

# An Efficient Tool for Mitigating Voltage Unbalance with Reactive Power Control of Distributed Grid-Connected Photovoltaic Systems

Malinwo Estone Ayikpa

**Abstract**—With the rapid increase of grid-connected PV systems over the last decades, genuine challenges have arisen for engineers and professionals of energy field in the planning and operation of existing distribution networks with the integration of new generation sources. However, the conventional distribution network, in its design was not expected to receive other generation outside the main power supply. The tools generally used to analyze the networks become inefficient and cannot take into account all the constraints related to the operation of grid-connected PV systems. Some of these constraints are voltage control difficulty, reverse power flow, and especially voltage unbalance which could be due to the poor distribution of single-phase PV systems in the network. In order to analyze the impact of the connection of small and large number of PV systems to the distribution networks, this paper presents an efficient optimization tool that minimizes voltage unbalance in three-phase distribution networks with active and reactive power injections from the allocation of single-phase and three-phase PV plants. Reactive power can be generated or absorbed using the available capacity and the adjustable power factor of the inverter. Good reduction of voltage unbalance can be achieved by reactive power control of the PV systems. The presented tool is based on the three-phase current injection method and the PV systems are modeled via an equivalent circuit. The primal-dual interior point method is used to obtain the optimal operating points for the systems.

**Keywords**—Photovoltaic generation, primal-dual interior point method, three-phase optimal power flow, unbalanced system.

## I. INTRODUCTION

POWER quality studies for the electric power systems have always been done with conventional methods based on allocation of capacitor [1], [2], reactive power and voltage control [3], [4], voltage stability [5], taps adjustment of OLTCs and voltage regulator [6], [7], state estimation [8], [9]. However, in the last decade much effort has been concentrated on the integration of distributed generation (DG) sources in the distribution networks. Some studies present the impact of DG on electric power systems, [10], [11], other, the planning and management policy of the distribution and transmission systems with DG [12], [13]. Among the DG sources, photovoltaic (PV) is part of those with greatest growth. The integration of PV systems in distribution networks may cause positive as well as negative impacts in the planning and operation of these networks. When the optimal placement and size are well defined, grid-connected PV systems can reduce

the losses, improve the voltage profile, reduce voltage unbalance, and increase system reliability [11], [14], [15]. Despite its numerous advantages overloaded by the grid-connected PV system, it appears that a large scale of PV systems connected in distribution system can generate several negative impacts. Some of this impacts presented in [16], [17], are related to reverse power flow, voltage increase, power losses, voltage unbalance and an increase of reactive power. The analysis of these impacts on distribution network is not an easy task. In most applications of grid-connected PV systems, the inverters are used only as active power source, thus operating with unit power factor [18]. However, it should be noted that during the hours of maximum generation with high penetration level of PV systems operating with unit power factor, overvoltage problem might occur along distribution feeders [15]. This situation has been a subject of recent researches in which distribution static compensator (DSTATCOM) is used to keep voltages within acceptable limit when PV systems operate during a peak solar irradiation with a high level penetration [19], [20]. With the development of voltage source inverter technology, overvoltage can be limited by voltage regulation function of the inverter, which can operate as reactive power compensator [15], [21]. Researches developed in the last years were principally focused on the reactive power generated by single-phase PV systems [22], [23]. In this paper, single-phase and three-phase PV systems were associated to provide reactive power on the grid. This consideration helps not only to control the size of the PVs installation but also to improve the power quality. Tools based only on the single-phase equivalent model may not produce the best operating solutions for three-phase systems, especially when circuits and/or loads are considerably unbalanced [24]. For these reasons, this work models the PV power plants in the three-phase optimal power flow (TOPF) from the PV module equivalent circuit and, with the TOPF, analyzes the impact of the PV generation on the voltage unbalance of distribution systems.

The three-phase representation of the distribution system was first made in power flow programs that use the backward/forward sweeping method to obtain system voltage [25]. However, in [26] a three-phase current injection method (TCIM) was proposed resulting in better convergence properties than the conventional method. The same formulation was used later in [27].

Several papers on the TOPF problem have been published in recent years. Among them can be cited [28] which proposes

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a solution for an unbalanced optimal power flow (OPF) via the Quasi-Newton method; [6] which considers discrete control operations such as capacitor switching and taps adjustment of OLTCs; [29], based on semidefinite programming; [30] which extends the TOPF based on current injections for optimization of n-conductors systems; [7] which achieves optimal adjustment of capacitor banks and voltage regulators to minimize losses and [31], based on locational marginal price concept applied to distribution systems. Some papers also present studies on the impact of distributed PV generation [14], [27], [32].

In this study, PV modules are represented by the five-parameter model [33], [34]. This representation was used in the power flow problem [35] and, more recently, in the single-phase OPF [36] and TOPF problems [37]. This work describes a TOPF model based on the proposed formulation in [27], in which the PV systems are represented by the five-parameter model. Single-Phase and Three-Phase PVs are connected in the distribution networks, providing active and reactive power.

The next section explains how the PV generation is calculated. Section III presents a formulation of the problem with PV systems. Section IV analyzes the results obtained and, finally, Section V summarizes the main conclusions of the study.

## II. PV GENERATION CALCULATION

### A. The Five-Parameter Model

The electrical power supplied by a PV module can be obtained from the equivalent electrical circuit of Fig. 1 [33].

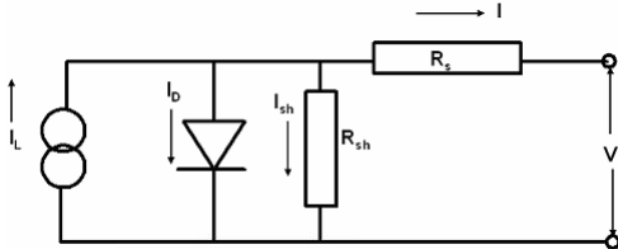


Fig. 1 Equivalent electrical circuit for PV module

Applying Kirchhoff's 1st Law to the circuit, the current injected by the module is:

$$I = I_L - I_o \left[ \exp \left( \frac{V + IR_s}{V_t \hat{a}} \right) - 1 \right] - \frac{V + IR_s}{R_{sh}} \quad (1)$$

where  $I$  is the injected current,  $V$  the terminal voltage,  $I_L$  is the current generated by the incident light,  $I_0$  is the saturation current of the diode,  $V_t = N_s k_b T_c / q$  is the thermal voltage of the array with  $N_s$  cells connected in series,  $k_b$  is the Boltzmann constant,  $T_c$  is the temperature (K),  $q$  is the electron charge and  $\hat{a}$  is the diode ideality constant. When one has the cells array,  $R_s$  and  $R_{sh}$  are the equivalent resistances.

Once the five parameters,  $I_L$ ,  $I_0$ ,  $R_s$ ,  $R_{sh}$  and  $\hat{a}$ , are known, the current, voltage and active power supplied by the module

can be calculated in each scenario of temperature and solar irradiation. However, not all parameters are available, since manufacturers' datasheet of PV modules only provide the nominal open-circuit voltage,  $V_{oc,n}$ , the nominal short-circuit current,  $I_{sc,n}$ , voltage and current at the maximum power point,  $V_{mp}$  and  $I_{mp}$ , the coefficients of temperature for open circuit voltage and short-circuit current,  $K_V$  and  $K_I$ , and the maximum power produced experimentally,  $P_{max,e}$ . The values are referred to STC conditions (standard test conditions). Some manufacturers also provide I-V curves for different levels of irradiation and temperature. In [34] an iterative method is proposed to obtain the parameters of equivalent circuit from the information provided by the manufacturers. This method is used to calculate PV generation.

### B. Five-Parameter and DC Power Calculation

The method is based on the fact that  $I_L$  varies with the temperature and the solar irradiation incident on the PV module according to:

$$I_{L,n} = (I_{L,n} + K_I \Delta T) \frac{G}{G_n} \quad (2)$$

where  $I_{L,n}$  is the current generated at nominal conditions,  $G$  the irradiation on the surface of the array,  $G_n$  the nominal irradiation and  $\Delta T = T - T_n$ , where  $T$  is the measured temperature and  $T_n$  the temperature at nominal conditions.

The saturation current of the diode is obtained in order to match the open circuit voltage with experimental data obtained for different temperatures. The starting point is the nominal saturation current

$$I_{o,n} = I_{sc,n} / \left[ \exp \left( \frac{V_{oc,n}}{V_{t,n} \hat{a}} \right) - 1 \right] \quad (3)$$

calculated on the thermal voltage,  $V_{t,n}$ , of the  $N_s$  cells connected in series at the temperature  $T_n$ .

$I_0$  is obtained from (4) employing the voltage and current coefficients  $K_V$  and  $K_I$  and the temperature correction factor  $\Delta T$ :

$$I_o = \frac{I_{sc,n} + K_I \Delta T}{\exp \left( \frac{V_{oc,n} + K_V \Delta T}{V_{t,n} \hat{a}} \right) - 1} \quad (4)$$

$\hat{a}$  is determined according to the type of technology used in the solar panel, and  $\hat{a} = 1,3$  for the module used (Hanwha polycrystalline) [38].  $R_s$  and  $R_{sh}$  are calculated from the assumption that only a pair  $\{R_s, R_{sh}\}$  leads to the maximum power experimentally measured,  $P_{max,e} = V_{mp} I_{mp}$ , in the conditions provided by the manufacturer. From (1), it follows that  $R_s$  and  $R_{sh}$  must respect:

$$P_{\max,e} = V_{mp} \left\{ I_L - I_o \left[ \exp \left( \frac{V_{mp} + R_s I_{mp}}{V_i \hat{a}} \right) - 1 \right] - \frac{V_{mp} + R_s I_{mp}}{R_{sh}} \right\} \quad (5)$$

To solve (5),  $R_s$  is incremented from zero.  $R_{sh}$  is initialized to the minimum value obtained by the inclination of the straight line segment joining the points on the I-V curve defined by the short-circuit and maximum power condition:

$$R_{sh,min} = \frac{V_{mp}}{I_{sc,n} - I_{mp}} - \frac{V_{oc,n} - V_{mp}}{I_{mp}} \quad (6)$$

For each pair  $\{R_s, R_{sh}\}$  the value of  $I_L$  is updated using the relation between the currents of the circuit:

$$I_{L,n} = \frac{R_s + R_{sh}}{R_s} I_{sc,n} \quad (7)$$

Fig. 2 shows how  $R_s$  and  $R_{sh}$  are obtained.

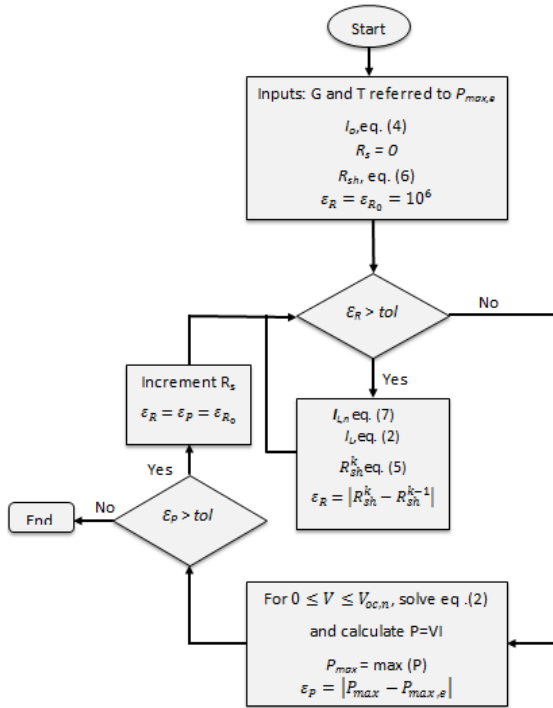


Fig. 2 Algorithm to determine  $R_s$ , and  $R_{sh}$

Having found  $R_s$  and  $R_{sh}$ ,  $I_L$ ,  $I_o$  and the DC power generated  $P_o$  can be calculated for each scenario of temperature and solar irradiation, according to [37].

### C. Inverter Modeling and AC Power Calculation

The PV modules are connected in series forming *strings* and these are connected in parallel to the inverters. The number of modules per *string*,  $N_{PV,s}$ , and the number of *strings*,  $N_{str}$ , are chosen in order to achieve a good efficiency of DC-AC conversion. The inverter efficiency was calculated

by means of exponential interpolation of the data indicated in the manufacturer's datasheet for each voltage curve, with determination coefficient  $R^2 = 0,99$ . The active AC power at the inverter output is:

$$P_{PV,inv,\omega} = \eta_{inv,\omega} N_{PV,s} N_{str} P_o \quad (8)$$

where the efficiency of the inverter is obtained by the function that interpolates the data of the manufacturer ( $\eta_{inv,\omega} = f_V(V_{mp,\omega}, I_{mp,\omega})$ ).

### III. THREE-PHASE OPTIMAL POWER FLOW

The TOPF problem is formulated from current injections [27]. In the problem, the active power supplied by the PV plants is not controllable. However, these power plants can generate or absorb reactive power since the inverters can operate with power factors (PF) less than 1.

#### A. Modeling of PV Plant in the TOPF

The three-phase PV power plants are connected to the grid via a transformer. Depending on the capacity of the PV plant, the connection can be done without a transformer. In this study, the single-phase PVs are connected in the grid without transformer. The AC power supplied by the PV plant is:

$$P_{PV,\omega} = P_{PV,inv,\omega} N_{inv} \quad (9)$$

where  $N_{inv}$  the number of inverters.

If the PV plant operates with adjustable power factor within the inverter limits, its reactive generation,  $Q_{PV,\omega}$  must respect the nominal inverter capacity,  $S_{inv}^{\max}$ , and its power factor,  $PF_{inv}$ , according to (10):

$$|Q_{PV}| \leq \min \left\{ \sqrt{\left( \frac{P_{PV,\omega}}{PF_{inv}} \right)^2 - (P_{PV,\omega})^2}, \sqrt{(S_{inv}^{\max} \times N_{inv})^2 - (P_{PV,\omega})^2} \right\} \quad (10)$$

#### B. TOPF Problem Formulation and Its Resolution

##### 1) Objective Function

The performance index studied is the minimum voltage unbalance for all buses of the system, defined by their negative sequence component.

$$F = \min \sum_{k=1}^n \left( V_{Re,k,-}^2 + V_{Im,k,-}^2 \right) \quad (11)$$

where,

$$V_{k,-} = V_k^a + a^2 V_k^b + a V_k^c, \quad a = 1 \angle 120^\circ$$

$$V_k^{abc} = V_{Re,k}^{abc} + j V_{Im,k}^{abc}$$

$V_k^{abc}$  is the three-phase voltage phasor of the bus  $k$  and the subscripts "Re" and "Im" indicate the real and imaginary part of the phasor.

## 2) Equality Constraints

The current balance in a given bus  $k$  of the system is made by adding the current injections per phase of the elements connected to this bus:

$$I_{g,k}^{abc} - I_{d,k}^{abc} - I_k^{abc} = 0 \quad (12)$$

where  $I_{g,k}^{abc}$  are the contributions of generators,  $I_{d,k}^{abc}$  the contributions of loads and  $I_k^{abc}$  the lines and transformers contributions connected to the phases  $a$ ,  $b$  and  $c$  of the bus  $k$ .

Separating the real and imaginary parts of (12), the current balance equations of the TOPF are obtained:

$$\begin{aligned} I_{Re,g,k}^{abc} - I_{Re,d,k}^{abc} - I_{Re,k}^{abc} &= 0 \\ I_{Im,g,k}^{abc} - I_{Im,d,k}^{abc} - I_{Im,k}^{abc} &= 0 \end{aligned} \quad (13)$$

The currents injected by the generators are expressed by:

$$I_{g,k}^{abc} = I_{Re,g,k}^{abc} + jI_{Im,g,k}^{abc} = \left( \frac{P_{g,k}^{abc} + jQ_{g,k}^{abc}}{V_k^{abc}} \right)^* \quad (14)$$

where  $P_{g,k}^{abc}$  and  $Q_{g,k}^{abc}$  are active and reactive powers generated. On the other hand, the currents consumed by the loads are:

$$I_{d,k}^{abc} = I_{Re,d,k}^{abc} + jI_{Im,d,k}^{abc} = \left( \frac{P_{d,k}^{abc} + jQ_{d,k}^{abc}}{V_k^{abc}} \right)^* \quad (15)$$

where  $P_{d,k}^{abc}$  and  $Q_{d,k}^{abc}$  are active and reactive powers consumed.

The current contributions of the lines are calculated by the network equations, expressed in matrix form as:

$$\begin{bmatrix} I_{Re,k}^{abc} \\ I_{Im,k}^{abc} \\ \vdots \\ I_{Re,m}^{abc} \\ I_{Im,m}^{abc} \end{bmatrix} = \begin{bmatrix} G^{abc} & -(B^{abc})^t & \dots & -G^{abc} & B^{abc} \\ (B^{abc})^t & G^{abc} & \dots & -B^{abc} & -G^{abc} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -G^{abc} & B^{abc} & \dots & G^{abc} & -(B^{abc})^t \\ -B^{abc} & -G^{abc} & \dots & (B^{abc})^t & G^{abc} \end{bmatrix} \begin{bmatrix} V_{Re,k}^{abc} \\ V_{Im,k}^{abc} \\ \vdots \\ V_{Re,m}^{abc} \\ V_{Im,m}^{abc} \end{bmatrix} \quad (16)$$

In (16),  $G^{abc}$  and  $B^{abc}$  are  $3 \times 3$  matrix composed of the real and imaginary parts of the elements of the admittance matrix of the system,  $I_{Re,k}^{abc}$ ,  $I_{Re,k}^{abc}$ ,  $I_{Im,k}^{abc}$  and  $I_{Im,m}^{abc}$  are vectors  $3 \times 1$  and  $t$  indicates the transposed matrix.

The transformers and voltage regulators are considered fixed and represented by the  $\pi$ -equivalent circuit [39].

So that the voltages in the reference bus ( $ref$ ) to be lagged of  $120^\circ$ , (17) is introduced into the formulation of the problem

$$\begin{aligned} V_{Im,ref}^a &= 0 \\ V_{Im,ref}^b - V_{Re,ref}^b \tan(-2\pi/3) &= 0 \\ V_{Im,ref}^c - V_{Re,ref}^c \tan(2\pi/3) &= 0 \end{aligned} \quad (17)$$

Finally, in order to keep the voltage magnitude of the reference bus equal in the three phases, (18) can be included in the TOPF model:

$$\begin{aligned} V_{Re,ref,a}^2 + V_{Im,ref,a}^2 - V_{Re,ref,b}^2 - V_{Im,ref,b}^2 &= 0 \\ V_{Re,ref,a}^2 + V_{Im,ref,a}^2 - V_{Re,ref,c}^2 - V_{Im,ref,c}^2 &= 0 \end{aligned} \quad (18)$$

## 3) Inequality Constraints

They represent operating limits and/or security aspects of the system. For each bus  $k$  and phases  $a$ ,  $b$  and  $c$ , they are expressed by (19):

$$\begin{aligned} V_{min,k}^{2abc} \leq V_{Re,k}^{2abc} + V_{Im,k}^{2abc} \leq V_{max,k}^{2abc} \\ P_{gmin,k}^{abc} \leq P_{g,k}^{abc} \leq P_{gmax,k}^{abc} \\ Q_{gmin,k}^{abc} \leq Q_{g,k}^{abc} \leq Q_{gmax,k}^{abc} \end{aligned} \quad (19)$$

At the bus  $k$ , if there is a three-phase PV plant, in (19),  $P_{gmin,k}^{abc} = P_{gmax,k}^{abc} = P_{PV,\omega}^{abc}$  and  $Q_{gmin,k}^{abc} = Q_{gmax,k}^{abc} = Q_{PV,\omega}^{abc}$  with limits given in (10).

The TOPF problem was solved by the primal-dual interior point method [40].

## IV. RESULTS

The studies were performed with the IEEE34 bus system [41] in which the mutual impedances are not considered and the loads modeled as a constant power and a 70-bus system (SIS70) [42], a single-phase equivalent system transformed in a three-phase system by the addition of unbalance in the loads, maintaining the system's load in phase A and affecting the coefficients 0.8 and 0.6 respectively for the load in phase B and C. Solar irradiation and temperature data were obtained at the INMET station in Santa Marta – SC in January 2014. Data from the Hanwha SF220-30-1P240L (240Wp) panel and the SUNNY TRIPOWER 12000TL-US (12kVA, FP $\geq$ 0.8) inverter are used. The barrier parameter and the tolerance adopted for all simulations were  $10^{-10}$  and  $10^{-6}$  respectively.

### A. Output Power Generation of the Inverter

Fig. 3 shows the output power of the inverter on the day 23/01/2014, affected by the efficiency of the inverter, which varies from about 80% to about 97% depending on the irradiation and temperature. The maximum power point is achieved at 13 h.

### B. Impact of PV Generation on the System

The PV plants were formed by connecting 48 SF220-30-1P240L panels per inverter, 24 of which were connected in series ( $N_{PV,s} = 24$  and  $N_{str} = 2$ ). The number of inverters was chosen according to the capacity of the plant. The single-phase PV has a capacity of 12kW and the three-phase PV a

capacity of 500 kW. One (01) three-phase PV connected to the bus 890 with 15 single-phase PVs were used for IEEE34 bus system and two (02) three-phase PVs connected to the buses 58 & 62 with 30 single-phase PVs for 70-bus system (SIS70). Single-Phase PV systems are distributed equally among the three phases of the system. These configurations have been adopted considering the loading of each system.

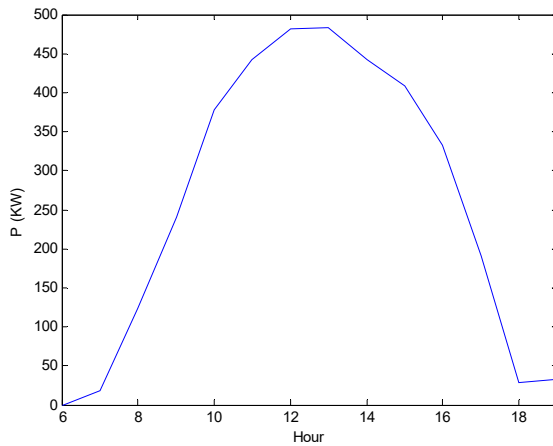


Fig. 3 Inverter Output Power on the day 23/01/2014

### C. Simulation Results for a Specific Scenario of Irradiation and Temperature

The following results were obtained with PV systems operating at maximum power on the day 23/01/2014. Tables I and II indicate voltage unbalance and losses status for (i) operation mode without PV (base case), (ii) only three-phase PVs are allocated in the network with the unit power factor (PF=1) and (iii) single-phase and three-phase PVs operating with PF≠1. We can note in Table I that when single-phase and three-phase PV systems operate jointly with PF≠1, voltage unbalance level is considerably reduced. This operation mode contributes to improve the results presented in [37], wherein only three-phase PV systems were allocated. Table II shows the reduction in system losses when voltage unbalance is minimized.

TABLE I  
IMPACT OF PV SYSTEMS ON VOLTAGE UNBALANCE  
Min. Unbal. ( $10^{-4}$ )

Systems	Base	PV/PF=1	PV/ PF≠1
IEEE34	2,98	3,33	1,81
SIS70	5,7	6,83	4,66

TABLE II  
IMPACT OF PV SYSTEMS ON LOSSES  
Losses (kW)

Systems	Base	PV/PF=1	PV/ PF≠1
IEEE34	100,74	52,24	46,04
SIS70	38,11	22,66	16,48

Figs. 4 and 5 present voltage profile for (i) only three-phase PV systems operating with PF=1, (ii) only three-phase PV systems operating with PF≠1 and (iii) association of single-phase and three-phase PV systems operating with PF≠1. The

simulation results show that when the reactive power control of single-phase and three-phase PV systems association is taken into account, voltage profile almost coincides with its nominal value.

TABLE III  
ACTIVE AND REACTIVE POWER INJECTIONS, IEEE34

Active Power (kW) and Reactive Power (kVar)						
<b>Without PV</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	640,07	615,82	613,85	138,47	123,11	124,83
<b>PV/PF=1</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	493,58	468,13	465,28	131,54	115,14	117,18
	P <sub>PVa</sub>	P <sub>PVb</sub>	P <sub>PVc</sub>	Q <sub>PVa</sub>	Q <sub>PVb</sub>	Q <sub>PVc</sub>
	161	161	161	-	-	-
<b>PV/PF≠1</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	406,05	382,8	378,5	154	135,82	150,56
	P <sub>PVa</sub>	P <sub>PVb</sub>	P <sub>PVc</sub>	Q <sub>PVa</sub>	Q <sub>PVb</sub>	Q <sub>PVc</sub>
	161	161	161	-39,5	-39,5	-39,5

TABLE IV  
ACTIVE AND REACTIVE POWER INJECTIONS, SIS70

Active Power (kW) and Reactive Power (kVar)						
<b>Without PV</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	1127	898,5	671,5	889,62	710,31	531,70
<b>PV/PF=1</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	853,51	615,63	379,45	930,75	743,34	556,68
	P <sub>PVa</sub>	P <sub>PVb</sub>	P <sub>PVc</sub>	Q <sub>PVa</sub>	Q <sub>PVb</sub>	Q <sub>PVc</sub>
	161	161	161	-	-	-
<b>PV/PF≠1</b>	P <sub>grefa</sub>	P <sub>grefb</sub>	P <sub>grefc</sub>	Q <sub>grefa</sub>	Q <sub>grefb</sub>	Q <sub>grefc</sub>
	684,88	459,01	235,89	847,21	652,30	511,28
	P <sub>PVa</sub>	P <sub>PVb</sub>	P <sub>PVc</sub>	Q <sub>PVa</sub>	Q <sub>PVb</sub>	Q <sub>PVc</sub>
	161	161	161	-3,22	-3,22	-3,22
	161	161	161	22,07	22,07	22,07

Tables III and IV show the active and reactive powers provided by the substation and the three-phase PV plants. Active power provided by the substation decreases with PV system operating. Depending on the grid utility, reactive power can be generated or absorbed when PV systems operate with PF≠1. Fig. 6 shows the reactive power provided by the substation for different operation modes of PV systems and Fig. 7, the contribution in the reactive power control of single-phase PVs, for IEEE34 feeder test. Note that the reactive power provided by the substation increases when PV systems operate with PF≠1. This result can be justified by the fact that the PV systems absorb more reactive power in the grid due to a very high current in the circuit. Also, this network consists of a very long and slightly charged feeder, which could lead to strong degradation of the voltage level. Table III shows reactive power absorbed by the three-phase PV plant and Fig. 7 shows that the single-phase PVs can generate or absorb reactive power depending on their location in the network. On the other hand, Fig. 8 shows that the reactive power injected by the PV power plants into the network has like consequence the reduction of the reactive power injected in the substation bus. This can be observed through Fig. 9, where there is a strong contribution of the reactive power injected by the single-phase PV plants, which contributes significantly in reducing the voltage unbalance and therefore losses as well.

Table IV also shows a distinct participation in the reactive power control of three-phase PVs for the 70-bus system.

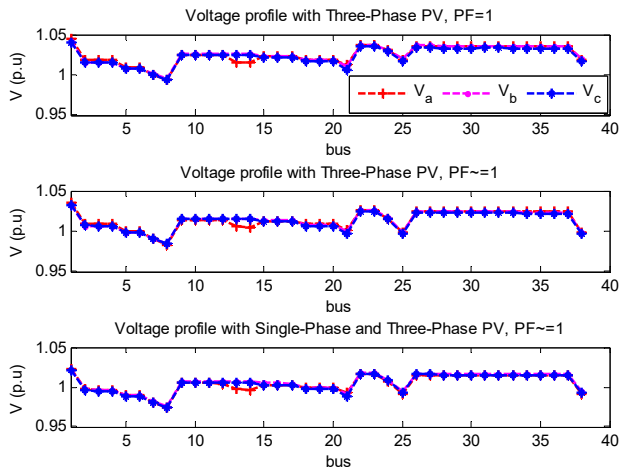


Fig. 4 Voltage profile, IEEE34

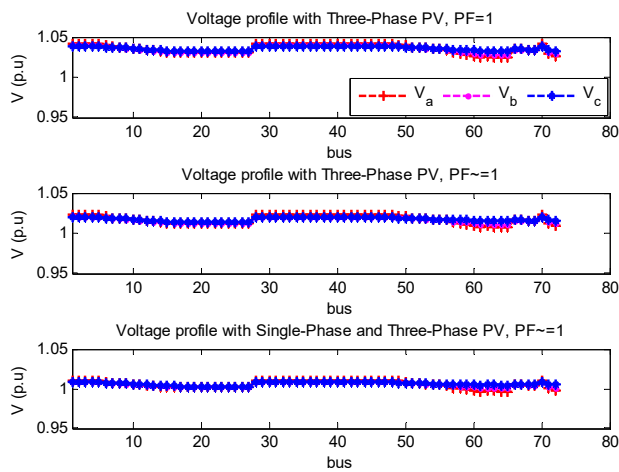


Fig. 5 Voltage profile, SIS70

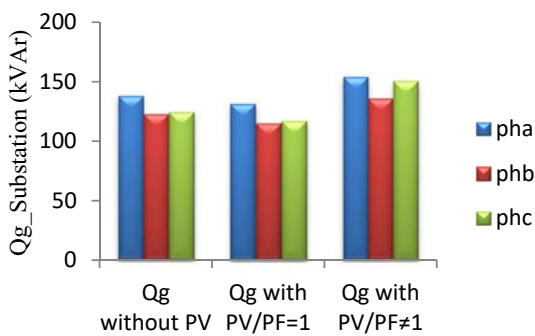


Fig. 6 Reactive power injection in the substation bus, IEEE34

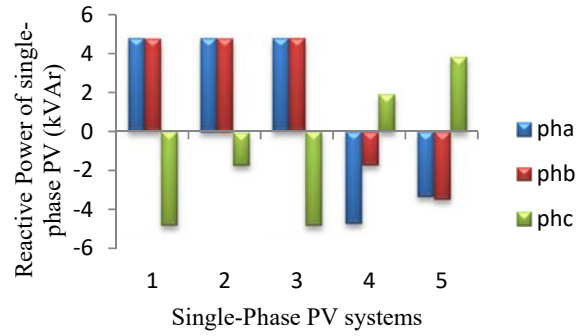


Fig. 7 Reactive power injection of single-phase PVs, IEEE34

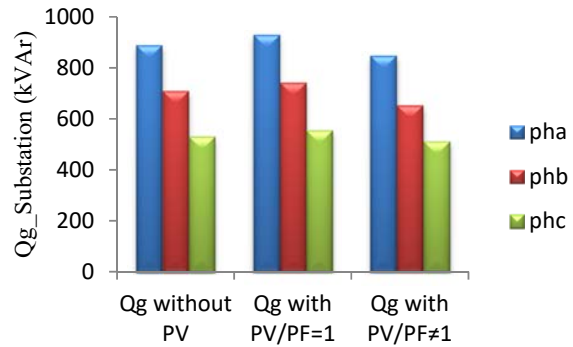


Fig. 8 Reactive power injection in the substation bus, SIS70

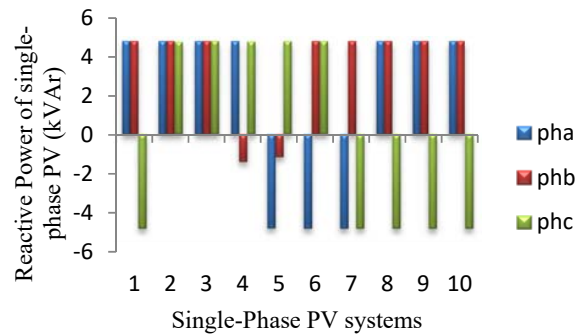


Fig. 9 Reactive power injection of single-phase PVs, SIS70

To better understand the impact of distributed allocation of PV systems on the voltage unbalance, we consider only the single-phase PVs allocated to the distribution systems studied, operating with unit power factor or with  $PF \neq 1$ . Figs. 10 and 11 show that when the single-phase PVs provide the grid with the reactive power, voltage profile becomes closer to the nominal. This aspect confirms the importance of distributed applications of PV systems and their contribution on the reduction of voltage unbalance when these are well located in the network.

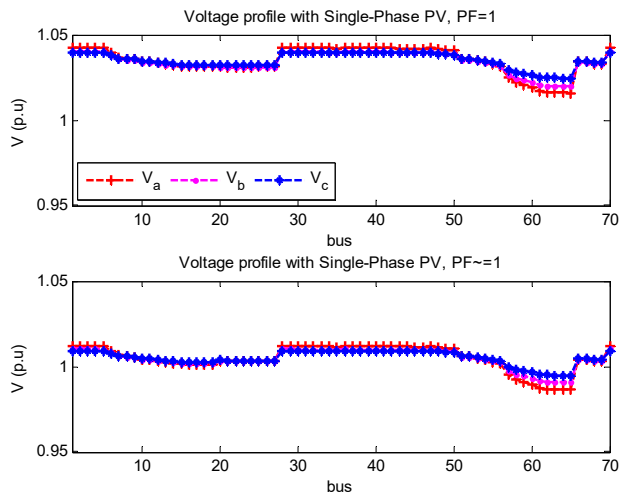


Fig. 10 Voltage profile with only single-phase PVs, SIS70

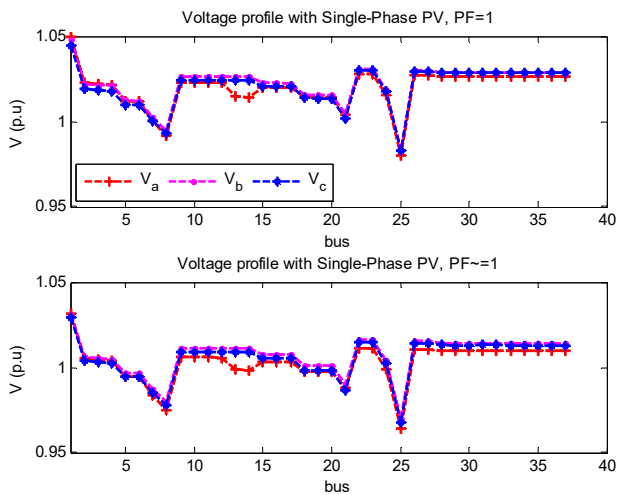


Fig. 11 Voltage profile with only single-phase PVs, IEEE34

## V.CONCLUSION

In this paper, a tool for the analysis and simulation of unbalanced electrical networks was presented. The proposed tool took into account the modeling of single-phase and three-phase PV systems and used the primal-dual interior point method as an optimization technique for electrical networks. Compared to the results obtained in the previous research and taking into account the single-phase PV systems distributed between the phases of the grid, the OPF model has had a very significant effect in reducing the voltage unbalance and the losses in the distribution networks. The approach used for the positioning of single-phase PV installations in the network was based on the selection of the load buses, whereas for the three-phase PV systems, the choice was randomly made. This approach could affect the simulation results if the position of the PV plants is changed in the network. The use of a robust algorithm for optimal allocation of PV systems with other DG sources is currently under investigation.

## APPENDIX

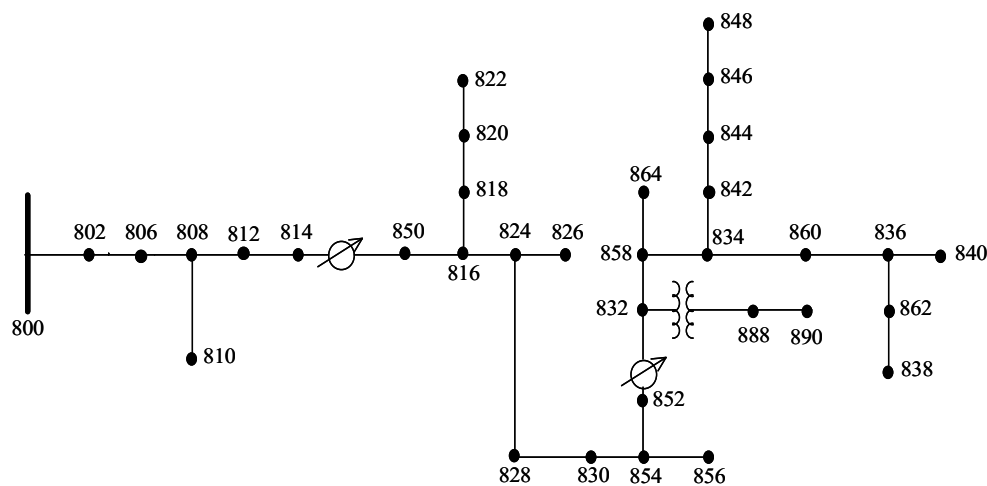


Fig. 12 IEEE34-bus



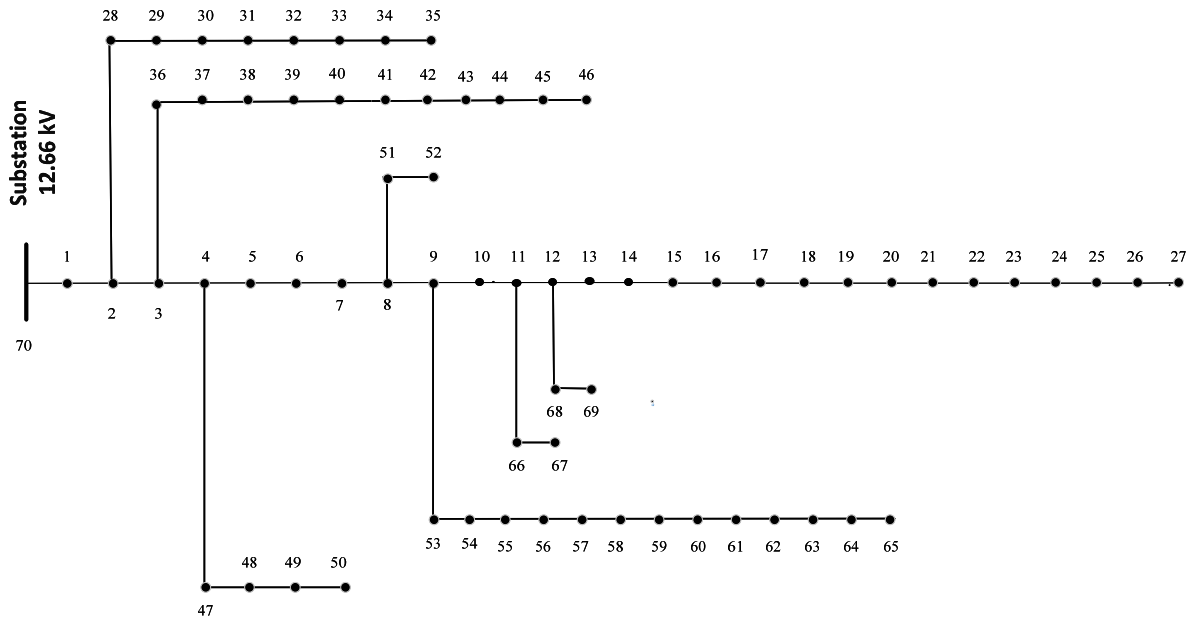


Fig. 13 SIS70-bus

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