Active Suspension - Case Study on Robust Control

Kruczek A., Stříbrský A., Honcù J., Hlinovský M.

Abstract—Automotive suspension system is important part of car comfort and safety. In this article automotive active suspension with linear motor as actuator is designed using H-infinity control. This paper is focused on comparison of different controller designed for quart, half or full-car model (and always used for "full" car). Special attention is placed on energy demand of the whole system. Each controller configuration is simulated and then verified on the hydraulic quarter car test bed.

Keywords—active suspension, linear motor, robust control

I. INTRODUCTION

RECENTLY there is an increased demand on automotive suspension system design. The basic function of the vehicle suspension is to provide comfort to passengers, maximize the friction between the tires and the road surface and provide steering stability with good handling. In order to improve handling and comfort performance instead of a conventional static spring and damper system, semi-active and active systems are being developed. There are, of course, numerous variations and different configurations of suspension. In experimental active systems, the force input is usually provided by hydraulic actuators. As an alternative approach to active suspension system design electromechanical actuators are being studied. Such actuators would provide a direct interface between electronic control and the suspension systems.

In the time of growing interest of renewable energy resources the minimization of energy consumption can be small contribution to better utilization of energy resources. Especially for the car application the energy consumption play important role of the design process. In this paper the linear electric motor is used as an actuator and then there is possibility to recuperate energy during specific movement of suspension, accumulate it and use it later when necessary.

All suspension systems are designed to meet specific requirements. In suspension systems, usually two most important features are expected to be improved - disturbance absorbing (i.e. passenger comfort) and attenuation of the disturbance transfer to the road (i.e. car handling). The first requirement could be presented as an attenuation of the damped mass acceleration or as a peak minimization of the damped mass vertical displacement. The second one is characterized as an attenuation of the force acting on the road or - in simple car model - as an attenuation of the unsprung mass acceleration. It is obvious that there is a contradiction between these two requirements. Effort devoted to passive suspension design is ineffective, because there is a contradiction between both requirements. The best result (in sense of requirements improvement) can be achieved by active suspension, which means that some additional force can act on system.

With respect to these contradictory requirements the best results can be achieved using active suspension systems generating variable mechanical force acting in the system using a linear electrical motor as the actuator. Compared to traditional drives using rotational electro-motors and lead screw or toothed belts, the direct drive linear motor exhibits the property of contact-less transfer of electrical power according to the laws of magnetic induction. The electromagnetic force is applied directly without the intervention of a mechanical transmission. Low friction and no backlash resulting in high accuracy, high acceleration and velocity, high force, high reliability and long lifetime enable not only effective usage of modern control systems but also represent the important attributes needed to control vibration suspension efficiently.

II. MODEL AND EXPERIMENT CONFIGURATION

Most important part of controller design is to develop and use appropriate mathematic model, which is later validated during experiments. This paper is prepared to compare different system configurations – with quarter, half and full-car model controller.

A. Quarter car suspension model

Let's start with the simplest quarter car model (see Fig. 1). It contains two springs (one in suspension and second representing car tires), one dumper and source of power as actuator.

In fact a linear electric motor is used as an actuator but presumption to use ideal source of power can be achieved, because the linear motor in its working range has nearly linear characteristic and very small time constants.

The model is described by the following motion equations:

$$m_b \ddot{z}_b = F - k_b(z_b - z_w) - b_b(\dot{z}_b - \dot{z}_w)$$
$$m_w \ddot{z}_w = -F + k_b(z_b - z_w) - k_w(z_w - z_r) + b_b(\dot{z}_b - \dot{z}_w) \quad (1)$$

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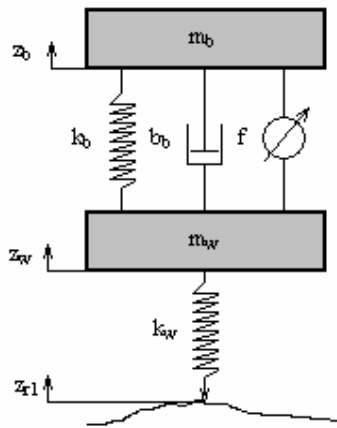
Fig. 1 Quarter car model schema

B. Half car suspension model

Natural expansion of quarter car model is half car model for the left and right part of the car. Model is shown in the Fig. 2. Then the following motion equations are added for pitching, centre of gravity movement and body speed, respectively:

$$m_b \ddot{z}_{b1} L_1 - m_b \ddot{z}_{b2} L_2 - J_P \dot{\omega} = 0$$
$$m_b \ddot{z}_{b1} + m_b \ddot{z}_{b2} - m_b \dot{v}_T = 0$$
$$\dot{z}_{b1} = v_T + \omega L_1$$
$$\dot{z}_{b2} = v_T - \omega L_2 \tag{2}$$

Of course, there is possibility to prepare model for the front and rear part also. In fact we can use the same equations and rotate the model for 90°.
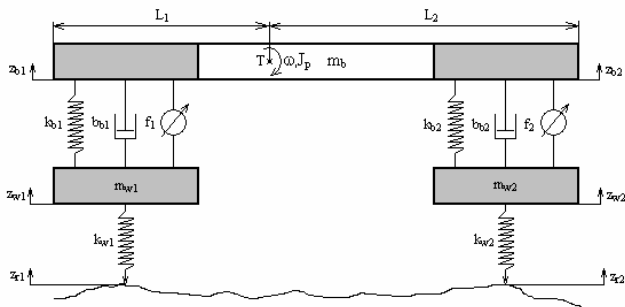

Fig. 2 Half car model schema

C. Full car suspension model

Final model in this paper is full car model. It means two half car model are linked together. Then the following motion equations are added for pitching, rolling, centre of gravity movement and body speed, respectively:

$$F_1 L_1 - F_2 L_2 + F_3 L_1 - F_4 L_2 - J_P \dot{\omega} = 0$$
$$F_1 D_1 + F_2 D_2 - F_3 D_1 - F_4 D_2 - J_{PR} \dot{\Omega} = 0$$
$$F_1 + F_2 + F_3 + F_4 - m_b \dot{v}_T = 0$$
$$\dot{z}_{b1} = v_T + \omega L_1 + \Omega D_1$$
$$\dot{z}_{b2} = v_T - \omega L_2 + \Omega D_2$$
$$\dot{z}_{b3} = v_T + \omega L_3 - \Omega D_3$$
$$\dot{z}_{b4} = v_T - \omega L_4 - \Omega D_4 \tag{3}$$

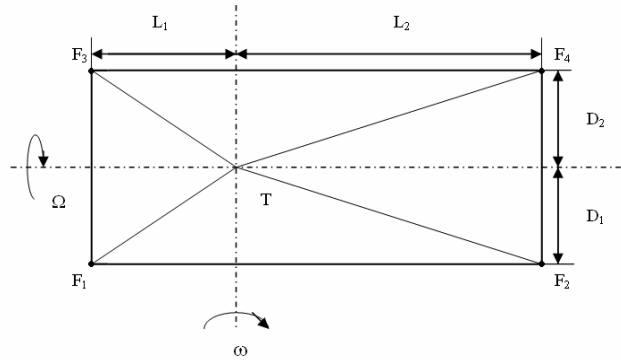More about car behaviour and modelling can be found in [1].

Fig. 3 Full car model schema

D. Experiment configuration

Special quarter car test bed has been prepared for described model validation. Mechanical configuration is obvious from the Fig. 4. The test bed consists of hydraulic source of power (as a input road disturbance), one quarter of the car and linear electric motor (as actuator).

Fig. 4 Experimental test bed

III. LINEAR ELECTRIC MOTOR

Linear electric motor has been used as an actuator generating required forces. Fig.5 represents the basic principle and configuration of the linear motor. The beauty of linear motors is that they directly translate electrical energy into usable linear mechanical force and motion, and vice versa. The motors are produced in synchronous and asynchronous versions. Compared to conventional rotational electro motors, the stator and the shaft (translator) of direct-drive linear motors are linear-shaped. One can imagine such a motor taking infinite stator diameter.
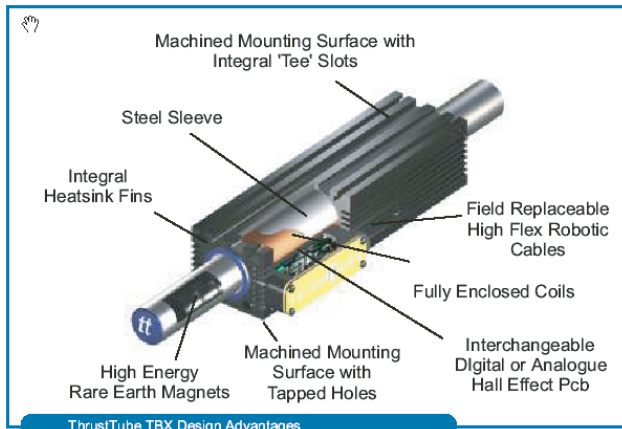

Fig. 5 Linear electric motor TBX3810

Linear motor translator movements take place with high velocities (up to approximately 200m/min), large accelerations (up to g multiples), and forces (up to kN). As mentioned above, the electromagnetic force can be applied directly to the payload without the intervention of a mechanical transmission, what results in high rigidity of the whole system, its higher reliability and longer lifetime. In practice, the most often used type is the synchronous three-phase linear motor. In this research the ThrustTube TBX3810 motor is used (see [2]).

It is necessary to answer one important question – if it is more advantageous to include the model of the linear electric motor in the model for active suspension synthesis or if it should be used only for simulations.

Comparing advantages and disadvantages of the model inclusion, it can be said that the closed-loop provides more information so that better control results can be achieved. Unfortunately, there are also some significant disadvantages in such a solution (complexity, nonlinearity etc.).

There is another important question whether the linear motor model could be omitted and a linear character of the desired force could be supposed. The answer is "yes". Both the mechanical and the electrical constants are very small – just about 1ms.

The detailed non-linear motor model used for simulation (not for synthesis) is described in [3] or [4].

IV. CONTROLLERS

According to mathematical models described in the previous section, three different controllers have been designed. Starting with easiest quarter car and ending with full car model controller.

The controllers have been developed using H-infinity control theory (for more details see [5]). More details about controller design (described in the following sections) can be found in [6].

Standard closed loop schema is plotted in Fig. 6. System has two inputs – road disturbance and acting signal for linear motor. The first output is used to modify performance and robustness behaviour of resulted closed loop. Second output is the feedback signal for controller.
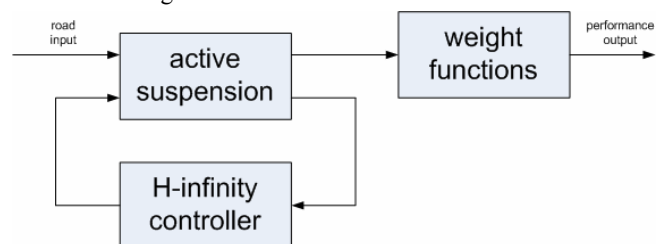

Fig. 6 Standard H-infinity configuration

The first output can be weighted by either by constant or by frequency function to achieve the desired closed loop characteristic. How the function or constant was developed is described the following sections (they are different for each controller configuration).

A. Quarter car controller

The first developed controller is for quarter car model. Then the same controller is used four times for each suspension element (front – rear, left – right). Configuration is obvious from the Figure 7. Of course the disadvantage is the controller has not the information about other elements. It means there is only small (and indirect) possibility to influence rolling and pitching.

Following weights have been used:
- road to wheel deviation – help to improve steady state deviation, influence stability of the car
- body acceleration – improve comfort of passenger, weighted by frequency function
- acting signal – provide possibility to limit desired actuator force, weighted by function
- wheel acceleration

As mentioned above, two outputs are weighted by function. First the body acceleration which is weighted according to different passenger sensitivity in different frequency ranges. Second the acting signal where the higher frequencies are limited by this function. In fact high pass filter (because of inverse) is used for weighting.
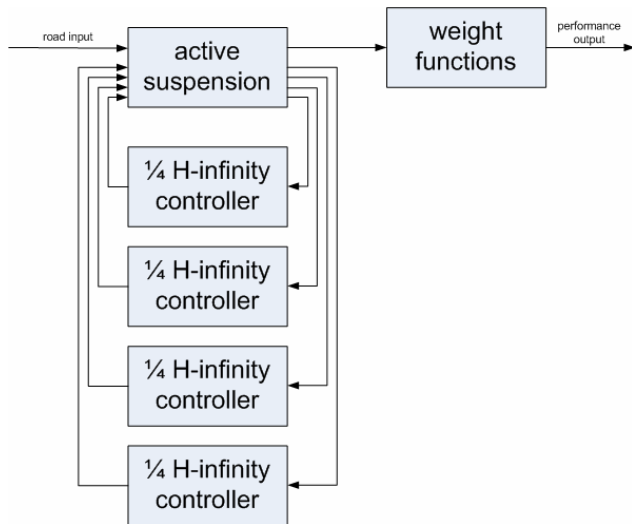
Fig. 7 Quarter car controller configuration

B. Half car controller

Half car controller has one advantage against quarter car – it can directly influence either the rolling or pitching of the car. It implies we have two possibilities during the half car controller design. Configuration is drawn in Figure 8.

The first possibility is to divide the car to the left and right half. Then the pitching can be influenced and resulted controller has been used two times for each element (left – right). For weighting the same outputs have been used as for quarter car (of course the values for weighting are different! because of influence between front and rear part of the car). Additional weighting constant is added – angle of pitching.

The second possibility is to divide the car to the front and rear half. Then the rolling can be influenced and resulted controller has been used two times for each element (front – rear). Additional weighting constant is added – angle of rolling.
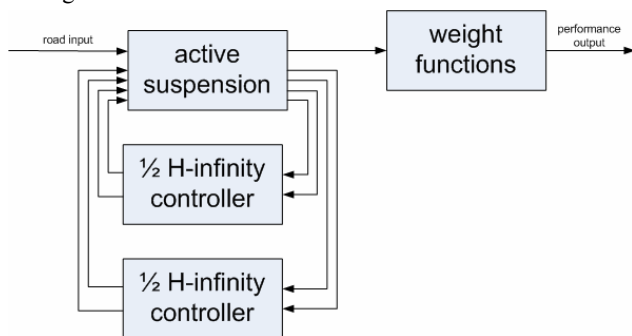

Fig. 8 Half car controller configuration

C. Full car controller

Of course last possibility how to control full suspension configuration is to develop controller for exactly this model. Then there is possibility to control all – each suspension element properties, rolling and pitching. Obviously such a controller is most difficult to design. Configuration is in
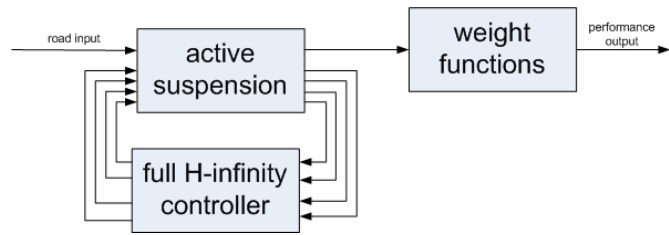

Fig. 9 Full car controller configuration

V. INPUT SIGNAL

There are many opportunities how to simulate road disturbance and there exists many possible models. In this paper two of them have been used.

A. Slow-down retarder

One specific situation important for car comfort and road friendliness has been chosen – the slow down retarder. Size of this jump is 0.5m x 0.45m x 0.05m.

As a mathematical model the half of sinus function $\sin(75,36\ t)$ is used and of course, the longitudal velocity has to be taken into consideration.

B. Common road (deterministic)

One possibility how to model common road is to use deterministic "random" signal. It can be described by the following equation:

$$z_r = \sum_{i=1}^{n} \sqrt{\frac{\dot{\omega}}{\pi \cdot v_x}} \left\{ \operatorname{Re}\left(\frac{b_0}{-\omega^2 + a_1 j\omega + a_0}\right) \cdot \cos(\omega t + \alpha_i) + \operatorname{Im}\left(\frac{b_0}{-\omega^2 + a_1 j\omega + a_0}\right) \cdot \sin(\omega t + \alpha_i) \right\}$$

$$b_0 = 0.121 \cdot v_x \qquad (4)$$
$$a_0 = 2.249 \cdot v_x$$
$$a_1 = 30.36 \cdot v_x$$

VI. RESULTS

The main objective of the paper was to compare different H-infinity controller. If we are thinking about each condition in controllers then the results are not surprising, but some aspects are at least important from the practical aspects. Let's take a look at it.

A. Results quantification

Again as for road model there are many possibilities how to quantify simulation or experimental results. Of course the results are only one so the quantification always must give the same comparison. In this paper the RMS value for body acceleration (comfort) and wheel-road deviation (stability). Finally the stability has not been affected by the different controllers, so only comfort is evaluated bellow.

The RMS for comfort is defined as:

$$J_{comf} = \sqrt{\int_0^T G_w * \ddot{z}_b \, dt}$$

(5)

Where Gw in (5) is transfer function with the response respecting comfort sensitivity of human being and * means the convolution.

B. Final results and comparison

For closed-loop illustration there is a body acceleration (in the Fig. 10) response on a driving through slow-down retarder described in section 5.1. for quarter car model configuration (full controller = green, half controller=red, passive=blue).
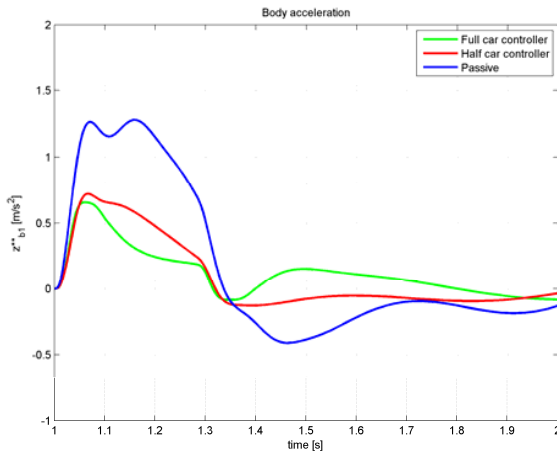

Fig. 10 Body acceleration on a slow-down retarder

Next figure (Fig. 11) illustrates the angular velocity of pitching of passive suspension (blue), half car controller (red) and full car controller (green) as a response during driving through slow-down retarder. There is an obvious improvement of half and full car controller against passive suspension and there is minimum difference between half and full controller (as is proven in Table II).Finally is the time to compare each method of control used for automotive active suspension. Both described signal was tested during simulations (and for quarter car during experiment). The results are summarized in the Table 1. The RMS for comfort (mentioned above) is compared for each controller, whereas "passive" controller means without control.
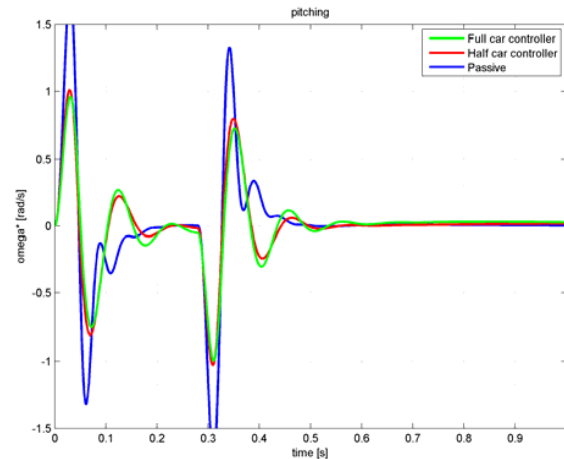

Fig. 11 Angular velocity on a slow-down retarder

TABLE 1
COMFORT (RMS) COMPARISON

	Slow-down retarder	Common road
Passive	0.2738	0.3860
¼ controller	0.2201	0.2931
½ controller	0.2674	0.3438
Full controller	0.2698	0.3622

Second table compares the rolling of the car for each controller. This gives us the picture how seems the results in case there are not (or there are) information about each element. Results are in Table II.

Both tables acknowledge the results illustrated on slow down retarder in Figures 9 and 10. Similar results has been obtained for simulations on "common road" described in section 5.2.

TABLE II
PITCH ANGLE (RMS) COMPARISON

	Slow-down retarder	Common road
Passive	0.0207	0.0362
¼ controller	0.0190	0.0254
½ controller	0.0133	0.0194
Full controller	0.0138	0.0196

VII. CONCLUSION

In this paper several H-infinity controllers (with different complexity) for active suspension with linear electric motor have been designed and then compared together. The experiment signal for real road simulation has been developed and then it has been used for simulations experiments.

It has been proved that it is not necessary to design the complex controller to achieve the best result. It can be observed that quarter car controller in full car simulations shows the better results for passenger comfort at a cost of the worse pitching (or rolling) of the car.

This is true for H-infinity control due to its robustness, but probably it will not be valid for some type of predictive control, where the signal from the front wheel can be used in the rear of the car. This should be inspiration for future research.

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