

# Adaptive Impedance Control for Unknown Non-Flat Environment

Norsinnira Zainul Azlan and Hiroshi Yamaura

**Abstract**—This paper presents a new adaptive impedance control strategy, based on Function Approximation Technique (FAT) to compensate for unknown non-flat environment shape or time-varying environment location. The target impedance in the force controllable direction is modified by incorporating adaptive compensators and the uncertainties are represented by FAT, allowing the update law to be derived easily. The force error feedback is utilized in the estimation and the accurate knowledge of the environment parameters are not required by the algorithm. It is shown mathematically that the stability of the controller is guaranteed based on Lyapunov theory. Simulation results presented to demonstrate the validity of the proposed controller.

**Keywords**—Adaptive impedance control, Function Approximation Technique (FAT), impedance control, unknown environment position.

## I. INTRODUCTION

THE robotic fingers or hands tasks are mainly classified into two categories, which are the noncontact tasks and contact tasks. In noncontact tasks, the finger or hand is assigned to follow a predefined position trajectory in an unconstrained environment. In contact tasks, it is desirable for the finger or hand to exert a certain amount of force on the environments. Teleoperation, object manipulation, massaging, polishing, wiping and assembling operations are some the examples of contact tasks.

Force control strategies are important in regulating the amount of force applied on the environment. Two main approaches adopted in force control are the hybrid position/force control [1] and impedance control [2]. Hybrid position/force control allows the direct control of position and force. However, the method is susceptible to unstable transition between the unconstrained and constrained motions.

Impedance control aims in regulating a desired dynamic relationship between the robot's end-effector position and contact force. It provides a unified framework for free and contact spaces; as a result a stable transition is achievable as the robot travels from the free space to constraint space. The formulation of an impedance control with force tracking capability has attracted the interest of many researchers [3]-[5]. However, in this technique, the controller requires the precise knowledge of the environment location and stiffness

beforehand, so that the reference position that is needed to produce the desired force can be generated. Nevertheless, in practical, it is difficult to obtain these parameters accurately in advance [3].

Seraji and Colbaugh [4] developed two simple adaptive force based impedance control schemes to compensate for unknown environment position and stiffness, but the method is limited to flat environment shape or constant environment location only. Jung et al. [3] developed a two-phased adaptive impedance control to cater for time-varying environment parameters, but the force convergence analysis presented is limited to the force errors due to the estimation of the environment position, which are represented by the step function only. There is no force convergence analysis for the cases where the environment location is time-varying presented in the study.

On the other hand, FAT based adaptive impedance have been introduced by Huang and Chien [6] to deal with time-varying uncertainties in the plant parameter. However, to the best of the authors' knowledge there is no study that has been conducted in deriving a solution based on FAT to overcome unknown time-varying environment location problem.

Therefore, this paper proposes a new FAT based adaptive impedance control for controlling a robotic finger to exert the desired force in the force controllable direction, while tracking a required position trajectory in the position controllable direction, under unknown non-flat environment shape. In this paper, unknown non-flat environment shape is also referred to as unknown time-varying environment location. FAT based target compensators are incorporated in the target impedance, all of the uncertainties are expressed in terms of FAT expression without any differentiation with respect to time and the force error feedback is utilized in the update law. The stability of the proposed algorithm and the convergence of the force error have been analyzed based on Lyapunov stability theory. The simulation results are presented to confirm the effectiveness of the proposed strategy.

This paper is organized as follows; the FAT-based adaptive impedance control, including the target impedance in the force controllable direction, the target impedance in the position controllable direction and the impedance control law are presented in Section II. The simulation results are discussed in Section III and finally conclusion is drawn in Section IV.

## II. FAT-BASED ADAPTIVE IMPEDANCE CONTROL

The objective of the controller is to control the robotic finger to exert the desired force on the environment in the

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force controllable direction, while tracking a reference position trajectory in the position controllable direction, in an environment with unknown time-varying location (non-flat environment shape). Different target impedances are implemented in the force controllable direction and position controllable direction as described in the following subsections.

#### A. Target Impedance for the Force Controllable Direction

In the force controllable direction, the environment location,  $x_e$  is needed to calculate the reference trajectory,  $x_d$ , so that the desired force,  $f_d$  can be achieved [4]. However, the accurate expression of  $x_e$  may not be known exactly in advance. Therefore, this study proposes an FAT based target impedance to compensate for the uncertainty in the environment location.

For simplicity, consider that force is applied in one direction only. In this case, the FAT based target impedance can be described as

$$b_d \left( \dot{e}' + \hat{\delta}_{xe}' \right) + k_d \left( e' + \hat{\delta}_{xe} \right) + \frac{-b_d \dot{f}_d - k_d f_d}{k_e} = -k_f e_f \quad (1)$$

where

$$e' = x - x_e' \quad (2)$$

$$x_e' = x_e + \delta_{xe} \quad (3)$$

$$e_f = f_e - f_d \quad (4)$$

$b_d$  is the positive desired damping in the force controllable direction,  $k_d$  is the positive desired stiffness in the force controllable direction, and  $k_f$  is the positive force error factor in the force controllable direction which can be specified by the designer;  $x$  is the robot's end effector location in the force controllable direction,  $x_e$  is the time-varying true environment location,  $x_e'$  is the initial estimate of the environment location which can be selected by the designer,  $\delta_{xe}$  is the time-varying inaccuracy in the initial environment location estimate,  $\hat{\delta}_{xe}$  is the estimation of  $\delta_{xe}$  by FAT-based adaptive law,  $k_e$  is the true environment stiffness,  $f_e$  is the force exerted by the robot on the environment in the force controllable direction,  $e_f$  is the force error in the force controllable direction,  $\dot{e}'$ ,  $\hat{\delta}_{xe}'$  and  $\dot{f}_d$  are the time derivative of  $e'$ ,  $\hat{\delta}_{xe}$  and  $f_d$  respectively.

The time-varying uncertain inaccuracy in the initial

environment location estimate,  $\delta_{xe}$ , its estimation  $\hat{\delta}_{xe}$ , their respective derivatives,  $\dot{\delta}_{xe}$  and  $\dot{\hat{\delta}}_{xe}$  can be expressed using FAT as

$$\delta_{xe} = W_{xe} Z_{xe} + \varepsilon_{\delta_{xe}} \quad (5)$$

$$\hat{\delta}_{xe} = \hat{W}_{xe} Z_{xe} \quad (6)$$

$$\dot{\delta}_{xe} = \dot{W}_{xedot} Z_{xedot} + \varepsilon_{\delta_{xedot}} \quad (7)$$

$$\dot{\hat{\delta}}_{xe} = \dot{\hat{W}}_{xedot} Z_{xedot} \quad (8)$$

where  $W_{(\cdot)} \in \mathfrak{R}^{1 \times \beta_{(\cdot)}}$  are the vectors of true weighting value of the FAT representation,  $\hat{W}_{(\cdot)} \in \mathfrak{R}^{1 \times \beta_{(\cdot)}}$  are their respective estimations,  $Z_{(\cdot)} \in \mathfrak{R}^{\beta_{(\cdot)} \times 1}$  are vector of the basis functions of the FAT representation,  $\varepsilon_{\delta_{(\cdot)}}$  are the approximation errors and  $\beta_{(\cdot)}$  are the numbers of basis functions implemented.  $\varepsilon_{\delta_{(\cdot)}}$  can be neglected since it is assumed that sufficient number of basis functions is utilized in the expression

In comparison to the previous study in [7], in this paper, the derivative,  $\dot{\delta}_{xe}$  is represented by another FAT expression instead of differentiating it directly from (6).

The adaptive update law to estimate the weighting functions,  $\hat{W}_{(\cdot)}$  are derived as

$$\dot{\hat{W}}_{xe} = Q_{xe}^{-1} k_d k_e Z_{xe} e_f \quad (9)$$

$$\dot{\hat{W}}_{xedot} = Q_{xedot}^{-1} b_d k_e Z_{xedot} e_f \quad (10)$$

where  $Q_{(\cdot)}^{-1} \in \mathfrak{R}^{\beta_{(\cdot)} \times \beta_{(\cdot)}}$  are the diagonal positive definite symmetric matrices of the adaptive gain.

The stability of the proposed strategy and the convergence of the force error,  $e_f$  to zero can be analyzed by substituting (5) - (8) into (1), resulting in

$$b_d (\dot{x} - \dot{x}_e) - b_d Z_{xedot} \tilde{W}_{xedot} + k_d (x - x_e) - k_d Z_{xe} \tilde{W}_{xe} + \frac{-b_d \dot{f}_d - k_d f_d}{k_e} = -k_f e_f \quad (11)$$

where  $\tilde{W}_{(\cdot)}$  is the estimation error in weighting function, described by

$$\tilde{W}_{(i)} = W_{(i)} - \hat{W}_{(i)} \quad (12) \quad \text{into (20), } \dot{V} \text{ can be reduced to}$$

The force exerted by the robotic finger on the environment,  $f_e$  and its derivative  $\dot{f}_e$  can be described from the environment model as [4]

$$f_e = k_e (x - x_e) \quad (13)$$

$$\dot{f}_e = k_e (\dot{x} - \dot{x}_e) \quad (14)$$

The inaccurate force,  $f_{e_e}$  due to the initial environment location guess,  $x_e$  and its derivative,  $\dot{f}_{e_e}$  can be defined as

$$\begin{aligned} f_{e_e} &= k_e (x - (x_e + \delta_{xe})) \\ &= f_e - k_e \delta_{xe} \end{aligned} \quad (15)$$

$$\begin{aligned} \dot{f}_{e_e} &= k_e (\dot{x} - (\dot{x}_e + \dot{\delta}_{xe})) \\ &= \dot{f}_e - k_e \dot{\delta}_{xe} \end{aligned} \quad (16)$$

Rearranging (15) and (16),  $x$  and  $\dot{x}$  can be expressed as

$$x = \frac{f_e}{k_e} + x_e + \delta_{xe}, \quad \dot{x} = \frac{\dot{f}_e}{k_e} + \dot{x}_e + \dot{\delta}_{xe} \quad (17)$$

Substituting  $x$  and  $\dot{x}$  from (17) into (11) and multiplying the resulting equation with  $k_e$  gives

$$\begin{aligned} b_d \dot{e}_f + (k_d + k_e k_f) e_f \\ - b_d k_e Z_{x\dot{e}} \tilde{W}_{x\dot{e}} - k_d k_e Z_{xe} \tilde{W}_{xe} = 0 \end{aligned} \quad (18)$$

The Lyapunov-like function candidate can be defined as

$$\begin{aligned} V &= \frac{1}{2} e_f^T b_d e_f + \frac{1}{2} (\tilde{W}_{xe}^T Q_{xe} \tilde{W}_{xe}) \\ &\quad + \frac{1}{2} (\tilde{W}_{x\dot{e}}^T Q_{x\dot{e}} \tilde{W}_{x\dot{e}}) \end{aligned} \quad (19)$$

Differentiating (19) with respect to time,  $\dot{V}$  can be obtained as

$$\dot{V} = e_f^T b_d \dot{e}_f - \tilde{W}_{x\dot{e}}^T Q_{x\dot{e}} \dot{\tilde{W}}_{x\dot{e}} - \tilde{W}_{xe}^T Q_{xe} \dot{\tilde{W}}_{xe} \quad (20)$$

Substitute  $b_d \dot{e}_f$  from (18) and the update laws (9) – (10)

$$\dot{V} = -e_f^T (k_d + k_e k_f) e_f \quad (21)$$

From (19) and (21), it can be seen that  $V > 0$  and  $\dot{V} \leq 0$ . This implies that the force error,  $e_f$  and the parameter estimation errors,  $\tilde{W}_{xe}$  and  $\tilde{W}_{x\dot{e}}$  are bounded.

The convergence of  $e_f$  to zero can be evaluated by differentiating  $\dot{V}$  in (21) with respect to time, yielding

$$\ddot{V} = -2e_f^T (k_d + k_e k_f) \dot{e}_f \quad (22)$$

From (18), (19), (21) and (22), it can be seen that  $\ddot{V}$  is bounded. Therefore,  $\dot{V}$  is uniformly continuous. From the application of Barbalat's lemma, this indicates that

$$\lim_{t \rightarrow \infty} \dot{V} = 0 \quad (23)$$

From (21), this implies that the condition in (23) leads to

$$\lim_{t \rightarrow \infty} e_f = 0 \quad (24)$$

Hence, provided that the target impedance is achieved, the implementation of adaptive target impedance (1) and update laws (9) and (10), on a robotic finger working under uncertain time-varying environment location, leads to the convergence of the force error to zero, and  $f_e \rightarrow f_d$  as  $t \rightarrow \infty$ .

#### B. Target Impedance for the Position Controllable Direction

In position controllable direction, the exact information of the environment position is not required and the standard target impedance can be used in this direction.

For simplicity, consider that position is controlled in one direction only; thus, the target impedance for this direction can be written as

$$m_{dy} \ddot{e} + b_{dy} \dot{e} + k_{dy} e = -k_{fy} e_{fy} \quad (25)$$

where

$$e = y - y_d \quad (26)$$

$y_d$  is the reference position for the robot end-effector in the position controllable direction, which can be selected by the designer,  $y$  is the position of the robot's end effector in the position controllable direction,  $\dot{e}$  and  $\ddot{e}$  are the first and

second time derivative of  $e$  respectively,  $m_{dy}$  is the positive desired inertia in the position controllable direction,  $b_{dy}$  is the positive desired damping in the position controllable direction,  $k_{dy}$  is the positive desired stiffness in the position controllable direction, and  $k_{fy}$  is the positive force error factor in the position controllable direction which can be specified by the designer;  $y$  is the robot's end effector location in the position controllable direction,  $\dot{y}$  is the robot's end effector velocity in the position controllable direction,  $\ddot{y}$  is the robot's end effector acceleration in the position controllable direction and  $e_{fy}$  is the force error in the position controllable direction.

### C. Impedance Control Law

The impedance control law in [8] is implemented to drive the system so that the impedance error converges to zero. The augmented error in the control law in [8] is modified according to the target impedance in (1) for the force controllable direction, and the target impedance in (25) for the position controllable direction.

Considering that position and force are applied in one direction only for simplicity, the control input force to the robotic finger under the control scheme, is governed by

$$F = \begin{bmatrix} f_{in-x} \\ f_{in-y} \end{bmatrix} \quad (27)$$

where  $f_{in-x}$  and  $f_{in-y}$  are the control input in the force and position controllable directions respectively.

The control input force  $f_{in-x}$  is described by

$$f_{in-x} = f_{s-x} - k_x z_x + f_e \quad (28)$$

where  $f_{s-x}$  is the switching control force in the force controllable direction,  $k_x$  is the positive scalar and  $z_x$  is augmented impedance error which is obtained from the target impedance in (1), by multiplying (1) with  $b_d^{-1}$  and rearranging the equation, giving

$$z_x = \dot{e}' + \hat{\delta}_{xe}' + b_d^{-1} k_d \left( e' + \hat{\delta}_{xe}' \right) + b_d^{-1} \left( \frac{-b_d \dot{f}_d - k_d f_d}{k_e} \right) + b_d^{-1} k_f e_f \quad (29)$$

The control input force  $f_{in-y}$  is described by

$$f_{in-y} = f_{s-y} - k_y z_y \quad (30)$$

where  $f_{s-y}$  is the switching control force in the position controllable direction,  $k_y$  is the positive scalar and  $z_y$  is obtained from the target impedance in (25), which is described as

$$z_y = \dot{e} + \lambda e + k_{fy} e_{fy} \quad (31)$$

where

$$\dot{e}_{fy} + \gamma e_{fy} = m_{dy}^{-1} e_{fy} \quad (32)$$

and  $\lambda$  and  $\gamma$  are positive definite matrices chosen such that

$$\lambda + \gamma = m_d^{-1} b_d, \quad \dot{\lambda} + \lambda \gamma = m_d^{-1} k_d. \quad (33)$$

## III. SIMULATION RESULTS

### A. Force and Position Tracking

Simulation study has been conducted on the robotic finger in [9] to investigate the effectiveness of the proposed controller. It is assumed that the contact occurs at the end of the finger's third finger bone for simplicity, and the contact position and time can be obtained from the experiment. In this study, the environment stiffness is assumed to be known accurately and is set to 40000N/m.

The robotic finger is required to exert a desired force in the x direction, on a sinusoidal shaped environment, while tracking a reference position trajectory along y direction.

The desired force,  $f_d$  in the force controllable or x direction can be described as

$$f_d = \begin{cases} 0\text{N} & 0 \leq t < 0.0833 \text{ s} \\ 400(t - 0.0833)\text{N} & 0.0833 \text{ s} < t < 0.1333 \text{ s} \\ 20\text{N} & t > 0.1333 \text{ s} \end{cases} \quad (34)$$

and the reference position trajectories,  $y_d$  in the position controllable or y direction is set as

$$y_d = -60 \times 10^{-3} t + 15 \times 10^{-3} \text{ m} \quad (35)$$

The true sinusoidal shaped environment location can be expressed as

$$x_e = 0.004 \sin \frac{2\pi(t - 0.0833)}{2/3} + 0.0375 \text{ m} \quad (36)$$

However,  $x_e$  is assumed to be unknown in advance and is initially estimated as a constant, where  $\dot{x}_e$  and  $\ddot{x}_e$  in (1) are set as 0.042 m and 0 m/s respectively.

Several orthonormal functions as described in [6] can be utilized to represent the estimation of the uncertainties. In this study, Fourier Series has been chosen due to its ease in programming. The number of basis functions has been chosen as 11 ( $\beta_{xe} = \beta_{x\dot{e}} = 11$ ) and the period of the Fourier Series has been selected as 1.3 s to ensure the valid range of the orthonormal function [6]. The values of the adaptation gain have been set to  $Q_{xe}^{-1} = 10^{-7} I_{11}$  and  $Q_{x\dot{e}}^{-1} = 10^{-7} I_{11}$ .

The value of  $\hat{W}_{xe}$  has been initialized as

$$\hat{W}_{xe}(0) = \hat{W}_{x\dot{e}}(0) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0], \quad (37)$$

and the controller parameters have been tuned to

$$\begin{aligned} k_x &= 4200, & k_y &= 500, \\ b_d &= 1270, & k_d &= 3000, \\ k_f &= 10, & m_{dy} &= 1, \\ b_{dy} &= 110, & k_{dy} &= 3000, \\ k_{fy} &= 0. \end{aligned} \quad (38)$$

The resulting force tracking response in the force controllable direction is shown in Fig. 1 and the position tracking response in the position controllable direction is depicted in Fig. 2. From the figures, it can be seen that the robotic finger has precisely exerted the desired force on the environment, where  $f_e$  converges to  $f_d$ , while tracking the reference path in y direction accurately, even though the exact environment location is unknown in advance. The result verifies that proposed controller is effective in compensating the unknown environment location and driving the robotic finger to accomplish the desired task.

The estimation of the inaccuracy in the initial environment location estimate,  $\hat{\delta}_{xe}$  and its derivative,  $\dot{\hat{\delta}}_{xe}$  are shown in Figures 3 and 4 respectively. It can be seen that the estimations remain bounded as desired, but do not converge to the true value. However this does not affect the performance of the whole control system since the parameters are only need to be bounded to guarantee the system's stability and the convergence of the force error to zero.

In comparison to the method introduced in [7], the method in this study requires two adaptation gains to be tuned, which are  $Q_{xe}^{-1}$  and  $Q_{x\dot{e}}^{-1}$ , whereas the method in [7] need only one adaptation gain. However, in terms of performance, it can be

seen from the simulation results that both methods give excellent outputs in controlling the finger to operate in an environment with uncertain time-varying location.

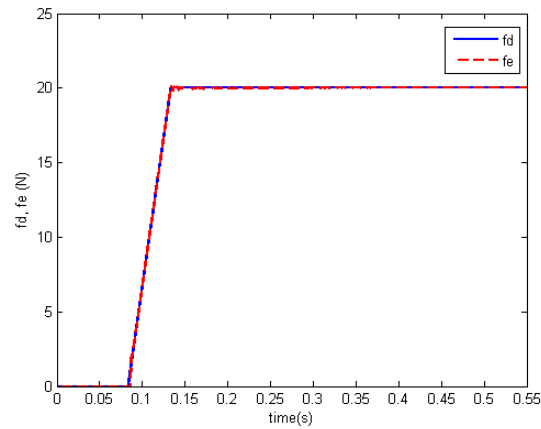


Fig. 1 Force response in the force controllable direction

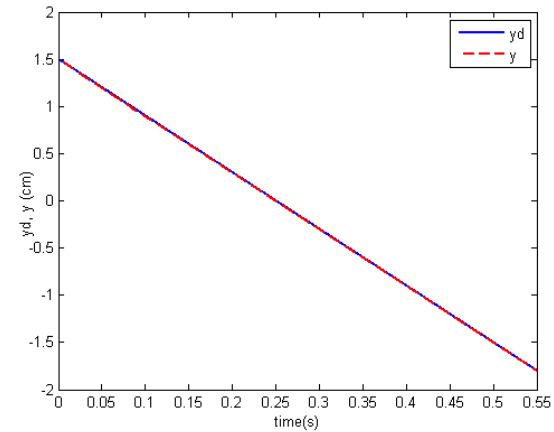


Fig. 2 Position response in the position controllable direction

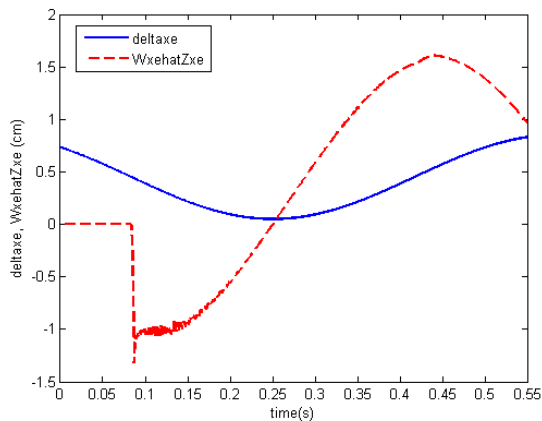
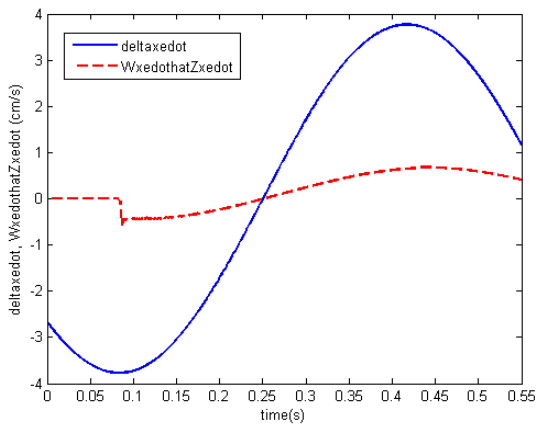


Fig. 3 Approximation of  $\delta_{xe}$  by FAT

Fig. 4 Approximation of  $\dot{\delta}_{xe}$  by FAT

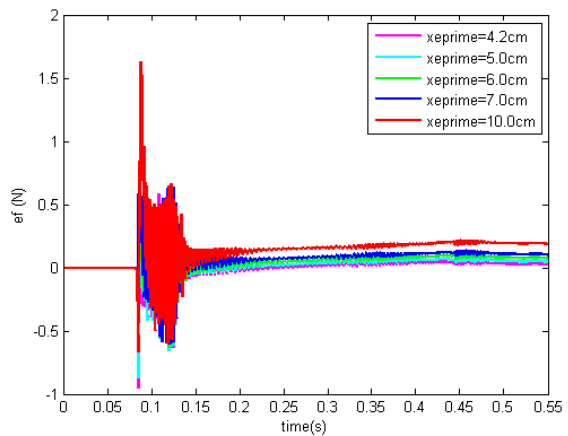
### B. Effect of the Initial Environment Location Estimate, $x_e'$ Selection

In the proposed approach, the value of the initial estimation of the environment location,  $x_e'$  in the target impedance (1) has to be selected before the control algorithm can be executed. The performance of the system has been evaluated for different values of inaccurate environment location estimate, where  $x_e' = 0.042$  m, 0.050 m, 0.060 m, 0.070 m and 0.100 m. The same true environment,  $x_e$  as in (36) and simulation setting as in the previous subsection have been implemented in this study.

Fig. 5 illustrates the force error of the system for different values of  $x_e'$ . From the result it can be seen that, small steady state error can be observed for all the tested values of  $x_e'$ , where  $e_f$  is less than 0.25 N at steady state. It can also be observed that under the same adaptation gain, the tracking accuracy increases if  $x_e'$  is chosen closer to the true value,  $x_e$ .

## IV. CONCLUSION

An adaptive impedance control based on Function Approximation Technique (FAT) to compensate for unknown time-varying environment location is presented in this paper. The target impedance for the force controllable direction has been modified by including FAT-based adaptive compensators, where the uncertainties are expressed by FAT. The estimation of the weighting function has been updated based on the force error feedback. The simulation results show that the proposed method has successfully controlled the robotic finger to exert the desired force while following the reference position trajectory, even though the exact knowledge of the environment position is unavailable a priori. The investigation on the selection initial environment location

Fig. 5 Force error response in x direction for different values of  $x_e'$ 

estimate,  $x_e'$  reveals that for the same value of  $Q_{xe}^{-1}$ , selecting  $x_e'$  closer to  $x_e$  results in a more accurate tracking performance.

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