An Expert System Designed to Be Used with MOEAs for Efficient Portfolio Selection

K. Metaxiotis, K. Liagkouras

Abstract—This study presents an Expert System specially designed to be used with Multiobjective Evolutionary Algorithms (MOEAs) for the solution of the portfolio selection problem. The validation of the proposed hybrid System is done by using data sets from Hang Seng 31 in Hong Kong, DAX 100 in Germany and FTSE 100 in UK. The performance of the proposed system is assessed in comparison with the Non-dominated Sorting Genetic Algorithm II (NSGAII). The evaluation of the performance is based on different performance metrics that evaluate both the proximity of the solutions to the Pareto front and their dispersion on it. The results show that the proposed hybrid system is efficient for the solution of this kind of problems.

Keywords—Expert Systems, Multiobjective optimization, Evolutionary Algorithms, Portfolio Selection.

I. INTRODUCTION

THE portfolio selection problem is a classical multi-objective optimization problem as there are two conflicting objectives (*return* and *risk*). The last years has been observed an increase in paper output concerning multi-objective techniques applied to the portfolio selection problem, particularly multi-objective evolutionary algorithms (MOEAs) [8]. The majority of the authors in the field focus their attention in the development of more efficient MOEAs for handling the complexity of difficult problems like the cardinality constrained portfolio selection problem (CCPSP) for which no efficient deterministic algorithm exists.

In this study we decided to follow a different approach. In particular, we propose a Hybrid System that consist the synergy between an Expert System (ES) and the NSGAII. In the first stage the Expert System performs a scrutiny in the available equities, based on their performance in a number of indicators. Through this evaluation we will manage to restrict the available pool of equities to the most promising ones. Next, we apply as normally a state-of-the-art MOEA like the NSGAII. The robustness of the proposed Hybrid System is validated by using data sets from the publicly available OR-Library retained by Beasley, based on a number of performance metrics that assess both the proximity of the solutions to the Pareto front and their dispersion on it.

The paper is organized as follows. In Section II we present the problem and we provide a formal introduction to the proposed Expert System by analyzing the methodological framework for the evaluation of the available pool of equities. Additionally, we present the interrelation between the ES and the MOEA. Section III presents the experimental environment and the parameters' setup. In Section IV we present the performance metrics. In Section V, through a number of computational experiments we test the performance of the proposed Hybrid System for the solution of the CCPSP. Section VI analyses the relevant results and Section VII concludes the paper.

II. A HYBRID SYSTEM FOR THE SOLUTION OF THE CARDINALITY CONSTRAINED PORTFOLIO SELECTION PROBLEM

As we mentioned earlier this study proposes a Hybrid System which is the synergy between an Expert System (ES) and a state-of-the-art MOEA namely the NSGAII, for efficient portfolio management. The term ES refers to a computer program that applies a substantial knowledge of specific areas of expertise to a problem-solving process [9]. A major difficulty in developing an ES is extracting the required expertise to develop the knowledge base [7]. It can also be challenging the codification of the knowledge into a format that can be used in an automated application. Regarding the development of Expert Systems for the portfolio management domain there are some very good studies in the available literature [7], [10], [12].

The purpose of this study is not restricted solely to the presentation of another ES for portfolio management, but firstly and foremostly, this paper focuses on the development of a simple to implement and efficient Hybrid System that benefit from the collaboration between the Expert System and the MOEA. The portfolio construction problem can be separated into two distinct phases [6]. Phase 1, focuses on the evaluation of the available equities based on preset criteria and selection of the better performing stocks. And Phase 2, that deals with the determination of the appropriate weight for each of the equities selected in the first phase.

The majority of studies regarding Expert Systems for portfolio management focus their attention in Phase 1, i.e. the selection of the equities to be included in the portfolio. A considerably smaller number of studies provide connectivity, usually with a multi-criteria decision support system that calculates weights for each of the equities selected in Phase 1. [11], [12]-[14], [17]. The same methodology applied in this paper. In particular during phase one the Expert System performs an initial evaluation on the available pool of securities based on a set of rules. Based on the results of this evaluation a number of stocks proceed to phase two where the

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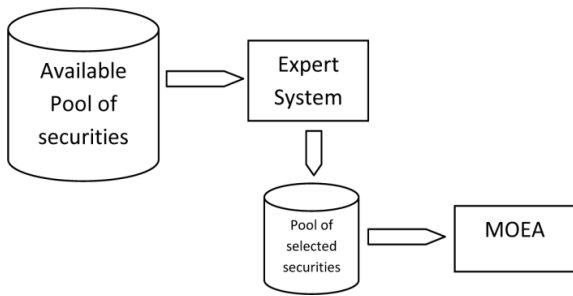
selected stocks are feed to the MOEA.

Fig. 1 A Hybrid System for Portfolio management

Thanks to the filtering provided by the Expert System, the MOEA is better able to focus its computational effort to the most promising equities in terms of return and risk. Naturally, it comes to the readers mind which is the mechanism used by the Expert System to filter the pool of available securities and how efficient is that mechanism. To answer this question we will start our analysis from the MOEA. The portfolio selection problem is a bi-objective problem as there are two conflicting objectives, return and risk. Thus, the MOEA evaluates the various securities based on their corresponding return and risk combinations. Since, the evaluation of the various securities from the MOEA will become on the basis of their return and risk combinations, for maintaining consistency, but also for practical reasons that have to do with the availability of data, on the same ground will become the scrutiny of the various securities from the Expert System.

Below, we provide an analysis of the proposed Expert System. The validation of the proposed Hybrid System is done by using data sets from the publicly available OR-Library retained by Beasley. These data sets regard 3 test problems, port1, port2 and port3 that initially were used from the Chang et al. [1], and contain the mean return, the standard deviation and the correlation of assets. Fig. 2 provides a sketch of OR-Library data for the Portfolio Selection problem and their relation with the objectives of the MOEA. The Return of assets corresponds to the first objective and Standard deviation corresponds to the second objective of the MOEA.
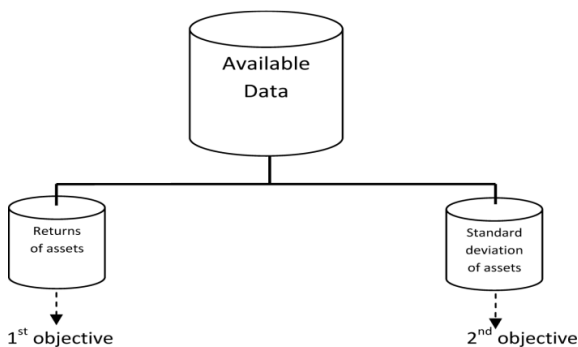

Fig. 2 OR-Library data for Portfolio Selection and the corresponding Objectives

TABLE I
THE CRITERIA FOR THE EVALUATION OF THE POOL OF AVAILABLE EQUITIES

Criterion	Definition	Criterion direction	Measuring unit
Return on equity	Net income divided by shareholders equity	Max	%
Equity STD	Dispersion of a set of share prices around their mean	Min	%

Formally, in its bi-objective form, the portfolio optimization problem can be formulated as follows.

Let Ω be the search space. Consider 2 objective functions f_1, f_2 where $f_i : \Omega \to \Re$ and $\Omega \subset \Re$.

Optimize	$f(w) = (f_1(w), f_2(w))$	(1)
Maximize portfolio return	$f_1(w) = \sum_{i=1}^{m} w_i \bar{r}_i$	(2)
Minimize portfolio risk	$f_2(x) = \sum_{i=1}^{m}\sum_{j=1}^{m} w_i w_j \sigma_i \sigma_j \rho_{ij}$	(3)

s.t.

- *Budget constraint or summation constraint $\sum_{i=1}^{m} w_i = 1$, requires all portfolios to have non-negative weights ($0 \le w_i \le 1 \quad i=1,2,..,m$) that sum to 1.*

- *Floor and ceiling constraint $a_i \le w_i \le b_i, \forall = 1,2,...,m$. Where a_i is the minimum weighting that can be held of asset i (i = 1, ..., m), b_i = the maximum weighting that can be held of asset i (i = 1, ...,m) and $0 \le a_i \le b_i \le 1$, $\forall = 1,2,...,m$.*

- *Cardinality constraint $C_{min} \le \sum_{i=1}^{m} q_i \le C_{max}$, where C_{min} is the minimum number of assets that a portfolio can hold, C_{max} is the maximum number of assets that a portfolio can hold, $q_i = 1$, for $w_i > 0$ and $q_i = 0$, for $w_i = 0$.*

<u>*where:*</u>

Decision variables $w = (w_1,.., w_m)$ subject to $w \subset \Omega$ and m equal to the number of stocks.

Rate of return of assets: $r_1, r_2.., r_m$.

ρ_{ij} is the correlation between asset i and j and $-1 \le \rho_{ij} \le 1$.

σ_i, σ_j represent the standard deviation of stocks returns i and j

From the formal presentation of the portfolio optimization problem become clear that it is a bi-objective problem where the first objective corresponds to the Return of assets and the second objective corresponds to the portfolio's variance.

Table I presents the criteria for the evaluation of the pool of available equities. According to the criteria, as shown in Table I, the higher the stock's return the better and the lower the stock's standard deviation (std) the better. Table II presents the complete scheme for the evaluation of the various assets by the Expert System. Stocks are ranked from best performing to worse performing, for each one of the evaluation criterion. Top performance gets zero points and worst performance gets *n-1* points, where *n* is the number of stocks.

TABLE II
AN EXAMPLE OF THE TWO EVALUATION CRITERIA: RETURN AND STD AS
IMPLEMENTED BY THE EXPERT SYSTEM

Criterion: **Return**		Criterion: **STD**	
Direction: **Max to Min**		Direction: **Min to Max**	
Returns	Points	STD	Points
$r_B = 0.04$	0	$Std_C = 0.001$	0
$r_E = 0.031$	1	$Std_B = 0.003$	1
$r_A = 0.023$	2	$Std_E = 0.010$	2
$r_C = 0.01$	3	$Std_D = 0.021$	3
$r_D = -0.01$	4	$Std_A = 0.032$	4

As soon as we complete the ranking process we assign weight to each one of the two objectives. We assign an equal weight of 0.50 to each objective. Subsequently, the weight is multiplied by the corresponding points assigned during the previous stage and thus results the score for each objective. Finally, by adding the score of both objectives we obtain the evaluation for the performance of each stock. Table III illustrates the aforementioned process.

TABLE III
EVALUATION OF THE PERFORMANCE OF THE AVAILABLE POOL OF STOCKS

Calculation of score							
1st Objective: **Return**			2nd Objective: **Risk**			Score of Stocks	
Stock	Weight * Points		Stock	Weight * Points		Stock	Points
A	0.5*2	+	A	0.5*3.50	=	A	2.75
B	0.5*0	+	B	0.5*0.75	=	B	0.375
C	0.5*3	+	C	0.5*0	=	C	1.5
D	0.5*4	+	D	0.5*2.50	=	D	3.25
E	0.5*1	+	E	0.5*1.75	=	E	1.375

As soon as, we calculate the scores of the available pool of stocks (see Table III), we sort them according to their corresponding performance, as shown in Fig. 3. Next, we need to determine how many of these stocks will feed the MOEA. In our example that we deal with only a handful of stocks this is not a real issue. However, for big instances of the portfolio optimization problem, selection of the most promising stocks from the available pool of stocks is of utmost importance, as we can save valuable computational time and on top of that, as we will show in Section V, also generates better results.

Score of Stocks Criterion: Min		
Stock		Points
B		0.375
E		1.375
C		1.5
A		2.75
D		3.25

(Stocks sorted from best to worst)

Fig. 3 Stocks' ranked from best to worst

Fig. 3 provides the complete list of available pool of stocks, sorted according to the evaluation criteria, from the best performing stock to worst performing stock. For a better understanding of the proposed methodology Fig. 4 provides a graphical representation of the architecture of the entire Expert System.
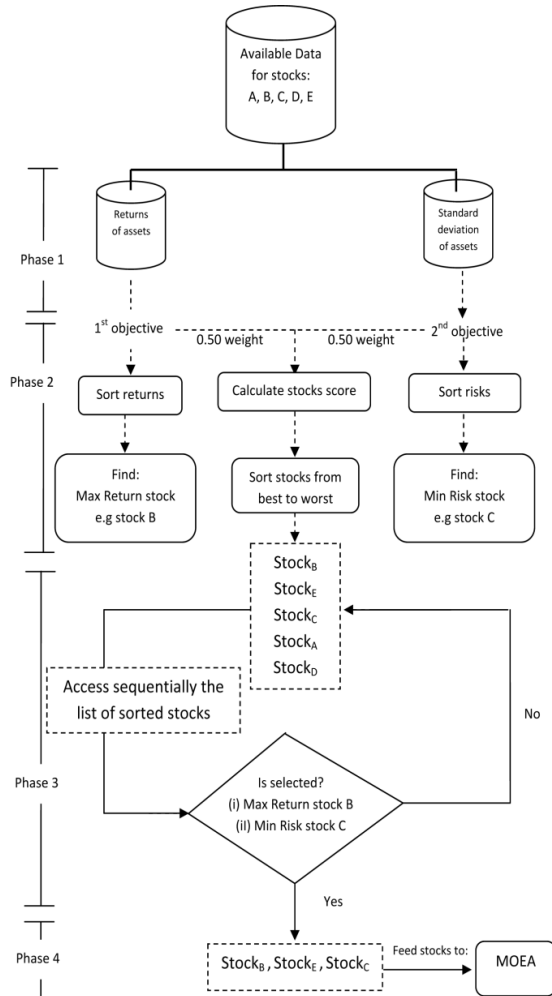

Fig. 4 Structure of the proposed Expert System

We have already explained Phase 1 of the ES in the previous pages. In Phase 2, for each objective i.e. Return and Risk, we determine the top performing stock. Obviously, for Return objective the top performing stock is the one with the highest mean return, and for the Risk objective the top performing stock is the one with the lowest risk. During Phase 2, we also sort the available pool of stocks according to their performance in both objectives, from best performing stocks towards the worst performing stock.

In Phase 3, we set the termination criterion for inclusion of stocks to the group of selected stocks that will feed the MOEA. The termination criterion is the inclusion in the group of selected stocks of the top performing stock for the Return and Risk objective. Please, notice that the access to the sorted list of stocks is sequential which entails that all previous stocks have to be included, in the list of selected stocks till to locate the top performing stocks for both objectives. Finally in Phase 4, as shown in Fig. 4 the selected stocks are fed to the MOEA.

In the case of our example, for the Return objective, stock B is the top performing stock and respectively for Risk

objective, stock C is the top performing stock, as shown in Fig. 4. That means in order to satisfy the termination criterion, we should feed the MOEA with the first three stocks, as stock C, the top performing stock for the Risk objective is ranked 3rd in the relevant list, as shown in Phase 3 of Fig. 4. The choice of this particular termination criterion has not been made randomly. Actually, the selection of the termination criterion is related with the usage of the Pareto optimality framework by the MOEA for the determination of solutions, as portfolio optimization is a multi-objective problem. The inclusion in the group of selected stocks for feeding the MOEA of the top performing stocks for Return and Risk objective alike, allow us to create portfolios that yield a wide range of non-dominated solutions.

III. EXPERIMENTAL ENVIRONMENT

A. Parameter Setup

All algorithms have been implemented in Java and run on a personal computer Core 2 Duo at 1.83 GHz. The jMetal [9] framework has been used to compare the performance of the proposed Hybrid System against a state-of-the-art MOEA, namely the NSGAII. We set the maximum cardinality of the portfolio to ten ($K_{max} = 10$) for all test problems. The participation of each stock in the portfolio is determined by the lower and upper bounds. We set the lower bound $l_i = 0.01$ and the upper bound $u_i = 1$, for each asset i, where $i = 1, \ldots, n$. In all tests we use polynomial mutation, binary tournament and simulated binary crossover (SBX) as mutation, selection and crossover operator, respectively. The crossover probability is $P_c = 0.9$ and mutation probability is $P_m = 1/n$, where n is the number of decision variables. The distribution indices for the crossover and mutation operators are $\eta_c = 20$ and $\eta_m = 20$, respectively. Population size is set to 100, using $25,000$ function evaluations with 30 independent runs.

IV. PERFORMANCE METRICS

A. Hypervolume

Hypervolume [5] is an indicator of both the convergence and diversity of an approximation set. Thus, given a set S containing m points in n objectives, the hypervolume of S is the size of the portion of objective space that is dominated by at least one point in S.

The hypervolume of S is calculated relative to a reference point which is worse than (or equal to) every point in S in every objective. The greater the hypervolume of a solution the better considered the solution. One of the main advantages of hypervolume [15] is that it is able to capture in a single number both the closeness of the solutions to the optimal set and, to some extent, the spread of the solutions across objective space. According to a number of studies Hypervolume, also has nicer mathematical properties than many other metrics. In particular, Zitzler et al. [16] state that hypervolume is the only unary metric of which they are aware that is capable of detecting that a set of solutions X is not worse than another set X'. Also Fleischer [4] has proved that hypervolume is maximized if and only if the set of solutions contains only Pareto optima.

B. Spread

Deb et al. [2] introduced the spread of solutions (Δ) as another indicator of the quality of the derived set of solutions. Spread indicator examines whether or not the solutions span the entire Pareto optimal region. First, it calculates the Euclidean distance between the consecutive solutions in the obtained non-dominated set of solutions. Then it calculates the average of these distances. After that, from the obtained set of non-dominated solutions the extreme solutions are calculated. Finally, using the following metric it calculates the nonuniformity in the distribution.

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}}$$

where d_f and d_l are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained nondominated set. The parameter $\bar{d}$ is the average of all distances d_i, $i = 1, 2, \ldots, (N - 1)$, where N is the number of solutions on the best nondominated front.

C. Epsilon Indicator I_ε

Zitzler et al. [16] introduced the epsilon indicator (I_ε). There are two versions of epsilon indicator the multiplicative and the additive. In this study we use the unary additive epsilon indicator as it has been implemented in jMetal framework. The basic usefulness of epsilon indicator of an approximation set A ($I_{\varepsilon+}$) is that it provides the minimum factor ε by which each point in the real front R can be added such that the resulting transformed approximation set is dominated by A. The additive epsilon indicator is a good measure of diversity, since it focuses on the worst case distance and reveals whether or not the approximation set has gaps in its trade-off solution set.

V. EXPERIMENTAL RESULTS

A number of computational experiments were performed to test the performance of the proposed Hybrid System for the Cardinality constrained Portfolio Selection Problem (CCPSP). The performance of the proposed Hybrid System is assessed in comparison with a well-known MOEA, namely NSGAII. The evaluation of the performance is based on a variety of metrics that assess both the proximity of the solutions to the Pareto front and their dispersion on it. For carrying out the experiments, we used data sets from the publicly available OR-Library retained by Beasley. These data sets correspond to three portfolio optimization problems from three different capital markets as shown in Table IV. More specifically, we used weekly price data for the period March 1992 to September 1997, from Hang Seng 31 in Hong Kong, DAX 100 in Germany and FTSE 100 in UK.

Problem Name	Stock Market Index	Assets
port1	Hang Seng	31
port2	DAX100	85
port3	FTSE100	89

As we have already explained in Section II the proposed Hybrid system comprised by an Expert System (ES) that performs the evaluation of the available pool of stocks and a state-of-the-art MOEA like the NSGAII. The evaluation of the available pool of stocks by the ES is based on criterion that is analyzed in Section II of this study. At the end of this evaluation the ES selects for each one of the test problems (port1 to port3), the number of stocks to feed the MOEA as shown in Table V.

TABLE V
NUMBER OF STOCKS SELECTED BY THE ES TO FEED THE MOEA

Problem Name	Stock Market Index	Available pool of Assets	No. of stocks selected by the ES to feed the MOEA	Selected stocks ÷ Available stocks %
port1	Hang Seng	31	15	48%
port2	DAX100	85	35	41%
port3	FTSE100	89	23	26%

Table V shows us, that the ES works as a filter for scrutinizing the available pool of equities. The output of the Expert System is a considerably smaller pool of selected equities that feed the MOEA. Thanks to the filtering provided by the ES, the MOEA is better able to focus its computational effort to the most promising assets in terms of return and risk. As shown in Table V the filtering of the available pool of stocks by the ES is considerable and varies in the examined test problems between 52% for port1 and up to 74% for port3.

A. Hang Seng (31 Stocks) Port1 in OR-Library

TABLE VI
PORT1 - MEAN STD, MEDIAN AND IQR FOR HV, SPREAD AND EPSILON

Problem		HYBRID 1	NSGAII
PORT1	HV. Mean and Std	$9.88e\text{-}01_{3.5e\text{-}03}$	$9.83e\text{-}01_{4.6e\text{-}03}$
	HV. Median and IQR	$9.89e\text{-}01_{5.8e\text{-}03}$	$9.84e\text{-}01_{3.9e\text{-}03}$
	SPREAD. Mean and Std	$8.65e\text{-}01_{1.4e\text{-}02}$	$8.96e\text{-}01_{1.3e\text{-}02}$
	SPREAD. Median and IQR	$8.63e\text{-}01_{1.2e\text{-}02}$	$8.95e\text{-}01_{2.0e\text{-}02}$
	EPSILON. Mean and Std	$4.81e\text{-}05_{1.4e\text{-}05}$	$7.11e\text{-}05_{1.9e\text{-}05}$
	EPSILON. Median and IQR	$4.46e\text{-}05_{2.4e\text{-}05}$	$6.81e\text{-}05_{1.6e\text{-}05}$

Where: Hybrid 1 = ES + NSGAII

TABLE VII
PORT1 - BOXPLOTS FOR HV, SPREAD AND EPSILON

HV		SPREAD		EPSILON	
HYBR.1	NSGAII	HYBR.1	NSGAII	HYBR.1	NSGAII
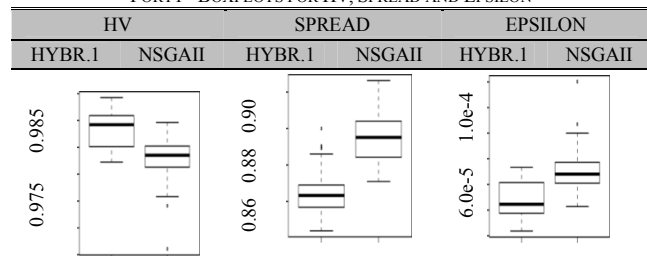
Where: Hybrid 1 = ES + NSGAII

TABLE VIII
WILCOXON TEST IN PORT1 FOR HV, SPREAD AND EPSILON

Problem			NSGAII
PORT1	HV. Mean and Std	HYBRID 1	▲
	HV. Median and IQR		▲
	SPREAD. Mean and Std		▲
	SPREAD. Median and IQR		▲
	EPSILON. Mean and Std		▲
	EPSILON. Median and IQR		▲

Where: Hybrid 1 = ES + NSGAII

The results in the Tables VI-XIV have been produced by using JMetal [9] framework. Tables VI-VIII present the results regarding the port1 problem (Hang Seng index) in OR-Library. In particular, Table VI compares the performance of Hybrid System 1 (HS1) with the NSGAII. In reality, what we examine with these tests is the effect of the proposed Expert System (ES) on the performance of MOEAs.

For the measurement of the performance of the tested algorithms we use a number of performance metrics that assess both the proximity of the solutions to the Pareto front and their dispersion on it. Regarding the HV [15] indicator the higher the value (i.e. the greater the hypervolume) the better the computed front. HV captures in a single number both the closeness of the solutions to the optimal set and to a certain degree, the spread of the solutions across objective space. The second indicator the Spread (Δ) [4] examines the spread of solutions across the pareto front. The smaller the value of this indicator, the better the distribution of the solutions is. Spread indicator takes a zero value for an ideal distribution of the solutions in the Pareto front. The third indicator, the Epsilon [16] is a measure of the smaller distance that a solution set A, needs to be changed in such a way that it dominates the optimal Pareto front of this problem. Obviously the smaller the value of this indicator, the better the derived solution set.

Table VII uses boxplots to present graphically, the performance of HS1 against the NSGAII for the three performance indicators, namely: HV, Spread and Epsilon. Boxplot is a convenient way of graphically depicting groups of numerical data through their quartiles.

Table VIII, presents if the results of HS1 compared with the results of the NSGAII, are statistically significant or not. For that reason, we use the Wilcoxon rank-sum test as it is implemented by the jMetal framework [3]. In Table VIII, three different symbols are used. In particular "—" indicates that there is not statistical significance between the algorithms. "▲" means that the algorithm in the row has yielded better results than the algorithm in the column with confidence (95%) and "▽" is used when the algorithm in the column is statistically better than the algorithm in the row. The same interpretation holds for the rest of the Tables of Section V.

The relevant results for port1 problem as presented by the Tables VI-VIII indicate that the proposed Hybrid System (HS) generates better results with confidence for all performance metrics when compared with the relevant results for the NSGAII. The results also indicate that the ES filters efficiently the available pool of equities, allowing the MOEA to focus its

computational effort to the most promising assets in terms of return and risk combinations.

### B. DAX100 (85 Stocks) Port2 in OR-Library

TABLE IX
PORT2 - MEAN STD, MEDIAN AND IQR FOR HV, SPREAD AND EPSILON

Problem		HYBRID 1	NSGAII
PORT2	HV. Mean and Std	$9.78\text{e-}01_{4.6\text{e-}03}$	$9.66\text{e-}01_{7.6\text{e-}03}$
	HV. Median and IQR	$9.79\text{e-}01_{5.6\text{e-}03}$	$9.65\text{e-}01_{1.3\text{e-}02}$
	SPREAD. Mean and Std	$9.42\text{e-}01_{1.6\text{e-}02}$	$1.02\text{e+}01_{4.2\text{e-}02}$
	SPREAD. Median and IQR	$9.41\text{e-}01_{2.3\text{e-}02}$	$1.01\text{e+}00_{4.8\text{e-}02}$
	EPSILON. Mean and Std	$5.98\text{e-}05_{1.3\text{e-}05}$	$9.18\text{e-}05_{2.1\text{e-}05}$
	EPSILON. Median and IQR	$5.61\text{e-}05_{1.5\text{e-}05}$	$9.33\text{e-}05_{3.6\text{e-}05}$

Where: Hybrid 1 = ES + NSGAII

TABLE X
PORT2 - BOXPLOTS FOR HV, SPREAD AND EPSILON
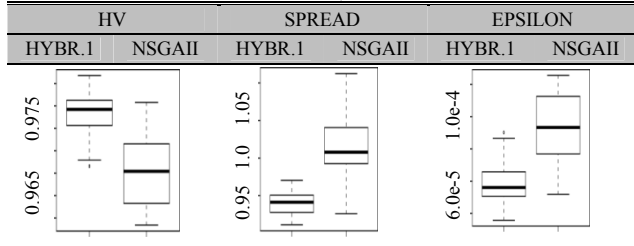
Where: Hybrid 1 = ES + NSGAII

TABLE XI
WILCOXON TEST IN PORT2 FOR HV, SPREAD AND EPSILON

Problem		NSGAII
PORT2	HV. Mean and Std	▲
	HV. Median and IQR	▲
	SPREAD. Mean and Std	▲
	SPREAD. Median and IQR	▲
	EPSILON. Mean and Std	▲
	EPSILON. Median and IQR	▲

(HYBRID 1)

Where: Hybrid 1 = ES + NSGAII

The port2 problem results indicate that the proposed Hybrid System although uses a fraction (41%) of the available pool of stocks as shown in Table V, still succeed to outperform the NSGAII in all performance metrics with confidence, according to the relevant results as revealed by the Tables IX - XI.

### C. FTSE100 (89 Stocks) Port3 in OR-Library

TABLE XII
PORT3 - MEAN STD, MEDIAN AND IQR FOR HV, SPREAD AND EPSILON

Problem		HYBRID 1	NSGAII
PORT3	HV. Mean and Std	$9.72\text{e-}01_{5.9\text{e-}03}$	$9.47\text{e-}01_{1.3\text{e-}02}$
	HV. Median and IQR	$9.74\text{e-}01_{8.7\text{e-}03}$	$9.49\text{e-}01_{1.7\text{e-}02}$
	SPREAD. Mean and Std	$8.94\text{e-}01_{1.1\text{e-}02}$	$9.67\text{e-}01_{4.0\text{e-}02}$
	SPREAD. Median and IQR	$8.93\text{e-}01_{1.9\text{e-}02}$	$9.65\text{e-}01_{5.5\text{e-}02}$
	EPSILON. Mean and Std	$3.63\text{e-}05_{7.8\text{e-}06}$	$6.97\text{e-}05_{1.7\text{e-}05}$
	EPSILON. Median and IQR	$3.44\text{e-}05_{1.1\text{e-}05}$	$6.68\text{e-}05_{2.2\text{e-}05}$

Where: Hybrid 1 = ES + NSGAII

TABLE XIII
PORT3 - BOXPLOTS FOR HV, SPREAD AND EPSILON
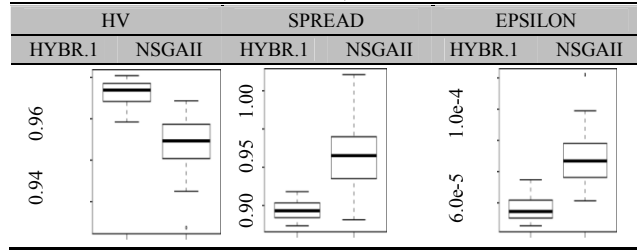
Where: Hybrid 1 = ES + NSGAII

TABLE XIV
WILCOXON TEST IN PORT3 FOR HV, SPREAD AND EPSILON

Problem		NSGAII
PORT3	HV. Mean and Std	▲
	HV. Median and IQR	▲
	SPREAD. Mean and Std	▲
	SPREAD. Median and IQR	▲
	EPSILON. Mean and Std	▲
	EPSILON. Median and IQR	▲

(HYBRID 1)

Where: Hybrid 1 = ES + NSGAII

The port3 problem uses data from FTSE100 index in the UK. The relevant results in Tables XII-XIV indicate that the proposed methodology generates better results with confidence for all performance metrics, compared with the results of NSGAII. The relevant results in Tables XII-XIV indicate that the Cardinality constrained Portfolio Selection Problem (CCPSP) in this particular case the port3 problem can be solved more efficiently by the synergy of an Expert System and a MOEA rather than the simple application of a stand-alone MOEA.

VI. ANALYSIS OF THE RESULTS

In this section, we analyze the results obtained by the implementation of the proposed Hybrid System to the solution of the Cardinality constrained Portfolio Selection Problem (CCPSP) for three different instances. Three well known performance indicators of MOEAs namely Hypervolume, Spread and Epsilon have been applied to assess the quality of the proposed Hybrid System. Examining the results of HV indicator, we notice that the Hybrid System performs better than the stand-alone NSGAII for the solution of the CCPSP.

The Wilcoxon rank-sum test validates that the observed difference in HV indicator performance between the Hybrid System and the stand-alone MOEA (i.e. NSGAII) is statistically significant with 95% confidence for all examined instances. Regarding the other two performance metrics, the Spread and Epsilon indicators, we also notice that the proposed Hybrid System outperforms with confidence the stand-alone NSGAII for all three examined test problems. Finally, the boxplots provide a graphical confirmation of the aforementioned results as shown in Tables VIII, XI and XIV.

It is clear from the presented experimental results that the proposed Hybrid System outperforms the stand-alone NSGAII in all three performance metrics with confidence in the solution of the CCPSP and generates higher quality solutions.

VII. CONCLUSIONS

Although there is an increase in papers' output in recent years regarding the implementation MOEAs techniques [8] to the solution of the cardinality constrained portfolio selection problem, still to the best of our knowledge have not been explored the potential benefits from the synergy between a specially designed Expert System and a MOEA for the solution of the cardinality constrained portfolio selection problem. The purpose of this study is to cover this gap in the relevant literature by presenting a Hybrid System for the solution of the cardinality constrained portfolio selection problem. The proposed Hybrid System is tested by using data from three different stock markets. The relevant results are evaluated by using three well-known performance metrics, namely HV, Spread and Epsilon indicators. In all three performance metrics (HV, Spread, Epsilon), the Hybrid System outperforms the stand-alone MOEA (i.e. the NSGAII) with confidence for all three examined instances.

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