

Adaptive Filtering in Subbands for Supervised Source Separation

Bruna Luisa Ramos Prado Vasques, Mariane Rembold Petraglia, Antonio Petraglia

Abstract—This paper investigates MIMO (Multiple-Input Multiple-Output) adaptive filtering techniques for the application of supervised source separation in the context of convolutive mixtures. From the observation that there is correlation among the signals of the different mixtures, an improvement in the NSAF (Normalized Subband Adaptive Filter) algorithm is proposed in order to accelerate its convergence rate. Simulation results with mixtures of speech signals in reverberant environments show the superior performance of the proposed algorithm with respect to the performances of the NLMS (Normalized Least-Mean-Square) and conventional NSAF, considering both the convergence speed and SIR (Signal-to-Interference Ratio) after convergence.

Keywords—Adaptive filtering, multirate processing, normalized subband adaptive filter, source separation.

I. INTRODUCTION

THE evolution of telecommunications has driven the development of efficient techniques for digital signal processing. Adaptive filtering techniques, in particular, have attracted a great deal of interest. Due to good performance, low computational complexity and high robustness, these techniques have been widely used in a variety of applications, such as system identification, channel equalization, echo cancellation and source separation [1]-[3]. This last application will be addressed in this article.

Most acquired audio signals correspond to mixtures of signals from various sources, such as speech, music, ambient and equipment noise. Source separation consists of retrieving the original source signals of interest from one or more mixing signals. Direct applications include real-time lectures with simultaneous translation and sampling of sounds for electronic music composition. Many derivative applications are aimed at identifying impulse responses and/or modifying the mixing signal, for example in speech enhancement within hearing devices and audio rendering for multichannel devices. In some applications, excerpts of the original signals present in the mixtures are known prior to the separation [4]. In these cases, one can use supervised adaptive algorithms, such as those that will be approached in this work, to obtain the coefficients of the separation system.

The blind audio source separation (BASS) technique has been a subject of intense research over the last few years. Several successful methods have emerged, such as Independent Component Analysis (ICA) [5],

Computational Auditory Scene Analysis (CASA) [6] and Sparse Decomposition (SD) [7]. However, it is still difficult to assess the characteristics and limitations of source separation algorithms due to the lack of adequate performance measures, specially in the challenging case of convolutive mixtures. The supervised source separation approach can be a good tool to aid in the refinement of blind source separation techniques.

In this paper we investigate the separation of sources from convolutive mixtures in a supervised way, through adaptive filtering, using two algorithms: Normalized Least-Mean-Square (NLMS) [1] and Normalized Subband Adaptive Filters (NSAF) [8]. The NSAF algorithm decomposes the input and desired signals, with the purpose of generating error signals in subbands that are used to adapt the coefficients of the applied filter over the entire frequency band. This procedure causes the NSAF to differ from its predecessor subband adaptive algorithms [9], which employ distinct subfilters and independent adaptation in the different subbands.

From the observation that there is correlation among the mixture signals, a modification in the NSAF algorithm is proposed for applications in supervised source separation procedures, by including the correlation matrices of the subband signals in the coefficient updating equation, thereby accelerating the convergence of the algorithm.

II. THE SOURCE SEPARATION PROBLEM

For a system with Q sources and P sensors, linear convolutive signal mixtures can be defined according to the equation

$$\mathbf{x}(n) = \mathbf{H}(n) * \mathbf{s}(n), \quad (1)$$

where “*” is the convolution operator,

$$\mathbf{H}(n) = \begin{bmatrix} h_{11}(n) & h_{12}(n) & \dots & h_{1Q}(n) \\ h_{21}(n) & h_{22}(n) & \dots & h_{2Q}(n) \\ \vdots & \vdots & \ddots & \vdots \\ h_{P1}(n) & h_{P2}(n) & \dots & h_{PQ}(n) \end{bmatrix} \quad (2)$$

is the mixture matrix of dimensions $P \times Q$, comprising the impulse responses $h_{ij}(n)$ (corresponding to the path from the j -th source to the i -th sensor) of the mixture filters,

$$\mathbf{s}(n) = [s_1(n) \ s_2(n) \ \dots \ s_Q(n)]^T \quad (3)$$

is the vector composed of the signals from the sources, and

$$\mathbf{x}(n) = [x_1(n) \ x_2(n) \ \dots \ x_P(n)]^T \quad (4)$$

is the vector formed by the signals arriving at the sensors.

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The task of separating linear convolutive mixed sources requires the determination of a so-called separation matrix, $\mathbf{W}(n)$, which is used with the purpose of estimating the source signals out of the mixed signals by computing

$$\mathbf{y}(n) = \mathbf{W}(n) * \mathbf{x}(n), \quad (5)$$

where

$$\mathbf{y}(n) = [y_1(n) \ y_2(n) \ \dots \ y_Q(n)]^T \quad (6)$$

and

$$\mathbf{W}(n) = \begin{bmatrix} w_{11}(n) & w_{12}(n) & \dots & w_{1P}(n) \\ w_{21}(n) & w_{22}(n) & \dots & w_{2P}(n) \\ \vdots & \vdots & \ddots & \vdots \\ w_{Q1}(n) & w_{Q2}(n) & \dots & w_{QP}(n) \end{bmatrix} \quad (7)$$

is the $Q \times P$ separation matrix. If the number of sensors P is equal to the number of sources Q , the separation problem is called determined, which is the case considered in this work.

III. ADAPTIVE ALGORITHMS

In this section the NLMS and NSAF adaptive algorithms are presented. It is also described how each algorithm was adequate to the problem of supervised source separation in the context of convolutive mixtures.

A. NLMS Algorithm

The NLMS algorithm is one of the most popular adaptive filtering approaches due to its computational simplicity, proven convergence in steady state environments with Gaussian noise, and robust behavior when implemented with finite precision arithmetic [2]. The NLMS algorithm updates the filter coefficients using the error between the desired output and the signal produced by the filter. The input vectors and coefficients in the instant n are denoted, respectively, by $\mathbf{x}(n) = [x(n) \ x(n-1) \ \dots \ x(n-N+1)]^T$ and $\mathbf{w}(n) = [w_0(n) \ w_1(n) \ \dots \ w_{N-1}(n)]^T$, where N is the adaptive filter length. Therefore, the output and error equations are

$$y(n) = \mathbf{w}^T(n) \mathbf{x}(n) \quad (8)$$

and

$$e(n) = d(n) - \mathbf{w}^T(n) \mathbf{x}(n), \quad (9)$$

respectively, where $d(n)$ is the reference signal.

The update of the coefficients is given by [10]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{\delta + \mathbf{x}^T(n) \mathbf{x}(n)} e(n) \mathbf{x}(n). \quad (10)$$

The adaptation step-size μ is introduced in order to control the misadjustment of the coefficients after convergence and the regularization parameter δ to avoid very large steps when $\mathbf{x}^T(n) \mathbf{x}(n)$ becomes very small. The range of suitable values for μ is [10]

$$0 \leq \mu \leq 2. \quad (11)$$

B. NLMS Algorithm for the Source Separation Problem

In order to adequate the NLMS algorithm to the problem of supervised source separation, assuming for simplicity the case of two sources and two sensors, we obtain the output signals, $y_1(n)$ and $y_2(n)$, and the error signals, $e_1(n)$ and $e_2(n)$, through the equations

$$y_i(n) = \mathbf{w}_i^T(n) \mathbf{x}(n) \quad (12)$$

and

$$e_i(n) = d_i(n) - y_i(n), \quad (13)$$

where the input vector $\mathbf{x}(n)$ is composed of the two mixed signal vectors

$$\mathbf{x}_i(n) = [x_i(n) \ x_i(n-1) \ \dots \ x_i(n-N+1)]^T, \quad (14)$$

for $i = 1, 2$, that is,

$$\mathbf{x}(n) = [\mathbf{x}_1^T(n) \ \mathbf{x}_2^T(n)]^T. \quad (15)$$

The coefficient vectors $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$ have coefficients of the two filters that generate each output, given by

$$\mathbf{w}_1(n) = [\mathbf{w}_{11}^T(n) \ \mathbf{w}_{12}^T(n)]^T \quad (16)$$

and

$$\mathbf{w}_2(n) = [\mathbf{w}_{21}^T(n) \ \mathbf{w}_{22}^T(n)]^T, \quad (17)$$

where

$$\mathbf{w}_{ij}(n) = [w_{ij,0}(n) \ w_{ij,1}(n) \ \dots \ w_{ij,N-1}(n)]^T. \quad (18)$$

The indices i and j are related to the sources and mixtures, respectively, and N is the number of coefficients of each separation filter. Therefore, at each iteration, the updates of the coefficient vectors $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$ are accomplished according to equation

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{\mu}{\delta + \mathbf{x}^T(n) \mathbf{x}(n)} e_i(n) \mathbf{x}(n). \quad (19)$$

C. NSAF Algorithm

The NLMS algorithm converges slowly when the input signal is colored. To solve this problem, a compelling approach is to use subband filtering in which the colored input signal is decomposed into almost mutually exclusive frequency bands and the decimated signal of each subband is approximately white [11].

The NSAF algorithm proposed in [8] decomposes the input and the desired signals into subbands, allowing the use of particular properties of each resulting signal. This algorithm exploits the principle of minimum perturbation, in which, from one iteration to the next, the coefficients of the adaptive filter must be altered in a minimum way, subject to restrictions imposed to the subband errors after the update (null subband *a posteriori* errors). Since the updating of the coefficients is carried out at a lower rate by decimating the error signals, computational cost increase is very small [12] when compared to the NLMS algorithm, whereas coefficient convergence is faster for colored input signals. A unique feature of the NSAF algorithm relies in the fact that the signal error is computed in subbands, but the updating of the

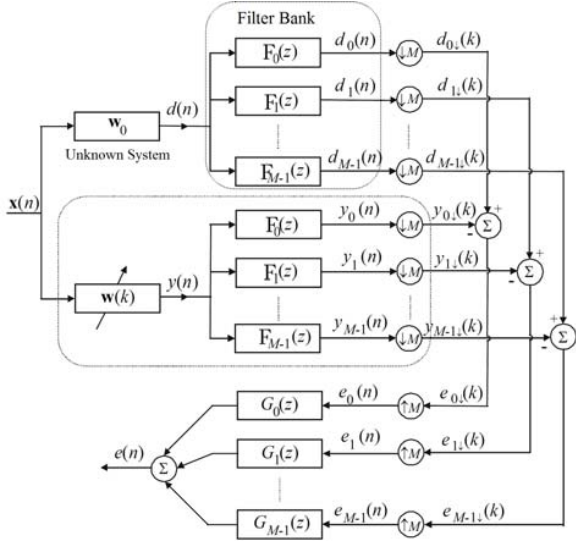


Fig. 1 NSAF structure

adaptive filter coefficients is performed in fullband, as shown in Fig. 1. In the NSAF algorithm, the desired signal $d(n)$ and the filter output $y(n)$ are decomposed into M subbands by the analysis filters $F_0(z), \dots, F_{M-1}(z)$ and are critically decimated. The decimation factor is equal to the number of subbands. Therefore, the decimated output signal of the m -th subband is given by

$$y_{m\downarrow}(k) = \sum_{l=0}^{N-1} \hat{w}_l(k) x_m(kM - l) = \hat{\mathbf{w}}^T(k) \mathbf{x}_m(k), \quad (20)$$

where

$$\mathbf{x}_m(k) = [x_m(kM) \ x_m(kM - 1) \ \dots \ x_m(kM - N + 1)]^T \quad (21)$$

and

$$\hat{\mathbf{w}}(k) = [\hat{w}_0(k) \ \hat{w}_1(k) \ \dots \ \hat{w}_{N-1}(k)]^T. \quad (22)$$

The corresponding error signal is given by

$$e_{m\downarrow}(k) = d_{m\downarrow}(k) - \hat{\mathbf{w}}^T(k) \mathbf{x}_m(k), \quad (23)$$

where $d_{m\downarrow}(k)$ is the desired decimated signal of the m -th subband.

Coefficient updates of the NSAF algorithm are given by

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu \sum_{m=0}^{M-1} \frac{\mathbf{x}_m(k)}{\delta + \|\mathbf{x}_m(k)\|^2} e_{m\downarrow}(k), \quad (24)$$

where μ and δ are introduced with similar purposes as those adopted for the NLMS algorithm.

D. NSAF Algorithm for the Source Separation Problem

To fit the NSAF algorithm to the supervised source separation problem, a procedure similar to the NLMS algorithm was used. Accordingly, (23) and (24) were modified to generate the error signals in subbands and to update the coefficients of the separation filters. Considering the case of

two sources and two sensors, the filter coefficient vectors that generate the source estimates are defined as

$$\hat{\mathbf{w}}_1(k) = [\hat{\mathbf{w}}_{11}^T(k) \ \hat{\mathbf{w}}_{12}^T(k)]^T \quad (25)$$

and

$$\hat{\mathbf{w}}_2(k) = [\hat{\mathbf{w}}_{21}^T(k) \ \hat{\mathbf{w}}_{22}^T(k)]^T, \quad (26)$$

where

$$\hat{\mathbf{w}}_{ij}(k) = [\hat{w}_{ij,0}(k) \ \hat{w}_{ij,1}(k) \ \dots \ \hat{w}_{ij,N-1}(k)]^T \quad (27)$$

is the N -th length filter coefficient vector that generates the portion of the estimation of the i -th source from the j -th mixture. Defining the vector with the samples of the mixture signals in the m -th subband as

$$\mathbf{x}_m(k) = [\mathbf{x}_{1,m}^T(k) \ \mathbf{x}_{2,m}^T(k)]^T, \quad (28)$$

where

$$\mathbf{x}_{i,m}(k) = [x_{i,m}(kM) \ x_{i,m}(kM - 1) \ \dots \ x_{i,m}(kM - N + 1)]^T, \quad (29)$$

the decimated subband error signal and the update equation of the separation system coefficients relative to the i -th source estimate are given, respectively, by

$$e_{i,m\downarrow}(k) = d_{i,m\downarrow}(k) - \hat{\mathbf{w}}_i^T(k) \mathbf{x}_m(k) \quad (30)$$

and

$$\hat{\mathbf{w}}_i(k+1) = \hat{\mathbf{w}}_i(k) + \mu \sum_{m=0}^{M-1} \frac{\mathbf{x}_m(k)}{\delta + \|\mathbf{x}_m(k)\|^2} e_{i,m\downarrow}(k). \quad (31)$$

E. Improved NSAF Algorithm for the Source Separation Problem

From the observation that there is correlation among the signals of the mixtures, we have introduced in the update equation of the NSAF algorithm estimates of the correlation matrices of the subband mixed signals, $\hat{\mathbf{R}}_m(k) = E[\mathbf{x}_m(k) \mathbf{x}_m^T(k)]$, in order to accelerate its convergence, obtaining

$$\hat{\mathbf{w}}_i(k+1) = \hat{\mathbf{w}}_i(k) + \mu \sum_{m=0}^{M-1} \hat{\mathbf{R}}_m^{-1}(k) \frac{\mathbf{x}_m(k)}{\delta + \|\mathbf{x}_m(k)\|^2} e_{i,m\downarrow}(k), \quad (32)$$

where

$$\hat{\mathbf{R}}_m(k) = \mathbf{x}_m(k) \mathbf{x}_m^T(k) = \begin{bmatrix} \hat{\mathbf{R}}_{m,11}(k) & \hat{\mathbf{R}}_{m,12}(k) \\ \hat{\mathbf{R}}_{m,21}(k) & \hat{\mathbf{R}}_{m,22}(k) \end{bmatrix}. \quad (33)$$

Considering that the decimated subband input signals are approximately white, the matrices $\hat{\mathbf{R}}_{m,ij}(k)$ can be approximated by diagonal matrices given by

$$\hat{\mathbf{R}}_{m,ij}(k) \approx \sigma_{m,ij}^2(k) \mathbf{I}, \quad (34)$$

where

$$\sigma_{m,ij}^2(k) = \mathbf{x}_{i,m}^T(k) \mathbf{x}_{j,m}(k). \quad (35)$$

Defining the inverse correlation matrix of the mixed signals in the m -th subband as

$$\hat{\mathbf{R}}_m(k)^{-1} = \begin{bmatrix} \hat{\mathbf{S}}_{m,11}(k) & \hat{\mathbf{S}}_{m,12}(k) \\ \hat{\mathbf{S}}_{m,21}(k) & \hat{\mathbf{S}}_{m,22}(k) \end{bmatrix}, \quad (36)$$

using the approximation (34) and noting that $\sigma_{m,12}^2(k) = \sigma_{m,21}^2(k)$, we obtain

$$\hat{\mathbf{S}}_{m,11}(k) = (\sigma_{m,11}^2(k) - \sigma_{m,12}^4(k)\sigma_{m,22}^{-2}(k))^{-1}\mathbf{I}, \quad (37)$$

$$\hat{\mathbf{S}}_{m,22}(k) = (\sigma_{m,22}^2(k) - \sigma_{m,12}^4(k)\sigma_{m,11}^{-2}(k))^{-1}\mathbf{I}, \quad (38)$$

$$\hat{\mathbf{S}}_{m,12}(k) = -\sigma_{m,11}^{-2}(k)\sigma_{m,12}^2(k)(\sigma_{m,22}^2(k) - \sigma_{m,12}^4(k)\sigma_{m,11}^{-2}(k))^{-1}\mathbf{I}, \quad (39)$$

$$\hat{\mathbf{S}}_{m,21}(k) = -\sigma_{m,22}^{-2}(k)\sigma_{m,12}^2(k)(\sigma_{m,11}^2(k) - \sigma_{m,12}^4(k)\sigma_{m,22}^{-2}(k))^{-1}\mathbf{I}. \quad (40)$$

The increase in the computational complexity resulting from the introduction of the inverse of the correction matrices in the updating equation (31) is of $3M^2 + 8M$ multiplications. Since, in general, the number of subbands, M , is much smaller than the length of the adaptive filters, N , this increase is not significant.

IV. DESIRED SIGNAL FOR THE EVALUATION OF THE BLIND SOURCE SEPARATION

In the problem of blind separation of audio signals in convolutive mixtures, it is generally impossible to recover the original signals from the sources, being allowed to obtain as a valid solution filtered versions of the original signals. Denoting by $H_{ij}(z)$ the corresponding transfer function of the i -th source to the j -th sensor, one can write in the z domain the mixture signals $X_i(z)$ and the outputs of the separation system $Y_i(z)$ for the problem of two sources and two sensors as

$$X_1(z) = H_{11}(z)S_1(z) + H_{12}(z)S_2(z), \quad (41)$$

$$X_2(z) = H_{21}(z)S_1(z) + H_{22}(z)S_2(z), \quad (42)$$

$$Y_1(z) = W_{11}(z)X_1(z) + W_{12}(z)X_2(z), \quad (43)$$

$$Y_2(z) = W_{21}(z)X_1(z) + W_{22}(z)X_2(z). \quad (44)$$

Substituting equations (41) and (42) in (43) and (44), we obtain

$$Y_1(z) = (W_{11}(z)H_{11}(z) + W_{12}(z)H_{21}(z))S_1(z) + (W_{11}(z)H_{12}(z) + W_{12}(z)H_{22}(z))S_2(z), \quad (45)$$

$$Y_2(z) = (W_{21}(z)H_{11}(z) + W_{22}(z)H_{21}(z))S_1(z) + (W_{21}(z)H_{12}(z) + W_{22}(z)H_{22}(z))S_2(z). \quad (46)$$

To find a possible solution to the problem, it was imposed on the system the condition that the desired output signal $D_i(z)$ should represent a filtered version of the original signal $S_i(z)$, for $i = 1, 2$. To this end, it is enough to equal to zero the portion of the signal corresponding to $S_2(z)$ in the equation of $Y_1(z)$ and the portion of the corresponding signal to $S_1(z)$ in the equation of $Y_2(z)$, that is,

$$W_{11}(z)H_{12}(z) + W_{12}(z)H_{22}(z) = 0, \quad (47)$$

$$W_{21}(z)H_{11}(z) + W_{22}(z)H_{21}(z) = 0. \quad (48)$$

The above equations have infinite solutions and the one adopted was $W_{11}(z) = H_{22}(z)$, $W_{12}(z) = -H_{12}(z)$, $W_{21}(z) = H_{21}(z)$ e $W_{22}(z) = -H_{11}(z)$. Thus, the signals $D_1(z)$ and $D_2(z)$ become

$$D_1(z) = (W_{11}(z)H_{11}(z) + W_{12}(z)H_{21}(z))S_1(z) = (H_{22}(z)H_{11}(z) - H_{12}(z)H_{21}(z))S_1(z), \quad (49)$$

$$D_2(z) = (W_{21}(z)H_{12}(z) + W_{22}(z)H_{22}(z))S_2(z) = (H_{12}(z)H_{21}(z) - H_{11}(z)H_{22}(z))S_2(z). \quad (50)$$

The desired signals from the above equations were used in the simulations, whose results are presented in the next section.

V. SIMULATION RESULTS

The adaptive algorithms for supervised source separation described in Section III were evaluated using two convolutive mixtures of two speech signals, corresponding to the signals acquired by two microphones separated 5 cm from each other in reverberant environment and sampled at 8 kHz. Simulations of acoustic room propagation were developed using the "Image-Source" model described in [13]. The simulations were performed for three different reverberation times (T_{60}): 0.1 s, 0.25 s and 0.5 s. The evaluation measure used was the SIR (Signal-to-Interference Ratio).

For the subband decomposition, a cosine-modulated filter bank [11] with $M = 4$ subbands and prototype filter of length 32 was used. The step-size and regularization parameters employed in all algorithms were $\mu = 0.5$ and $\delta = 0.1$, respectively.

Figs. 2-and 4 show the evolution of the SIR along the iterations obtained with the NLMS, NSAF and improved NSAF algorithms for $T_{60} = 0.1$ s, 0.25 s and 0.5 s, respectively, with separation filter lengths $N = 623, 1599$ and 3227, which are equal to the lengths of the mixing filters L .

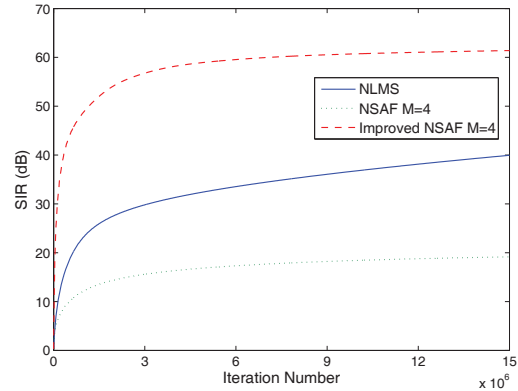
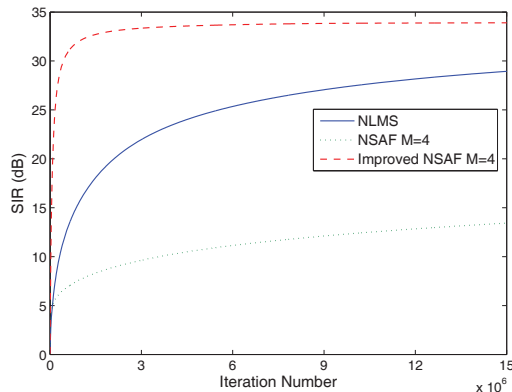
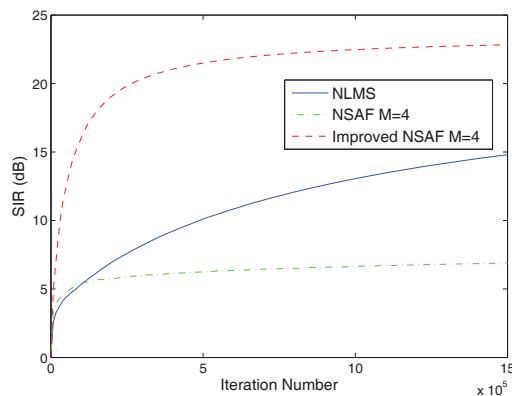


Fig. 2 SIR evolution (in dB) for $T_{60} = 0.1$ s

It can be observed in these figures that the improved NSAF algorithm presents a considerably higher convergence rate than those of the NLMS algorithm and of the conventional NSAF algorithm for the problem of supervised source separation. As expected, the introduction of the correlation matrix of the mixed signals into the coefficient update equation accelerates the convergence of the NSAF algorithm, which is slow due to the strong correlation among the signals of the mixtures. Subband processing exploits the characteristics of the mixed signals in the subbands by normalizing the adaptation step, resulting in higher SIR values than those obtained with the NLMS algorithm.

From Figs. 2-4, it can also be concluded that there is a limitation in the source separation model employing finite impulse response (FIR) filters, which produces smaller SIR

Fig. 3 SIR evolution (in dB) for $T_{60} = 0.25$ sFig. 4 SIR evolution (in dB) for $T_{60} = 0.5$ s

values as the reverberation time increases. Therefore, no matter how good the blind separation algorithm is, for high-throughput cases, the maximum SIR to be achieved with FIR filters of length $N = L$ is limited by the adopted separation system model (around 23 dB in the simulated scenario with $T_{60} = 0.5$ s), while for smaller reverberation times it is possible to obtain high SIR values (above 60 dB in the simulated scenario with $T_{60} = 0.1$ s).

VI. CONCLUSIONS

In this work, supervised adaptive algorithms were applied to the source separation problem, considering linear determined convolutive mixtures and separation system composed of FIR filters. Due to the correlation among the mixed signals, conventional adaptive algorithms, both in fullband and subband structures, have slow convergence and result in low SIR values. In order to improve the performance of the NSAF algorithm for source separation, correlation matrices of the mixed signals in the different subbands were introduced into the coefficient update equations, which can be approximated by block diagonal matrices using the whitening property of the decimated subband signals, resulting in a small increase in computational complexity over the conventional NSAF algorithm. Simulation results using speech signals in reverberant environments confirmed the improved performance of the proposed algorithm with respect to the conventional

algorithms, and quantified the limitations in the SIR obtained with the separation system model that employs FIR filters.

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