An EOQ Model for Non-Instantaneous Deteriorating Items with Power Demand, Time Dependent Holding Cost, Partial Backlogging and Permissible Delay in Payments

M. Palanivel, R. Uthayakumar

Abstract—In this paper, Economic Order Quantity (EOQ) based model for non-instantaneous Weibull distribution deteriorating items with power demand pattern is presented. In this model, the holding cost per unit of the item per unit time is assumed to be an increasing linear function of time spent in storage. Here the retailer is allowed a trade-credit offer by the supplier to buy more items. Also in this model, shortages are allowed and partially backlogged. The backlogging rate is dependent on the waiting time for the next replenishment. This model aids in minimizing the total inventory cost by finding the optimal time interval and finding the optimal order quantity. The optimal solution of the model is illustrated with the help of numerical examples. Finally sensitivity analysis and graphical representations are given to demonstrate the model.

Keywords—Power demand pattern, Partial backlogging, Time dependent holding cost, Trade credit, Weibull deterioration.

I. INTRODUCTION

DETERIORATION plays a significant role in many inventory systems. Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreased usefulness. Most physical goods undergo decay or deterioration over time, examples being medicines, volatile liquids, blood banks, and so on. So decay or deterioration of physical goods in stock is a very realistic factor and there is a big need to consider this in inventory modeling. The first attempt to describe the optimal ordering policies for such items was made by Ghare and Schrader [8]. Philip [27] developed an inventory model with a three parameter Weibull distribution rate without considering shortages. Deb [5] derived inventory model with time-dependent deterioration rate. A detailed review of deteriorating inventory literatures is given by Goyal [11]. Many researchers assume that the deterioration of the items in inventory starts from the instant of their arrival in stock. In fact, most goods would have a span of maintaining quality or original condition (e.g. vegetables, fruit, fish, meat and so on), namely, during that period, there is

no deterioration occurring. Wu et al. [40] defined the phenomenon as ''non-instantaneous deterioration''. In the real world, this type of phenomenon exists commonly such as firsthand vegetables and fruits have a short span of maintaining fresh quality, in which there is almost no spoilage. Afterwards, some of the items will start to decay. For this kind of items, the assumption that the deterioration starts from the instant of arrival in stock may cause retailers to make inappropriate replenishment policies due to overvalue the total annual relevant inventory cost. Therefore, in the field of inventory management, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. In this direction Ouyang [23] developed an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Liao [15] studied an EOQ model with non instantaneous receipt and exponential deteriorating item under two level trade credits. Uthayakumar [37] formulated a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payment is developed by Geetha [7]. Joint control of inventory and its pricing for non-instantaneous deteriorating items under permissible delay in payments and partial backlogging is developed by Maihami [17]. Dye [6] investigated the effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Palanivel [24] developed the finite horizon EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand and partial backlogging under inflation.

In real life situations the demand of an item towards the beginning of a period, e.g., a week or a month, can be greater or smaller than the demand at the end of the period. Jalbar et al. [12] investigated a two-echelon inventory/ distribution system with power demand pattern and backorders. Datta and Pal [4] investigated an inventory system with power demand pattern for items with variable rate of deterioration. An Economic Order Quantity model for Weibull deteriorating items with power demand and partial backlogging have been studied by Tripathy and Pradhan [36].

In most of the inventory model, mentioned above, holding cost has been considered as a constant function. But, in real-life situations, when the deteriorating and perishable items such as food products are kept in storage, the more

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complicated the storage facilities and services needed, and therefore, the higher the holding cost. Therefore, the holding cost is always not a constant function. It's varying according to time. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like Naddor [22], Veen [38], Muhlemann [20], and Goh [9]. Alfares [2] proposed an inventory model with stock-level dependent demand rate and variable holding cost. Roy [29] developed an inventory model for deteriorating items with time varying holding cost and demand is price dependent. Mishra [19] developed a deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Shah [32] studied an optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Pando [26] developed the maximizing profits in an inventory model with both demand rate and holding cost per unit time dependent on the stock level. Pando [25] gave an economic lot-size model with non-linear holding cost hinging on time and quantity.

The suppliers offer delay in payment to the retailers to buy more items and the retailers can sell the item before the closing of the delay time. As a result, the retailers sell the items and earn interests. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. This provides opportunities to the retailers to accumulate revenue and earn interest by selling their items during the delay period. This permissible delay in payment provides benefit to the supplier by attracting new customers who consider it to be a type of price reduction and reduction in sells outstanding as some customers make payments on time in order to take advantage of permissible delay more frequently. In this direction, Goyal [10] extended the EOQ model under the conditions of permissible delay in payments. Liao [14] developed an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. Teng [35] developed another approach on the EOQ model under conditions of permissible delay in payments. Khanra [13] developed an EOQ model for deteriorating items with time dependent quadratic demand under permissible delay in payment. Recently, Musa and Sani [21] studied an inventory ordering policies of delayed deteriorating items under permissible delay in payments. Rezaei and Salimi [28] developed an economic order quantity and purchasing price model for items with imperfect quality when inspection shifts from buyer to supplier. Lin [16] developed a joint optimal ordering and delivery policy for an integrated supplier–retailer inventory model with trade credit and defective items.

When the shortage occurs, some customers are willing to wait for back order and others would turn to buy from other sellers. Inventory model of deteriorating items with time proportional backlogging rate have been developed by Chang [3]. Wang [39] studied shortages and partial backlogging of items. Min [18] derived a perishable inventory model under stock-dependent selling rate and shortage-dependent partial backlogging with capacity constraint. Sarkar [31] studied an

optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles. Ahmed [1] considered an inventory model with ramp type demand rate, partial backlogging and general deterioration rate. Sarkar [30] developed an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. Taleizadeh [33] provided an economic order quantity model with a known price increase and partial backordering. Tan [34] developed the discrete-in-time deteriorating inventory model with time-varying demand, variable deterioration rate and waiting-time-dependent partial backlogging.

In the present work, a deterministic inventory model for non-instantaneous deteriorating items with power demand pattern and permissible delay in payments are proposed in which the deterioration is a Weibull two parameter distribution, and holding cost is expressed as linearly increasing functions of time. Shortages are allowed and partially backlogged in this model. We have shown the suitable numerical examples to illustrate the model. Sensitivity analysis of the optimal solution with respect to major parameters of the system is carried out. To the author's best of knowledge, such type of model has not yet been discussed in the inventory literature.

The rest of the paper is organized as follows: In Section II, the notations and assumptions, which are used throughout this article, are described. In Section III, the mathematical model to minimize the total annual inventory cost is established. Section IV presents solution procedure to find the optimal time length and optimal order quantity. Numerical examples are provided in Section V to illustrate the theory and the solution procedure. This is followed by sensitivity analysis and conclusion.

II. NOTATIONS AND ASSUMPTIONS

To develop the mathematical model, the following assumptions are being made:

A. Notations

The following notations are used throughout this paper:

A The ordering cost per order.

C_2 The deterioration cost per unit per year.

C_3 The shortage cost for backlogged items per unit per year.

C_4 The unit cost of lost sales per unit.

p The purchasing cost per unit.

s The selling price per unit, with $s > p$.

I_e The interest earned per dollar per year.

I_c The interest charged in stock by the supplier.

M Trade credit period.

μ The life time of the items per cycle.

t_1 Length of time in which the inventory has no shortage,

$t_1 > \mu$.

T The length of the order cycle.

S The initial inventory level.

Q The order size per cycle.

$q(t)$ The inventory level at any time $t, t \geq 0$.

TC The total cost of the system.

B. Assumptions

To develop the mathematical model, the following assumptions are being made:

1. A single item is considered over the fixed period T which is subject to Weibull deterioration rate.
2. Deterioration takes place after the life time of items.
3. There is no replenishment or repair of deteriorated items takes place in a given cycle.
4. $\theta(t) = \alpha \beta t^{\beta-1}$ is the Weibull two parameter deterioration, where $0 < \alpha < 1, \beta > 0$ are called scale and shape parameter respectively.
5. The replenishment takes place at an infinite rate.
6. The lead time is zero.
7. $C_1(t) = a + bt, a \geq 0, b \geq 0$ is the holding cost excluding interest charges, which is linear function of time.
8. $D(t)$ is the demand rate at any time t such that

$D(t) = \dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}$ where d is a positive constant, n may be any positive number, T is the planning horizon.

9. During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock.
10. Shortages are allowed and during stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is, $B(t) = \dfrac{1}{1 + \delta(T - t)}$, δ is backlogging parameter $0 < \delta < 1$ and $(T - t)$ is waiting time $(t_1 \leq t \leq T)$. And the remaining fraction $1 - B(t)$ is lost.

III. FORMULATION AND SOLUTION OF THE MODEL

The inventory system is developed as follows: Q units of items arrive at the inventory system at the beginning of each cycle. During the time interval $[0, \mu]$, the inventory level is decreasing only due to demand rate. The inventory level is dropping to zero owing to demand and deterioration during the time interval $[\mu, t_1]$. Finally, a shortage occurs due to demand and partial backlogging during the time interval

$[t_1, T]$. The behaviour of the inventory model is demonstrated in Fig. 1.
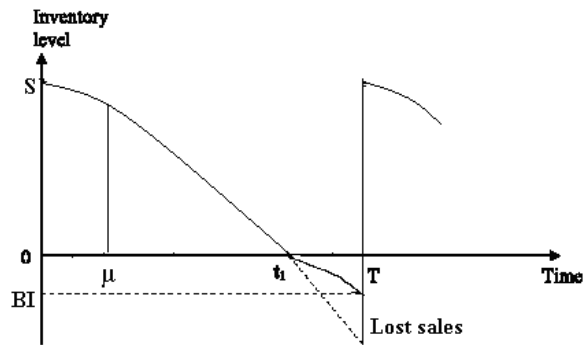

Fig. 1 Graphical representation of the inventory system

Based on the above description, during the time interval $[0, \mu]$, the differential equation representing the inventory status is given by

$$\frac{dq(t)}{dt} = -D(t), \qquad 0 \leq t \leq \mu \qquad (1)$$

With the condition $q(0) = S$, the solution of (1) is

$$q(t) = S - \frac{dt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \qquad 0 \leq t \leq \mu \qquad (2)$$

In the second interval $[\mu, t_1]$, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dq(t)}{dt} + \theta(t)q(t) = -D(t), \quad \mu \leq t \leq t_1 \qquad (3)$$

With the condition $q(t_1) = 0$, taking the first two terms of the exponential series and disregarding the terms containing α^2 and then integrating we get the solution of (3) is

$$q(t) = \frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - t^{1/n})(1 - \alpha t^{\beta}) + \frac{\alpha(t_1^{(1/n)+\beta} - t^{(1/n)+\beta})}{1 + n\beta} \right], \mu \leq t \leq t_1 \quad (4)$$

Put $t = \mu$ in (2) and (4) we find the value of S as

$$S = \frac{d\mu^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - \mu^{1/n})(1 - \alpha\mu^{\beta}) + \frac{\alpha(t_1^{(1/n)+\beta} - \mu^{(1/n)+\beta})}{1 + n\beta} \right] \quad (5)$$

Substituting (5) in (2) we get

$$q(t) = \frac{d}{T^{\frac{1}{n}}}\left[\mu^{1/n} - t^{1/n} + (t_1^{1/n} - \mu^{1/n})(1 - \alpha\mu^\beta) + \frac{\alpha(t_1^{(1/n)+\beta} - \mu^{(1/n)+\beta})}{1 + n\beta}\right]$$

$$0 \le t \le \mu \qquad (6)$$

During the third interval $[t_1, T]$, shortage occurred and the demand is partially backlogged. That is, the inventory level at time t is governed by the following differential equation:

$$\frac{dq(t)}{dt} = \frac{-D(t)}{1 + \delta(T - t)}, \qquad t_1 \le t \le T \qquad (7)$$

With the condition $q(t_1) = 0$, the solution of (7) is

$$q(t) = \frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - t^{1/n})(1 - \delta T) + \frac{\delta(t_1^{(1/n)+1} - t^{(1/n)+1})}{1 + n}\right],$$

$$t_1 \le t \le T \qquad (8)$$

The maximum backordered inventory BI is obtained at $t = T$, then from (8)

$$BI = -q(T) = -\frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - T^{1/n})(1 - \delta T) + \frac{\delta(t_1^{(1/n)+1} - T^{(1/n)+1})}{1 + n}\right] \quad (9)$$

Thus the order size during total time interval $[0, T]$ is,

$$Q = S + BI.$$

$$= \frac{d\mu^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - \mu^{1/n})(1 - \alpha\mu^\beta) + \frac{\alpha(t_1^{(1/n)+\beta} - \mu^{(1/n)+\beta})}{1 + n\beta}\right]$$

$$- \frac{d}{T^{\frac{1}{n}}}\left[(t_1^{1/n} - T^{1/n})(1 - \delta T) + \frac{\delta(t_1^{(1/n)+1} - T^{(1/n)+1})}{1 + n}\right] \qquad (10)$$

The holding cost HC during the period $[0, T]$ is

$$HC = \int_0^\mu C_1(t)q(t)dt + \int_\mu^{t_1} C_1(t)q(t)dt$$

$$HC = \frac{d}{T^{\frac{1}{n}}}\left[-\frac{a\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} - \frac{b\alpha t_1^{1/n}\mu^{\beta+2}\beta}{2(\beta+2)}\right.$$

$$+ \frac{a\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1} + \frac{b\alpha\mu^{(1/n)+\beta+2}\beta}{2((1/n)+\beta+2)}$$

$$+ \frac{a\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + \frac{b\alpha t_1^{(1/n)+\beta+2}\beta}{2n(\beta+2)((1/n)+\beta+2)}$$

$$\left. + \frac{at_1^{(1/n)+1}}{n+1} + \frac{bt_1^{(1/n)+2}}{2(2n+1)}\right] \qquad (11)$$

The deteriorating cost DC during the period $[0, T]$ is

$$DC = C_2 \int_\mu^{t_1} \theta(t)q(t)dt$$

Ignoring the terms containing α^2 or higher degree of it since $0 < \alpha < 1$, we get

$$DC = \frac{C_2 d}{T^{\frac{1}{n}}}\left[\frac{\alpha t_1^{(1/n)+\beta}}{1 + n\beta} + \frac{n\alpha\beta\mu^{(1/n)+\beta}}{1 + n\beta} - \alpha t_1^{(1/n)}\mu^\beta\right] \qquad (12)$$

Total shortage cost SC during the period $[0, T]$ is given by

$$SC = -C_3 \int_{t_1}^T q(t)dt$$

$$SC = -C_3 \frac{d}{T^{\frac{1}{n}}}\left[(T - \delta T^2)t_1^{1/n} + t_1^{(1/n)+1}\left(\frac{2\delta T - 1}{1 + n}\right)\right.$$

$$\left. - \frac{\delta t_1^{(1/n)+2}}{1 + 2n} + \frac{2n^2\delta T^{(1/n)+2}}{(1+n)(1+2n)} - \frac{n T^{(1/n)+1}}{(1+n)}\right] \qquad (13)$$

The lost sales cost LC during the period $[0, T]$ is

$$LC = C_4 \int_{t_1}^T \left(1 - \frac{1}{1 + \delta(T - t)}\right)D(t)dt = C_4 \frac{d\delta}{T^{\frac{1}{n}}}\left[\frac{n T^{(1/n)+1}}{(1+n)} + \frac{t_1^{(1/n)+1}}{(1+n)} - t_1^{1/n}T\right] \quad (14)$$

The total average cost of the system per unit time is given by

$$TC = \begin{cases} TC_1, 0 < M \le \mu \\ TC_2, \mu < M \le t_1 \\ TC_3, M > t_1 \end{cases} \qquad (15)$$

where TC_1, TC_2, and TC_3 are discussed as follows.
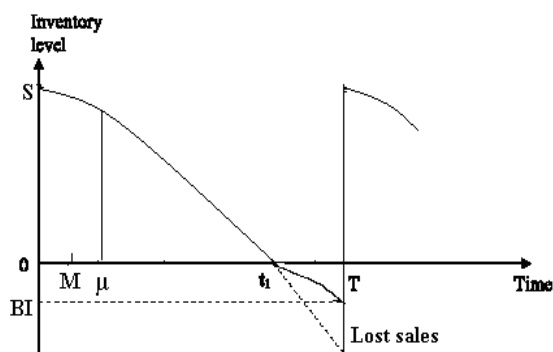
Case 1: $0 < M \le \mu$

Fig. 2 Inventory level as a function of time for case 1 $(0 < M \le \mu)$

In this case the length of delay in payment (M) is absolutely less than the length with no deterioration (μ). Since the interest is payable during the time $(t_1 - M)$, the interest payable in any cycle $[0, T]$ is

$$
\begin{aligned}
IC_1 &= pI_c \int_M^{t_1} q(t)dt = pI_c\left[\int_M^{\mu} q(t)dt + \int_{\mu}^{t_1} q(t)dt\right] \\
&= pI_c \frac{d}{T^{\frac{1}{n}}}\left[-\frac{\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} + \frac{\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1}\right.\\
&\quad + \frac{\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + \frac{t_1^{(1/n)+1}}{n+1} - \frac{\alpha\mu^{(1/n)+\beta}M\beta}{(1/n)+\beta}\\
&\quad \left. + t_1^{1/n}M(\alpha\mu^{\beta}-1) + \frac{M^{(1/n)+1}}{(1/n)+1} - \frac{\alpha t_1^{(1/n)+\beta}M}{1+n\beta}\right] \quad (16)
\end{aligned}
$$

During the permissible delay period when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with rate I_e. Therefore, the interest earned in the cycle period $[0, T]$ is

$$
IE_1 = sI_e \int_0^{t_1} D(t)tdt = sI_e \frac{d}{T^{\frac{1}{n}}}\frac{t_1^{(1/n)+1}}{n+1} \quad (17)
$$

Total average cost per cycle = replenishment cost + inventory holding cost + deterioration cost + shortage cost + lost sales cost + interest payable during the permissible delay period – interest earned during the cycle.

So, the total variable cost per unit time is

$$
TC_1 = \frac{1}{T}\left[A + HC + DC + SC + LC + IC_1 - IE_1\right]
$$

$$
\begin{aligned}
&= \frac{A}{T} + \frac{d}{T^{\frac{1}{n}+1}}\left[-\frac{a\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} - \frac{b\alpha t_1^{1/n}\mu^{\beta+2}\beta}{2(\beta+2)} + \frac{a\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1}\right.\\
&\quad + \frac{b\alpha\mu^{(1/n)+\beta+2}\beta}{2((1/n)+\beta+2)} + \frac{a\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + \frac{a t_1^{(1/n)+1}}{n+1}\\
&\quad \left. + \frac{b\alpha t_1^{(1/n)+\beta+2}\beta}{2n(\beta+2)((1/n)+\beta+2)} + \frac{b t_1^{(1/n)+2}}{2(2n+1)}\right]\\
&\quad + \frac{C_2 d}{T^{\frac{1}{n}+1}}\left[\frac{\alpha t_1^{(1/n)+\beta}}{1+n\beta} + \frac{n\alpha\beta\mu^{(1/n)+\beta}}{1+n\beta} - \alpha t_1^{(1/n)}\mu^{\beta}\right]\\
&\quad - \frac{C_3 d}{T^{\frac{1}{n}+1}}\left[(T-\delta T^2)t_1^{1/n} + t_1^{(1/n)+1}\left(\frac{2\delta T-1}{1+n}\right) - \frac{\delta t_1^{(1/n)+2}}{1+2n}\right.\\
&\quad \left. + \frac{2n^2\delta T^{(1/n)+2}}{(1+n)(1+2n)} - \frac{n\,T^{(1/n)+1}}{(1+n)}\right] + C_4 \frac{d\delta}{T^{\frac{1}{n}+1}}\left[\frac{n\,T^{(1/n)+1}}{(1+n)} + \frac{t_1^{(1/n)+1}}{(1+n)} - t_1^{1/n}T\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ pI_c \frac{d}{T^{\frac{1}{n}}}\left[\frac{t_1^{(1/n)+1}}{n+1} - \frac{\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} + \frac{\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1}\right.\\
&\quad + \frac{\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + t_1^{1/n}M(\alpha\mu^{\beta}-1) - \frac{\alpha\mu^{(1/n)+\beta}M\beta}{(1/n)+\beta}\\
&\quad \left. + \frac{M^{(1/n)+1}}{(1/n)+1} - \frac{\alpha t_1^{(1/n)+\beta}M}{1+n\beta}\right] - sI_e\frac{d}{T^{\frac{1}{n}}}\frac{t_1^{(1/n)+1}}{n+1} \quad (18)
\end{aligned}
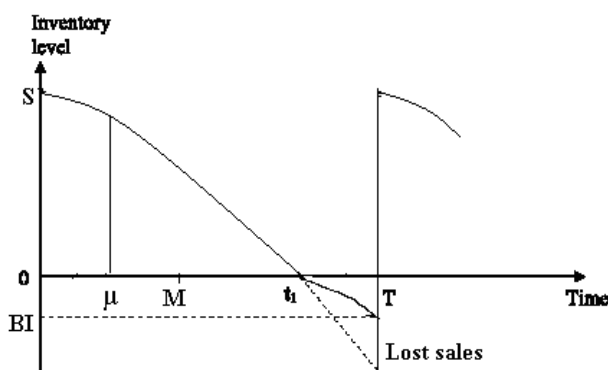$$

Case 2: $\mu < M \le t_1$

Fig. 3 Inventory level as a function of time for case $(\mu < M \le t_1)$

In this case the period of delay in payment (M) is more than the period with no deterioration (μ) but less than the period with positive inventory (t_1).

The interest payable in any cycle $[0, T]$ is

$$
\begin{aligned}
IC_2 &= pI_c \int_M^{t_1} q(t)dt = pI_c \frac{d}{T^{\frac{1}{n}}}\left[\frac{t_1^{(1/n)+1}}{n+1} + \frac{\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)}\right.\\
&\quad - \frac{\alpha M^{(1/n)+\beta+1}\beta}{((1/n)+\beta)((1/n)+\beta+1)} + \alpha t_1^{1/n}M\left(\frac{M^{\beta}}{\beta+1} - \frac{t_1^{\beta}}{1+n\beta} - \frac{1}{\alpha}\right)\\
&\quad \left. + \frac{M^{(1/n)+1}}{(1/n)+1}\right] \quad (19)
\end{aligned}
$$

Interest earned in the cycle period $[0, T]$ is

$$
IE_2 = sI_e \int_0^{t_1} D(t)tdt = sI_e \frac{d}{T^{\frac{1}{n}}}\frac{t_1^{(1/n)+1}}{n+1} \quad (20)
$$

Total average cost per cycle = replenishment cost + inventory holding cost + deterioration cost + shortage cost + lost sales cost + interest payable during the permissible delay period – interest earned during the cycle.

So, the total variable cost per unit time is

$$TC_2 = \frac{1}{T}[A + HC + DC + SC + LC + IC_2 - IE_2] = \frac{A}{T} +$$

$$\frac{d}{T^{\frac{1}{n}+1}}\left[-\frac{a\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} - \frac{b\alpha t_1^{1/n}\mu^{\beta+2}\beta}{2(\beta+2)} + \frac{a\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1} \right.$$

$$\left. + \frac{b\alpha\mu^{(1/n)+\beta+2}\beta}{2((1/n)+\beta+2)} + \frac{a\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + \frac{at_1^{(1/n)+1}}{n+1} \right.$$

$$\left. + \frac{b\alpha t_1^{(1/n)+\beta+2}\beta}{2n(\beta+2)((1/n)+\beta+2)} + \frac{bt_1^{(1/n)+2}}{2(2n+1)} \right]$$

$$+ \frac{C_2 d}{T^{\frac{1}{n}+1}}\left[\frac{\alpha t_1^{(1/n)+\beta}}{1+n\beta} + \frac{n\alpha\beta\mu^{(1/n)+\beta}}{1+n\beta} - \alpha t_1^{(1/n)}\mu^{\beta} \right]$$

$$- \frac{C_3 d}{T^{\frac{1}{n}+1}}\left[(T - \delta T^2)t_1^{1/n} + t_1^{(1/n)+1}\left(\frac{2\delta T-1}{1+n} \right) - \frac{\delta t_1^{(1/n)+2}}{1+2n} + \frac{2n^2\delta T^{(1/n)+2}}{(1+n)(1+2n)} \right.$$

$$\left. - \frac{n\,T^{(1/n)+1}}{(1+n)} \right] + C_4 \frac{d\delta}{T^{\frac{1}{n}+1}}\left[\frac{n\,T^{(1/n)+1}}{(1+n)} + \frac{t_1^{(1/n)+1}}{(1+n)} - t_1^{1/n}T \right] + pI_c \frac{d}{T^{\frac{1}{n}}}$$

$$\left[\frac{t_1^{(1/n)+1}}{n+1} + \frac{\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} - \frac{\alpha M^{(1/n)+\beta+1}\beta}{((1/n)+\beta)((1/n)+\beta+1)} \right.$$

$$\left. + \frac{M^{(1/n)+1}}{1/n+1} + \alpha t_1^{1/n}M\left(\frac{M^{\beta}}{\beta+1} - \frac{t_1^{\beta}}{1+n\beta} - \frac{1}{\alpha} \right) \right] - sI_e \frac{d}{T^{\frac{1}{n}}}\frac{t_1^{(1/n)+1}}{n+1} \qquad (21)$$
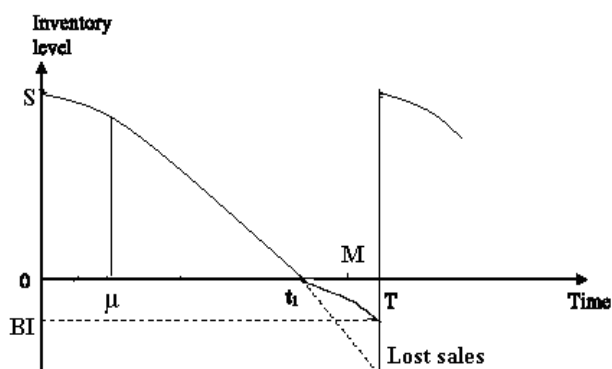
Case 3: $M > t_1$

Fig. 4. Inventory level as a function of time for case 3 $(M > t_1)$

In this case, the period of delay in payment (M) is more than period with positive inventory (t_1). In this case the retailer earns interest on the sales revenue up to the permissible delay period and no interest is payable during the period for the item kept in stock. Interest earned for the time period $[0, T]$ is

$$IE_3 = sI_e\left[\int_0^{t_1} D(t)t\,dt + (M - t_1)\int_0^{t_1} D(t)\,dt \right] = sI_e \frac{d}{T^{\frac{1}{n}}}\left[Mt_1^{1/n} - \frac{t_1^{(1/n)+1}n}{n+1} \right] \qquad (22)$$

Total average cost per cycle = replenishment cost + inventory holding cost + deterioration cost + shortage cost + lost sales cost − interest earned during the cycle.

So, the total variable cost per unit time is

$$TC_3 = \frac{1}{T}[A + HC + DC + SC + LC - IE_3] = \frac{A}{T} +$$

$$\frac{d}{T^{\frac{1}{n}+1}}\left[-\frac{a\alpha t_1^{1/n}\mu^{\beta+1}\beta}{\beta+1} - \frac{b\alpha t_1^{1/n}\mu^{\beta+2}\beta}{2(\beta+2)} + \frac{a\alpha\mu^{(1/n)+\beta+1}\beta}{(1/n)+\beta+1} \right.$$

$$\left. + \frac{b\alpha\mu^{(1/n)+\beta+2}\beta}{2((1/n)+\beta+2)} + \frac{a\alpha t_1^{(1/n)+\beta+1}\beta}{n(\beta+1)((1/n)+\beta+1)} + \frac{at_1^{(1/n)+1}}{n+1} \right.$$

$$\left. + \frac{b\alpha t_1^{(1/n)+\beta+2}\beta}{2n(\beta+2)((1/n)+\beta+2)} + \frac{bt_1^{(1/n)+2}}{2(2n+1)} \right]$$

$$+ \frac{C_2 d}{T^{\frac{1}{n}+1}}\left[\frac{\alpha t_1^{(1/n)+\beta}}{1+n\beta} + \frac{n\alpha\beta\mu^{(1/n)+\beta}}{1+n\beta} - \alpha t_1^{(1/n)}\mu^{\beta} \right]$$

$$- \frac{C_3 d}{T^{\frac{1}{n}+1}}\left[(T - \delta T^2)t_1^{1/n} + t_1^{(1/n)+1}\left(\frac{2\delta T-1}{1+n} \right) - \frac{\delta t_1^{(1/n)+2}}{1+2n} + \frac{2n^2\delta T^{(1/n)+2}}{(1+n)(1+2n)} \right.$$

$$\left. - \frac{n\,T^{(1/n)+1}}{(1+n)} \right] + C_4 \frac{d\delta}{T^{\frac{1}{n}+1}}\left[\frac{n\,T^{(1/n)+1}}{(1+n)} + \frac{t_1^{(1/n)+1}}{(1+n)} - t_1^{1/n}T \right]$$

$$- sI_e \frac{d}{T^{\frac{1}{n}}}\left[Mt_1^{1/n} - \frac{t_1^{(1/n)+1}n}{n+1} \right] \qquad (23)$$

While $M = \mu$, the result of TC_1 is equal to TC_2. Similarly while $M = t_1$, the result of TC_2 is equal to TC_3.

IV. SOLUTION PROCEDURE

In order to find the optimal solution t_1^* and to minimize the annual total relevant cost, we take the first and second order derivatives of $TC_i(t_1)$ with respect to t_1, where $i = \{1, 2, 3\}$. In other words, the necessary and sufficient conditions for minimization of $TC_i(t_1)$ are respectively $\frac{dTC_i(t_1)}{dt_1} = 0$ and

$\frac{d^2 TC_i(t_1)}{dt_1^2} > 0$ where $i = \{1, 2, 3\}$.

Case 1: $0 < M \le \mu$.

The necessary and sufficient conditions to minimize $TC_1(t_1)$ are respectively $\frac{dTC_1(t_1)}{dt_1} = 0$ and $\frac{d^2 TC_1(t_1)}{dt_1^2} > 0$.

Now $\frac{dTC_1(t_1)}{dt_1} = 0$ gives the following non − linear equations in t_1.

$$\frac{a\alpha\beta}{\beta+1}\left[-t_1^{(1/n)-1}\mu^{\beta+1}+t_1^{(1/n)+\beta}\right]+at_1^{1/n}+\frac{bt_1^{(1/n)+1}}{2}$$

$$+\frac{b\alpha\beta}{2(\beta+2)}\left[-t_1^{(1/n)-1}\mu^{\beta+2}+t_1^{(1/n)+\beta+1}\right]$$

$$+C_2\left[\alpha t_1^{(1/n)+\beta-1}-\alpha t_1^{(1/n)-1}\mu^{\beta}\right]$$

$$-C_3\left[(T-\delta T^2)t_1^{(1/n)-1}+(2\delta T-1)t_1^{1/n}-\delta t_1^{(1/n)+1}\right]$$

$$+C_4\delta\left[t_1^{1/n}-Tt_1^{(1/n)-1}\right]+pI_c\left[-\frac{\alpha t_1^{(1/n)-1}\mu^{\beta+1}\beta}{\beta+1}+\frac{\alpha t_1^{(1/n)+\beta}\beta}{(\beta+1)}\right.$$

$$\left.+t_1^{1/n}+t_1^{(1/n)-1}M(\alpha\mu^{\beta}-1)-\alpha t_1^{(1/n)+\beta-1}M\right]-sI_et_1^{1/n}=0 \quad (24)$$

and

$$\frac{d^2TC_1(t_1)}{dt_1^2}=\frac{d}{nT^{(1/n)+1}}\left\{\frac{a\alpha\beta}{\beta+1}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+1}\right.\right.$$

$$\left.+\left(\frac{1}{n}+\beta\right)t_1^{(1/n)+\beta-1}\right]+\frac{b\alpha\beta}{2(\beta+2)}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+2}\right.$$

$$\left.+\left(\frac{1}{n}+\beta+1\right)t_1^{(1/n)+\beta}\right]+\frac{1}{n}at_1^{(1/n)-1}+\left(\frac{1}{n}+1\right)\frac{bt_1^{(1/n)}}{2}$$

$$+C_2\left[\left(\frac{1}{n}+\beta-1\right)\alpha t_1^{(1/n)+\beta-2}+\left(1-\frac{1}{n}\right)\alpha t_1^{(1/n)-2}\mu^{\beta}\right]$$

$$+C_3\left[\left(1-\frac{1}{n}\right)(T-\delta T^2)t_1^{(1/n)-2}+\frac{1}{n}(1-2\delta T)t_1^{(1/n)-1}\right.$$

$$\left.+\delta\left(\frac{1}{n}+1\right)t_1^{1/n}\right]+C_4\delta\left[\frac{1}{n}t_1^{(1/n)-1}+T\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\right]$$

$$+pI_c\left[\left(1-\frac{1}{n}\right)\frac{\alpha t_1^{(1/n)-2}\mu^{\beta+1}\beta}{\beta+1}+\left(\frac{1}{n}+\beta\right)\frac{\alpha t_1^{(1/n)+\beta-1}\beta}{(\beta+1)}\right.$$

$$\left.+\frac{1}{n}t_1^{(1/n)-1}+\left(\frac{1}{n}-1\right)t_1^{(1/n)-2}M(\alpha\mu^{\beta}-1)\right.$$

$$\left.\left.-\alpha\left(\frac{1}{n}+\beta-1\right)t_1^{(1/n)+\beta-2}M\right]-sI_e\frac{1}{n}t_1^{(1/n)-1}\right\} \quad (25)$$

Since $\delta<1$ and $\alpha<1$ then $\dfrac{d^2TC_1(t_1)}{dt_1^2}>0$ when $\dfrac{1}{n}\leq 1$.

By solving (24) the optimal value of $t_1=t_1^*$ can be obtained and then from (5), (10) and (18), the optimal value of $S=S^*$, $Q=Q^*$ and $TC=TC_1^*$ can be found out respectively.

Case 2: $\mu<M\leq t_1$

The necessary and sufficient conditions to minimize $TC_2(t_1)$ are respectively $\dfrac{dTC_2(t_1)}{dt_1}=0$ and $\dfrac{d^2TC_2(t_1)}{dt_1^2}>0$.

Now $\dfrac{dTC_2(t_1)}{dt_1}=0$ gives the following non – linear equations in t_1.

$$\frac{a\alpha\beta}{\beta+1}\left[-t_1^{(1/n)-1}\mu^{\beta+1}+t_1^{(1/n)+\beta}\right]$$

$$+\frac{b\alpha\beta}{2(\beta+2)}\left[-t_1^{(1/n)-1}\mu^{\beta+2}+t_1^{(1/n)+\beta+1}\right]+at_1^{1/n}+\frac{bt_1^{(1/n)+1}}{2}$$

$$+C_2\left[\alpha t_1^{(1/n)+\beta-1}-\alpha t_1^{(1/n)-1}\mu^{\beta}\right]$$

$$-C_3\left[(T-\delta T^2)t_1^{(1/n)-1}+(2\delta T-1)t_1^{1/n}-\delta t_1^{(1/n)+1}\right]$$

$$+C_4\delta\left[t_1^{1/n}-Tt_1^{(1/n)-1}\right]$$

$$+pI_c\left[t_1^{1/n}+\frac{\alpha\beta t_1^{(1/n)+\beta}}{\beta+1}+\frac{\alpha t_1^{(1/n)-1}M^{\beta+1}}{\beta+1}\right.$$

$$\left.-\alpha Mt_1^{(1/n)+\beta-1}-t_1^{(1/n)-1}M\right]-sI_et_1^{1/n}=0 \quad (26)$$

and

$$\frac{d^2TC_2(t_1)}{dt_1^2}=\frac{d}{nT^{(1/n)+1}}\left\{\frac{a\alpha\beta}{\beta+1}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+1}\right.\right.$$

$$\left.+\left(\frac{1}{n}+\beta\right)t_1^{(1/n)+\beta-1}\right]+\frac{b\alpha\beta}{2(\beta+2)}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+2}\right.$$

$$\left.+\left(\frac{1}{n}+\beta+1\right)t_1^{(1/n)+\beta}\right]+\frac{1}{n}at_1^{(1/n)-1}+\left(\frac{1}{n}+1\right)\frac{bt_1^{(1/n)}}{2}$$

$$+C_2\left[\left(\frac{1}{n}+\beta-1\right)\alpha t_1^{(1/n)+\beta-2}+\left(1-\frac{1}{n}\right)\alpha t_1^{(1/n)-2}\mu^{\beta}\right]$$

$$+C_3\left[\left(1-\frac{1}{n}\right)(T-\delta T^2)t_1^{(1/n)-2}+\frac{1}{n}(1-2\delta T)t_1^{(1/n)-1}+\delta\left(\frac{1}{n}+1\right)t_1^{1/n}\right]$$

$$+C_4\delta\left[\frac{1}{n}t_1^{(1/n)-1}+T\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\right]$$

$$+pI_c\left[\left(\frac{1}{n}-1\right)\frac{\alpha t_1^{(1/n)-2}M^{\beta+1}}{\beta+1}+\left(\frac{1}{n}+\beta\right)\frac{\alpha t_1^{(1/n)+\beta-1}\beta}{(\beta+1)}\right.$$

$$\left.+\frac{1}{n}t_1^{(1/n)-1}-\left(\frac{1}{n}+\beta-1\right)t_1^{(1/n)+\beta-2}M\alpha\right.$$

$$\left.\left.+\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}M\right]-\frac{1}{n}sI_et_1^{(1/n)-1}\right\} \quad (27)$$

Since $\delta<1$ and $\alpha<1$ then $\dfrac{d^2TC_1(t_1)}{dt_1^2}>0$ when $\dfrac{1}{n}\leq 1$.

By solving (26) the optimal value of $t_1=t_1^*$ can be obtained and then from (5), (10) and (21), the optimal value of $S=S^*$, $Q=Q^*$ and $TC=TC_2^*$ can be found out respectively.

Case 3: $M > t_1$

The necessary and sufficient conditions to minimize $TC_3(t_1)$ are respectively $\dfrac{dTC_3(t_1)}{dt_1} = 0$ and $\dfrac{d^2TC_3(t_1)}{dt_1^2} > 0$.

Now $\dfrac{dTC_3(t_1)}{dt_1} = 0$ gives the following non – linear equations in t_1.

$$\frac{a\alpha\beta}{\beta+1}\left[-t_1^{(1/n)-1}\mu^{\beta+1} + t_1^{(1/n)+\beta}\right] + at_1^{1/n} + \frac{bt_1^{(1/n)+1}}{2}$$

$$+ \frac{b\alpha\beta}{2(\beta+2)}\left[-t_1^{(1/n)-1}\mu^{\beta+2} + t_1^{(1/n)+\beta+1}\right]$$

$$+ C_2\left[\alpha t_1^{(1/n)+\beta-1} - \alpha t_1^{(1/n)-1}\mu^{\beta}\right]$$

$$- C_3\left[(T - \delta T^2)t_1^{(1/n)-1} + (2\delta T - 1)t_1^{1/n} - \delta t_1^{(1/n)+1}\right]$$

$$+ C_4\delta\left[t_1^{1/n} - Tt_1^{(1/n)-1}\right] - sI_e\left[Mt_1^{(1/n)-1} - t_1^{1/n}n\right] = 0 \quad (28)$$

and

$$\frac{d^2TC_3(t_1)}{dt_1^2} = \frac{d}{nT^{(1/n)+1}}\left\{\frac{a\alpha\beta}{\beta+1}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+1}\right.\right.$$

$$+ \left(\frac{1}{n}+\beta\right)t_1^{(1/n)+\beta-1}\right] + \frac{b\alpha\beta}{2(\beta+2)}\left[\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\mu^{\beta+2}\right.$$

$$+ \left(\frac{1}{n}+\beta+1\right)t_1^{(1/n)+\beta}\right] + \frac{1}{n}at_1^{(1/n)-1} + \left(\frac{1}{n}+1\right)\frac{bt_1^{(1/n)}}{2}$$

$$+ C_2\left[\left(\frac{1}{n}+\beta-1\right)\alpha t_1^{(1/n)+\beta-2} + \left(1-\frac{1}{n}\right)\alpha t_1^{(1/n)-2}\mu^{\beta}\right]$$

$$+ C_3\left[\left(1-\frac{1}{n}\right)(T-\delta T^2)t_1^{(1/n)-2}\right.$$

$$+ \frac{1}{n}(1-2\delta T)t_1^{(1/n)-1} + \delta\left(\frac{1}{n}+1\right)t_1^{1/n}\right]$$

$$+ C_4\delta\left[\frac{1}{n}t_1^{(1/n)-1} + T\left(1-\frac{1}{n}\right)t_1^{(1/n)-2}\right]$$

$$+ sI_e\left[\left(1-\frac{1}{n}\right)Mt_1^{(1/n)-2} + t_1^{(1/n)-1}\right]\right\} \quad (29)$$

Since $\delta < 1$ and $\alpha < 1$ then $\dfrac{d^2TC_1(t_1)}{dt_1^2} > 0$ when $\dfrac{1}{n} \leq 1$.

By solving (28) the optimal value of $t_1 = t_1^*$ can be obtained and then from (5), (10) and (23), the optimal value of $S = S^*$, $Q = Q^*$ and $TC = TC_3^*$ can be found out respectively.

V. NUMERICAL EXAMPLES

The numerical examples given below cover all the three cases that arise in the model.

Example 1

Consider an inventory system with the following data: α = 0.1; β = 2; δ = 0.2; n = 4; d = 60; T = 1; A = 200; C_2 = 10; C_3 = 4; C_4 = 8; a = 0.4; b = 0.6; M = 0.2; μ = 0.4; I_e = 0.10; I_c = 0.15; p = 15; s = 18 in appropriate units.

Then we get the optimal values as t_1^* = 0.8143595, S^* = 57.1841, Q^* = 60.1295, TC_1^* = 3849.5908 in appropriate units. Fig. 5 shows that the function TC_1 is convex with respect to t_1.
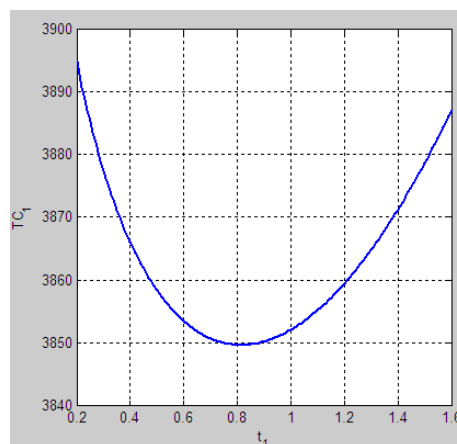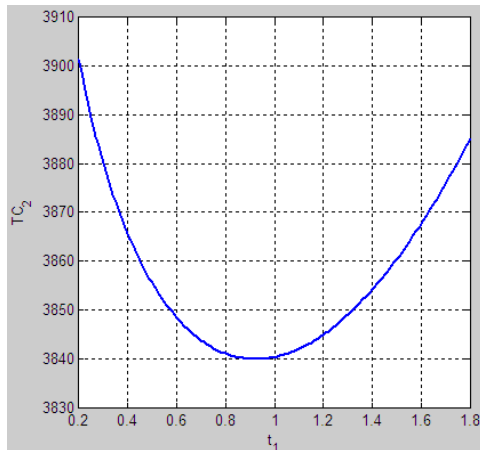

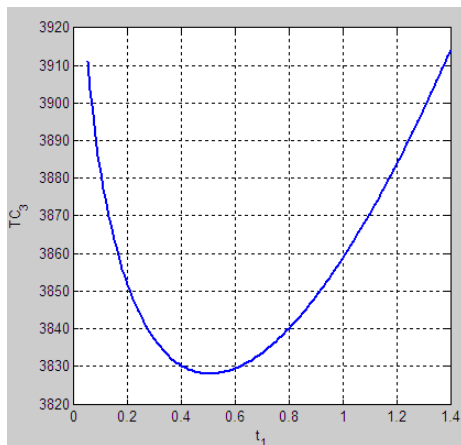Fig. 5 The total cost (Example 1) with respect to t_1.

Example 2

Consider an inventory system with the following data: α = 0.1; β = 2; δ = 0.2; n = 4; d = 60; T = 1; A = 200; C_2 = 10; C_3 = 4; C_4 = 8; a = 0.4; b = 0.6; M = 0.6; μ = 0.4; I_e = 0.10; I_c = 0.15; p = 15; s = 18 in appropriate units.

Then we get the optimal values as t_1^* = 0.9224980, S^* = 59.0958, Q^* = 60.2844, TC_2^* = 3839.8712 in appropriate units. The graph (Fig. 6) shows that the function TC_2 is convex with respect to t_1.

Fig. 6 The total cost (Example 2) with respect to t_1.

Example 3

Consider an inventory system with the following data: α = 0.1; β = 2; δ = 0.2; n = 4; d = 60; T = 1; A = 200; C$_2$ = 10; C$_3$ = 4; C$_4$ = 8; a = 0.4; b = 0.6; M = 0.8; μ = 0.4; I$_e$ = 0.10; I$_c$ = 0.15; p = 15; s = 18 in appropriate units.

Then we get the optimal values as $t_1^* = 0.5053002$, $S^* = 50.5997$, $Q^* = 59.5077$, $TC_3^* = 3828.0160$ in appropriate units. The graph (Fig. 7) shows that the function TC_3 is convex with respect to t_1.

Fig. 7 The total cost (Example 3) with respect to t_1.

Moreover, if μ = 0, this model becomes the instantaneous deteriorating item case, and the optimal total cost for the cases 2 & 3 can be found as $TC_2^* = 3842.5002$ and $TC_3^* = 3829.3129$ respectively. It can be seen that there is a decrease in total cost from the non-instantaneous deteriorating item model. This implies that if the retailer can convert the instantaneously deteriorating items to non-instantaneous deteriorating items by improving stock control, then the total cost per unit time will decrease. Also, when the supplier does not provide a credit period, the optimal retailer total cost can be found as $TC^* = 3856.4692$. It can be seen that optimal total

cost increases. So, retailers should try to get credit periods for their payments and if they wish to decrease their total cost.

VI. SENSITIVITY ANALYSIS

We now study the effects of changes in the values of the system parameters α, β, δ, μ, a, b and M on the optimal length of time in which there is no inventory shortage t_1^*, the optimal initial inventory level S^*, the optimal order quantity per cycle Q^* and the optimal total average cost TC^*. The sensitivity analysis is performed by changing each of the parameters by -60%, -40%, -20%, +20%, +40%, +60%, taking one parameter at a time and keeping the remaining parameters unchanged. The analysis is based on the Example 2 and the results are shown in Table I.

TABLE I
SENSITIVITY ANALYSIS FOR VARIOUS INVENTORY PARAMETERS

Parameter	% change	t_1^*	S^*	Q^*	TC^*
α	-60	0.9793	59.8311	60.1432	3837.8289
	-40	0.9585	59.5693	60.1983	3838.5578
	-20	0.9397	59.3249	60.2448	3839.2365
	+20	0.9066	58.8801	60.3182	3840.4670
	+40	0.8920	58.6763	60.3472	3841.0280
	+60	0.8784	58.4832	60.3720	3841.5578
β	-60	0.9550	59.5715	60.2550	3839.1679
	-40	0.9380	59.3397	60.2855	3839.6702
	-20	0.9280	59.1900	60.2913	3839.8617
	+20	0.9194	59.0388	60.2710	3839.7763
	+40	0.9187	59.0065	60.2545	3839.6247
	+60	0.9188	59.9905	60.2371	3839.4457
δ	-60	0.9136	58.9444	60.2704	3839.8223
	-40	0.9168	58.9986	60.2810	3839.8398
	-20	0.9197	59.0489	60.2827	3839.8560
	+20	0.9250	59.1396	60.2861	3839.8853
	+40	0.9274	59.1805	60.2878	3839.8985
	+60	0.9297	59.2189	60.2895	3839.9109
μ	-60	0.9072	59.0292	60.4584	3841.8624
	-40	0.9108	59.0335	60.4052	3841.2689
	-20	0.9159	59.0542	60.3457	3840.5894
	+20	0.9305	59.1613	60.2244	3839.1523
	+40	0.9399	59.2528	60.1685	3838.4642
	+60	0.9509	59.3720	60.1186	3837.8339
a	-60	0.9490	59.5451	60.3198	3837.1766
	-40	0.9400	59.3938	60.3079	3838.0858
	-20	0.9312	59.2440	60.2961	3838.9840
	+20	0.9139	58.9492	60.2728	3840.7477
	+40	0.9054	58.8040	60.2612	3841.6137
	+60	0.8971	58.6604	60.2498	3842.4694
b	-60	0.9409	59.4079	60.3090	3838.8277
	-40	0.9346	59.3017	60.3006	3839.1808
	-20	0.9284	59.1977	60.2924	3839.5286
	+20	0.9166	58.9559	60.2765	3840.2087
	+40	0.9109	58.8978	60.2687	3840.5415
	+60	0.9053	58.8016	60.2610	3840.8695
M	-60	0.8251	57.3811	60.1458	3848.1546
	-40	0.8581	57.9757	60.1946	3844.5339
	-20	0.8906	58.5470	60.2407	3841.8404
	+20	0.9537	59.6228	60.3259	3838.4987
	+40	0.9842	60.1284	60.3653	3837.6338
	+60	1.0140	60.6132	60.4029	3837.2096

From the above table we can conclude the following:

1. t_1^* and S^* decreases while Q^* and TC^* increases with increase in the value of the parameter α.

2. t_1^*, S^*, Q^* and TC^* changes variably with increase in the value of the parameter β.

3. t_1^*, S^*, Q^* and TC^* increases with increase in the value of the parameter δ.

4. t_1^* and S^* increases while Q^* and TC^* decreases with increase in the value of the parameter μ.

5. t_1^*, S^* and Q^* decreases while TC^* increases with increase in the value of the parameter a.

6. t_1^*, S^* and Q^* decreases while TC^* increases with increase in the value of the parameter b.

7. t_1^*, S^* and Q^* increases while TC^* decreases with increase in the value of the parameter M.

VII. CONCLUSION

In this paper, a model for determining the optimal length of time in which there is no inventory shortage and the optimal order quantity for non – instantaneous deteriorating items are developed where delay in payment is allowed. Power pattern demand, Weibull two parameter deterioration rate and holding cost is expressed as linearly increasing functions of time are considered in this model. This type of power pattern demand requires a different policy than the conventional policy based on general Weibull pattern. In cases where large portion of demand occurs at the beginning of the period we use $n > 1$ and when it is large at the end we use, $0 < n < 1$. Similarly $n = 1$ and $n = \infty$ corresponds to constant demand and instantaneous demand respectively. The model is very practical for the industries in which the holding cost is depending upon the time. Also, shortage is allowed and can be partially backlogged, where the backlogging rate is dependent on the time of waiting for the next replenishment. The results show that there is decrease in total cost from the non-instantaneously deteriorating items compared with instantaneously deteriorating items. Also, when a delay in payments is allowed, the total cost for the retailer also decrease. Finally, numerical examples and sensitivity analysis are provided to illustrate the model and the solution procedure.

The proposed model incorporates some realistic features that are likely to be associated with some kinds of inventory. Furthermore, this model can be adopted in the inventory control of retail business such as food industries, seasonable cloths, domestic goods, etc.

This paper can be extended in several ways, for instance, we may extend the model by considering the non – zero lead time. Also, we may consider inflation and time value of money in the model. Finally, we can extend the model by considering demand function as stochastic.

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