An Implementation of a Dual-Spin Spacecraft Attitude Reorientation Using Properties of Its Chaotic Motion

Anton V. Doroshin

Abstract—This article contains a description of main ideas for the attitude reorientation of spacecraft (small dual-spin spacecraft, nanosatellites) using properties of its chaotic attitude motion under the action of internal perturbations. The considering method based on intentional initiations of chaotic modes of the attitude motion with big amplitudes of the nutation oscillations, and also on the redistributions of the angular momentum between coaxial bodies of the dual-spin spacecraft (DSSC), which perform in the purpose of system's phase space changing.

Keywords-Spacecraft, Attitude Dynamics and Control, Chaos.

I. INTRODUCTION

As it was showed [1]-[14], the attitude dynamics of the dual-spin spacecraft (DSSC) is liable to the homo/heteroclinic chaos captures.

The DSSC at the action of internal perturbations [1]-[14] can perform some complicated attitude motion modes with big changing of spatial angles amplitudes (and, first of all, we must underline the possibility of the nutation amplitude increasing). This circumstance usually is considered as the space mission accident. But, it is possible to suggest the application of this choatization phenomenon as the method of the attitude reorientation of the small simple DSSC [1].

In [1] the possible change of the phase portrait form was analyzed – this change can be implemented at the spin-up (and at the spin-down) process of the rotor-body. Also it is important to note that the chaotic motion will arise after the perturbation (the internal perturbation [1]) initiations.

So, taking into account these described features of the perturbed attitude motion, we can suggest the intentional initiations of chaotic modes in the purpose of the small DSSC attitude reorientation.

II. MATHEMATICAL MODEL

The DSSC motion equation can be written in the form [1]:

$$\begin{aligned} A\dot{p} + (C_2 - B)qr + q\Delta &= 0; \\ B\dot{q} + (A - C_2)pr - p\Delta &= 0; \\ C_2\dot{r} + \dot{\Delta} + (B - A)pq &= 0; \\ \dot{\Delta} &= M_{\Lambda}, \end{aligned} \tag{0}$$

where $[p,q,r]^{T}$ – are components of the absolute angular velocity of the main body (platform-body) in the connected

frame $Ox_2y_2z_2$, σ - the relative angular velocity of the rotorbody; $\Delta = C_1(r+\sigma)$; $A = A_1 + A_2$, $B = A_1 + B_2$; $diag[A_1, A_1, C_1]$, $diag[A_2, B_2, C_2]$ - are the inertia tensors of the coaxial bodies (## *i*=1, 2) in the corresponding connected frames $Ox_iy_iz_i$, M_{Δ} - the internal torque of the rotor-engine.



Fig. 1 The coaxial bodies (DSSC)

The well-known Andoyer-Deprit variables are defined as follows (Fig. 1):

$$L = \frac{\partial T}{\partial i} = \mathbf{K} \cdot \mathbf{k};$$

$$I_2 = \frac{\partial T}{\partial \dot{\phi}_2} = \mathbf{K} \cdot \mathbf{s} = |\mathbf{K}| = K;$$

$$I_3 = \frac{\partial T}{\partial \dot{\phi}_3} = \mathbf{K} \cdot \mathbf{k}';$$

$$L \le I_2$$

$$K_{x_2} = Ap = \sqrt{I_2^2 - L^2} \sin l;$$

$$K_{y_2} = Bq = \sqrt{I_2^2 - L^2} \cos l;$$

$$K_{y_2} = C_2 r + \Delta = L,$$

where \mathbf{K} is the DSSC angular momentum's vector. Then, in the Andoyer-Deprit variables we have the well-known Hamiltonian form [1]:

A.V. Doroshin is the associate professor at the Space Engineering Department of Samara State Aerospace University (National Research University), Samara, Russia (e-mail: doran@inbox.ru).

$$H = H_0 + \varepsilon H_1;$$

$$H_0 = T = \frac{I_2^2 - L^2}{2} \left[\frac{\sin^2 l}{A_1 + A_2} + \frac{\cos^2 l}{A_1 + B_2} \right] + \frac{1}{2} \left[\frac{\Delta^2}{C_1} + \frac{(L - \Delta)^2}{C_2} \right], (1)$$

where *T* is the system kinetic energy, H_1 – a perturbed part of the Hamiltonian and ε – small dimensionless parameter corresponding to perturbations.

III. SIMULATION RESULTS

Now it is possible to simulate perturbed motion of DSSC using (0) in addition of the well-known Euler kinematical equations for nutation (θ), precession (ψ), intrinsic rotation (φ) and DSSC-rotor's relative rotation (δ):

$$\dot{\theta} = p\cos\varphi - q\sin\varphi; \quad \dot{\psi} = \frac{1}{\sin\theta} \cdot (p\sin\varphi + q\cos\varphi); \quad (2)$$
$$\dot{\varphi} = r - ctg\theta \cdot (p\sin\varphi + q\cos\varphi); \quad \dot{\delta} = \sigma$$

We consider the equations system (0) and (2) in the perturbed case of the DSSC motion at the following simple internal harmonic torque

$$\dot{\Delta} = M_{\Delta} = \varepsilon \sin(\nu t)$$
.

The following parameters were used at the numerical simulations:

$$A_{2} = 15, B_{2} = 8, C_{2} = 7, A_{1} = 5, C_{1} = 4 [kg \cdot m^{2}];$$

$$I_{2} = 20, \quad \Delta = 4 [kg \cdot m^{2} / s];$$

$$\theta_{0} = 0.62, \quad \psi_{0} = 0, \quad \varphi_{0} = 1.57 \ [rad];$$

$$\varepsilon = 1 \ [dimensionless]; \quad v = 1 \ [1 / s].$$

Figures (Figs. 2-4) illustrate the main properties of the DSSC chaotic motion. As we can see, the broad interval of the nutation angle can be obtained – this can be consider as a passive way of the attitude reorientation based only on the internal properties of the DSSC chaotic motion. The chaotic motion from this point of view is the "essential passive reorientation driver of the DSSC".

So, in this paper we present only the main idea description and numerical modeling results.



Fig. 2 Angular velocity components: p(t) – black, q(t) – red, r(t) – blue







Fig. 4 The chaotic evolution of the nutation angle: the green line corresponds to the level $\theta = \pi/2$ [*rad*]

For the "chaotic DSSC attitude reorientation" we must fulfill the following steps:

- 1) The "mono-body" scheme of the DSSC placing into the orbit;
- 2) The spin-up DSSC maneuver realization [1];
- The internal perturbation initiation and the DSSC capture into the chaotic motion mode with the big amplitude of the nutation angle;
- 4) The receiving of the required value of the nutation angle at the realization of the chaotic motion;
- 5) The shutdown of the internal perturbation and the passage to the regular motion;
- The spin-down DSSC maneuver realization (this step may be ignored).

So, as the general idea, to obtain the necessary attitude (spatial) DSSC reorientation we need to intentionally initiate the small internal perturbation of the DSSC-rotor (to "switch on" the internal harmonic torque $M_{\Delta} \rightarrow \varepsilon \sin(vt)$), to reach

the desirable attitude orientation at the uncontrolled realization of the chaotic motion, and to "switch off" the internal perturbation $(M_{\Lambda} \rightarrow 0)$.

IV. CONCLUSION

The paper indicates the possibility of the DSSC attitude reorientation using initiation of chaos motion. This scheme can be used in the small simple satellites. For realization of the reorientation we need to perform the capture into chaos motion, and shutdown the internal perturbation (the internal spin-up engine) after receiving of the required value of the nutation angle. We can develop this idea in details in separated publications.

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Anton V. Doroshin received his Ph.D. degree in theoretical mechanics from the Samara State Aerospace University in 2002. He is the Head of the Educational Programs Department and an Associate Professor at the Space Engineering Department (Division of Flight Dynamics and Control Systems) in the Samara State Aerospace University (National Research University). His research interests include space flight dynamics and dynamical systems theory. He is especially concerned with the analysis of non-linear regular and chaotic dynamical systems, attitude dynamics and control of dual-spin spacecrafts, gyrostats and multi-rotor rigid bodies systems. He is the author of numerous publications in these areas. Also Dr. Doroshin is the head of the several research projects and grants of the Russian Foundation for Basic Research, the Russian Federation Presidential Program for scientists and leading scientific schools of Russian Federation, International Soros Science Education Program. In 2010 he was awarded the Medal of Russian Academy of Science (Direction: Machinery, Mechanical Science and Control Systems). Also he is a member/reviewer of IAE (International Association of Engineers), WSEAS (World Scientific and Engineering Academy and Society), ELSEVIER, SPRINGER, and SAGE journals.