonal matrix $\Lambda_n^{\alpha} = diag\left\{\sigma_1^{\alpha}, \sigma_2^{\alpha} \cdots \sigma_n^{\alpha}\right\}$ consists of small eigenvalues of the noise subspace, so the small eigenvalue of the noise subspace is updated to $\sigma_i^{\alpha}, i = 1,2\cdots n$; due to $\alpha \in [0,1]$, the divergence of $\sigma_i^{\alpha}, i = 1,2\cdots n$ is improved, and when $\alpha \to 0$, $\sigma_i^{\alpha} \to 1, i = 1,2\cdots n$, i.e. The small eigenvalue divergence of the noise subspace decreases as the correction index α decreases.

Further analysis: When $0 < \alpha < 1$. After exponentially correcting the $\boldsymbol{R}_x$, the divergence of small eigenvalues in the noise subspace is improved. Beamforming can still have better sidelobe characteristics in the case of low fast beats, and can form a point beam. But for the same reason, the ratio between the large eigenvalues of the interference subspace and the small eigenvalues of the noise subspace is reduced, resulting in weakened beam interference and clutter suppression. Therefore, the choice of the correction index α directly affects the performance of beamforming. From the simulation analysis in the next section, it is known that the performance of the algorithm is best when the index $\alpha = 0.6$ is corrected in the sense that the output SINR is the largest.

The index-corrected weight vector is as follows:

$$\mathbf{W}_{EXP-LCMV} = \boldsymbol{R}_x^{-\alpha} \boldsymbol{C} (\boldsymbol{C}^H \boldsymbol{R}_x^{-\alpha} \boldsymbol{C})^{-1} \mathbf{F} \tag{15}$$

Therefore, the beam pattern of adaptive beamforming based on exponential correction for multi-carrier FDA can be expressed as:

$$B(\theta, r) = \left| \mathbf{W}_{EXP-LCMV}^H \mathbf{A}(\theta, r) \right|^2 \tag{16}$$

IV. Results and Discussion

The MATLAB software is used to verify the scheme proposed in this paper. Assume that the FDA has 11 emitter elements, and the spacing between adjacent elements is half of the maximum wavelength. The signal transmitted by each array element contains eight subcarriers. The center frequency of the signal transmission f_0 is 10 GHz, and the fundamental frequency offset Δf is 3 kHz. It is assumed that the enemy target is in our 30° direction and the distance is 150 km, i.e. $(\theta_0, r_0) = (30^{\circ}, 150\text{km})$.

Under the above simulation conditions, the simulation results of the conventional ULA-FDA beam characteristics are firstly carried out. The results are as follows: As can be seen from Fig. 3, the conventional ULA-FDA beam is an S-shape beam, and the beam radiates energy in space, is not only related to angles, but also to distance. It can be clearly seen from the mark of Fig. 4 that the beam can be controlled to point the main beam to the desired target position by setting the position matching weight according to the specific position of the enemy. However, this position matching weighting method does not change the S-shape of the beam. In addition to being formed at the target position, a beam is formed at some other locations, as indicated by the arrow pointing in the figure.

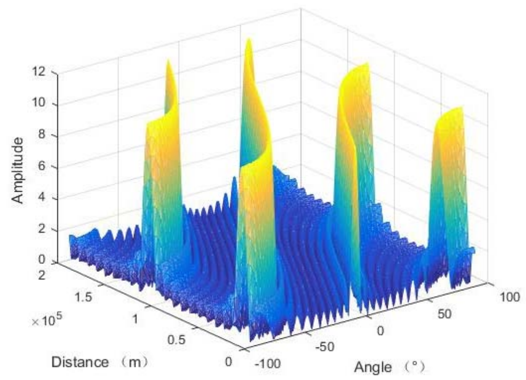

Fig. 3 Three-dimensional view of the beam pattern (ULA-FDA)

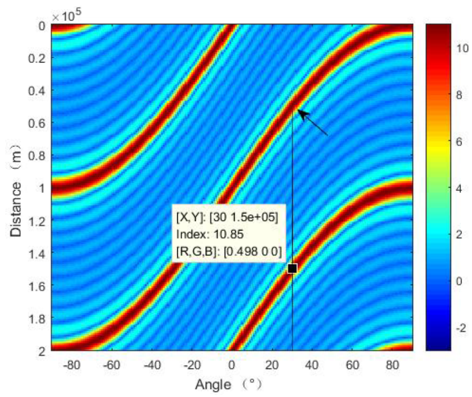

Fig. 4 Two-dimensional plane beam pattern (ULA-FDA)

In order to solve this problem, this paper proposes a multi-carrier FDA structure based on power exponential frequency offset. By changing the array structure and frequency offset, a concentrated point beam can be formed. Under the same simulation conditions, the simulation experiment is carried out on the multi-carrier FDA based on power exponential frequency offset which is compared with the frequency offset scheme in [13]. As can be seen from Fig. 5, the beam only points to a specific point in space, and the beam shape is an obvious dot-shape beam. Figs. 6 and 7 are the cutaway views of the angle dimension and the distance dimension respectively, it can be seen from the figure that in the distance dimension, the frequency offset scheme used in this paper has better energy concentration performance and higher resolution than the scheme in [13], and the main lobe width is 15 km and 100 km, respectively. Moreover, in the angle dimension, the frequency offset scheme adopted in this paper still has good sidelobe level and high resolution.

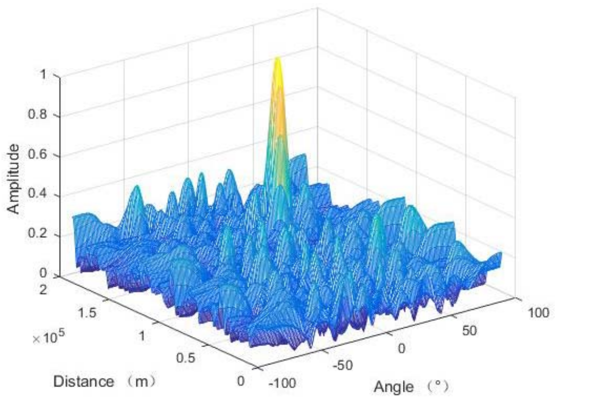

Fig. 5 Three-dimensional view of the multi-carrier FDA beam pattern

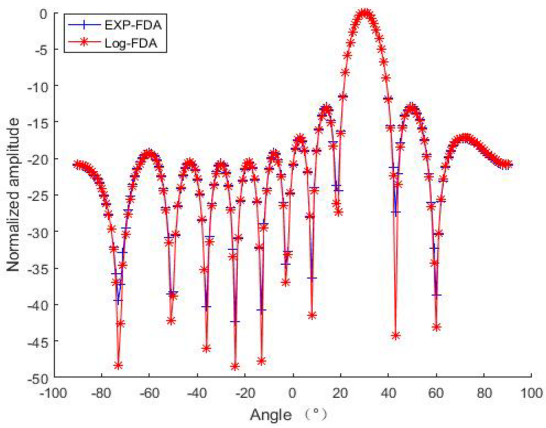

Fig. 6 Comparison of EXP-FDA and Log-FDA beam pattern (angle dimension)

Figs. 8 and 9 are a comparison of the beam pattern of the multi-carrier FDA in the LCMV algorithm with the EXP-LCMV algorithm beam pattern when the correction index is 0.6, and when the number of snapshots is 70000. It can be clearly seen from the figure that under such a large snapshot condition, the multi-carrier FDA can form a distinct point beam under the LCMV algorithm which has a good sidelobe level, and its beam performance is approximately the same as that in the EXP-LCMV algorithm.

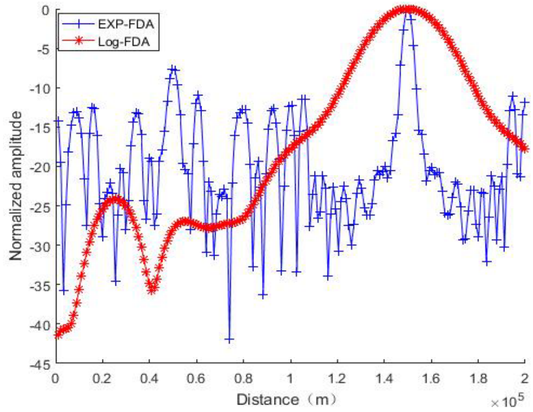

Fig. 7 Comparison of EXP-FDA and Log-FDA beam pattern (distance dimension)

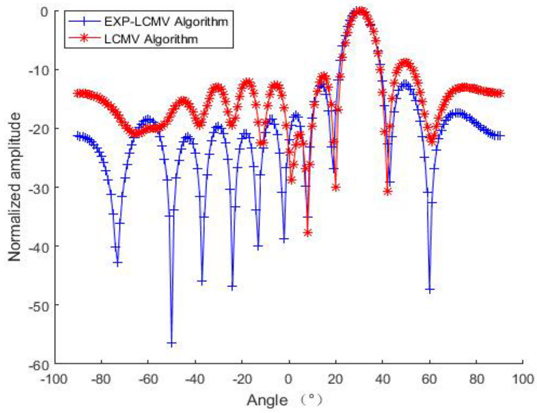

Fig. 8 Comparison of LCMV algorithm and EXP-LCMV algorithm beam pattern (angle dimension)

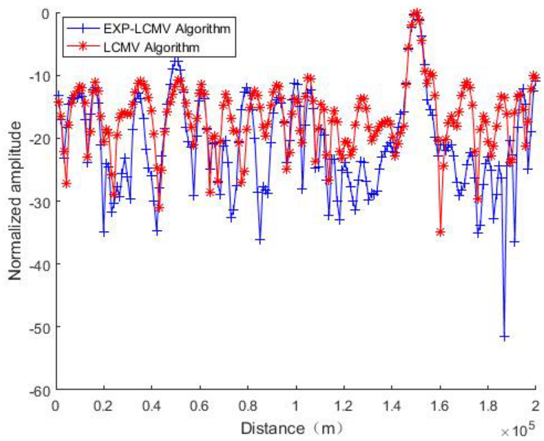

Fig. 9 Comparison of LCMV algorithm and EXP-LCMV algorithm beam pattern (distance dimension)

A huge number of snapshots make the system run too much memory and increase the computational complexity. Figs. 10 and 11 are a comparison of the beam pattern of the multi-carrier FDA under the LCMV algorithm and under the EXP-LCMV algorithm beam pattern when the correction index is 0.6, and when the number of snapshots is 200. It can be clearly seen from the figure that in the case of limited snapshot, the EXP-LCMV algorithm used in this paper can still generate spot beams with no offset in the main lobe direction, and the beam pattern of the LCMV algorithm is in limited snapshot conditions. The main lobe is deviated, the sidelobe level is high, and the beam is distorted.

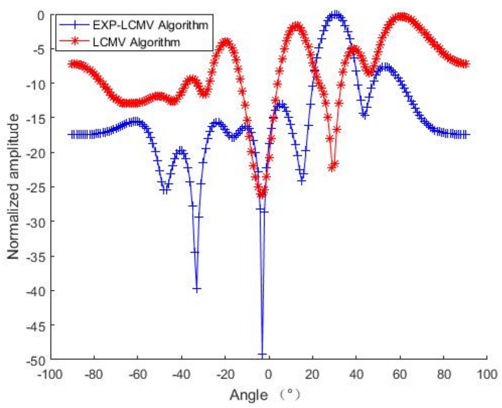

Fig. 10 Comparison of LCMV algorithm and EXP-LCMV algorithm beam pattern (angle dimension)

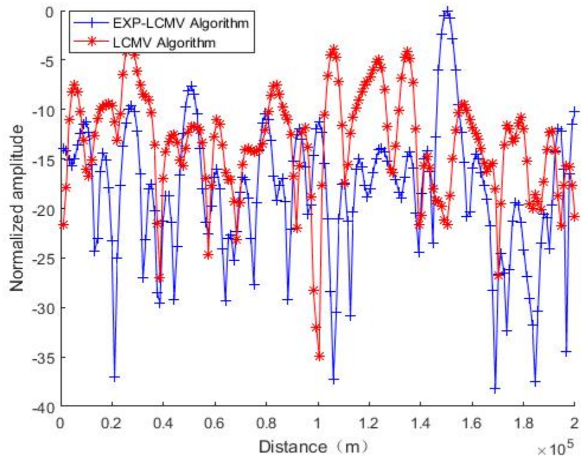

Fig. 11 Comparison of LCMV algorithm and EXP-LCMV algorithm beam pattern (distance dimension)

Assuming that the INR is 50 dB and the SNR is 10 dB, the robustness of the multi-carrier FDA under the LCMV algorithm and under the EXP-LCMV algorithm is compared by 500 Monte Carlo experiments. As can be seen from Fig. 12, The EXP-LCMV algorithm has reached a steady state when the number of snapshots is about 500, and the LCMV algorithm is far from reaching a steady state when the number of snapshots is 5000. Obviously, the multi-carrier FDA has higher beam robustness under the EXP-LCMV algorithm.

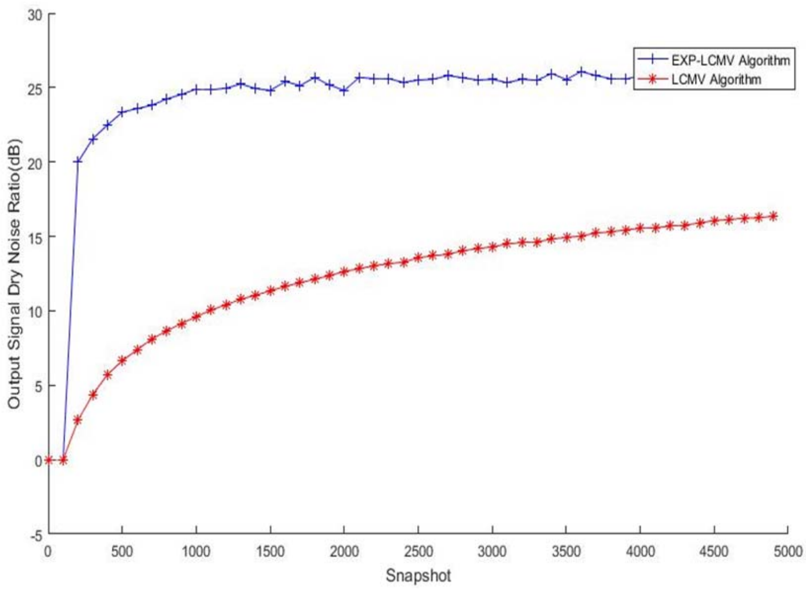

Fig. 12 Comparison of output SINR of LCMV algorithm and EXP-LCMV algorithm with the number of snapshots

Assuming that the INR is 50 dB and the SNR is 10 dB, a simulation experiment is performed on the output SINR of the EXP-LCMV algorithm with the modified index by 500 Monte Carlo experiments. As can be seen from Fig. 13, when the correction index is 0.6, the output SINR of the EXP-LCMV algorithm is the largest. Through theoretical analysis and simulation experiments, it can be seen that if the correction index is too large or too small, it will affect the divergence of small eigenvalues in the noise subspace, thus affecting the beam performance.

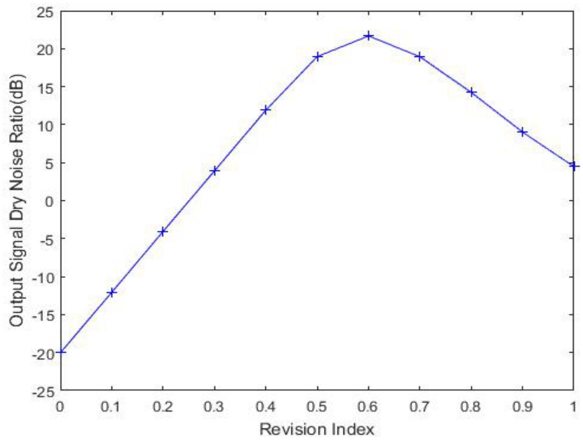

Fig. 13 Output SINR changes with correction index

V.CONCLUSION

As a new type of array structure, FDA has broad application prospects in target detection and interference in FDA. In this paper, the principle of beam diverse array beamforming and the method of point beamforming are analyzed. On the basis of that, an adaptive beamforming technique based on exponential correction for multi-carrier FDA is proposed to form a dot-shape beam in space and to improve the robustness of beamforming. The method firstly generates a point beam by using a power exponential frequency offset to generate a multi-carrier FDA, and then decomposes the data covariance matrix to obtain its interference subspace, noise subspace and a diagonal matrix of corresponding eigenvalues. Finally, the correction index is introduced to exponentially correct the small eigenvalues of the noise subspace, improve the divergence of small eigenvalues in the noise subspace, and promote the performance of beamforming. Through simulation experiments and the above results analysis, the method enables the multi-carrier FDA to form a dot-shape beam with limited snapshot, enhance beam flexibility, reduce sidelobe level, and improve beamforming robustness.

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