International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:8, No:1, 2014

Accurate Modeling and Nonlinear Finite Element Analysis of a Flexible-Link Manipulator

M. Pala Prasad Reddy, Jeevamma Jacob

***Abstract*—**Accurate dynamic modeling and analysis of flexible link manipulator (FLM) with non linear dynamics is very difficult due to distributed link flexibility and few studies have been conducted based on assumed modes method (AMM) and finite element models. In this paper a nonlinear dynamic model with first two elastic modes is derived using combined Euler/Lagrange and AMM approaches. Significant dynamics associated with the system such as hub inertia, payload, structural damping, friction at joints, combined link and joint flexibility are incorporated to obtain the complete and accurate dynamic model. The response of the FLM to the applied bang-bang torque input is compared against the models derived from LS-DYNA finite element discretization approach and linear finite element models. Dynamic analysis is conducted using LS-DYNA finite element model which uses the explicit time integration scheme to simulate the system. Parametric study is conducted to show the impact payload mass. A numerical result shows that the LS-DYNA model gives the smooth hub-angle profile.

***Keywords*—**Flexible link manipulator, AMM, FEM, LS-DYNA, Bang-bang torque input.

I. Introduction

FLEXIBLE link manipulators offer several advantages over rigid-link manipulators such as achieving high speed operation, lower energy consumption, and increase in payload carrying capacity. Robot manipulators have wide range of applications, from industrial automation and medical operations to hazardous environments like space, underwater, and nuclear power plants. However, the control of flexible link manipulator (FLM) is challenging, due to distributed link flexibility, which makes the system non-minimum phase, under-actuated and infinite dimensional. Due to the flexible nature of the system, the dynamics are significantly more complex and thus it is difficult to achieve accuracy in positioning of the end-effector, and above all it's difficult to obtain an accurate model of the system.

The goal in the modeling of any system is to achieve an accurate model representing the actual system behavior. Assumed modes method (AMM) and finite element methods (FEM) are commonly used to derive the dynamic models of complex dynamical systems. AMM looks at obtaining approximate models by solving the partial differential equation (PDE) characterizing the dynamic behavior of the system. Lagrangian Mechanics and the AMM have been used to derive the dynamic model of a single-link flexible manipulator. Most of the work on modeling of FLM using AMM approach is contributed by W J. Book [1]. In Book's method, the computational algorithms for dynamic modeling of flexible manipulators are derived by using the 4x4 homogeneous transformation matrices. These transformation matrices are developed to describe kinematics information. It has been reported that the link deflections can be described by a truncated modal expansion [2] and all the dynamics computations are performed in the link coordinate systems using forward and return recursion, in which joint deflections are not addressed. A comparative study of AMM and finite element method (FEM) based model behavior reported in [3], states that AMM with quadratic displacement yields more accurate hub angle response while FEM yields hub velocity and end point acceleration response to be more accurate. Also the effect of centrifugal force on the natural frequencies is presented in this paper. The different approaches while modeling the FLM will be more apparent and the dynamic behavior of the system is affected by various factors. A literature review on dynamic analysis of FLM is presented in [4]. In [5], a linear state space model for a single-link flexible manipulator is presented, which does not account for the distributed link flexibility. The dynamic model of a two link flexible manipulator system, incorporating the hub inertia, structural damping, and payload is derived in [6], but the effect of joint flexibility and friction are not considered in this paper. In [7], the significance of joint flexibility on the modeling of FLM is presented. The inclusion of joint flexibility cuts down the total deflections but increases the oscillatory behavior. A generalized modeling frame work has been described in [8], to obtain the closed-form dynamic equations of motion for a multi-link manipulator using AMM by considering flexibility in the links and joint, it does not account for friction effects. An approach to include both the internal and external damping effects of the beam into energy equations has been by Baker et al. [9]. The effect of internal structural damping on the dynamic modeling of a planar multi-link manipulator is presented in [10]. Design of accurate control for a flexible manipulator system requires the dynamic model, which describes the complete dynamic behavior of the system. The dynamic behavior of the system is affected by various factors such as flexibility in the links and joints, hub inertia, payload, structural damping and friction effects. So, to represent the actual system it is important to consider all significant effects while modeling. In this work the effect of link flexibility, joint flexibility, hub inertia, payload, friction and internal structural damping effects are considered simultaneously to derive the complete dynamic model of the

M. Pala Prasad Reddy is with the National Institute of Technology, Calicut, Kerala, India (phone: +91-9740336936; e-mail: prasadreddy.mule@gmail.com).

Jeevamma Jacob was with the National Institute of Technology, Calicut, Kerala, India (phone: +91-9895105978; e-mail: jeeva@nitc.ac.in).

FLM using combined Euler/Lagrange and assumed modes approaches.

Finite element model is very useful for static and dynamic analysis, which uses the implicit and explicit solvers to simulate the dynamics. There are different types of tools are available to solve the finite element problems. ANSYS and CAD/CAE tools find large applications in analysis of complex real world problems. Dynamic characterization of a linear finite element model using finite difference/ finite element approach is presented in [11]. Dynamic analysis of a rotating cantilever beam using FEM for variations in the rotating speed is reported in [12]. This paper account for stretch deformation instead of axial deformation and linear partial differential equations are derived using Hamilton principle. In [13] dynamic analysis of very flexible beams with large overall deflections has been performed by using ANSYS. Solutions of nonlinear equations are obtained by a Newton-Rapshon method using both direct as well as iterative solvers. In particular if different non-linearities are active difficulties may appear since contact element changes their status which is not differential for Newton's method. In such a cases lot of effort needed to achieve proper solution. In [14] integrated CAD/CAE procedures are utilized for dynamic analysis of two-link manipulator. I-DEAS procedure is used for solid modeling and vibration analysis; three programs are needed to create the samples of the nodal forces. Dynamic structural analysis of a fast shutter with pneumatic actuator and stress analysis of a rail joint with the use of LS-DYNA is presented in [15], [16]. Finite element limit load analysis of thin walled structures by using ANSY and LS-DYNA presented in [17]. Finite element analysis of a sheet metal air bending using hyper form LS-DYNA presented in [18]. Advanced robust and efficient contact algorithms are the heart of most LS-DYNA applications. The main advantage of LS-DYNA is the absence of convergence problems inherent to the solution algorithm. Literature survey shows that the LS-DYNA platform is useful for simulating highly nonlinear complex real world problems. The finite element system LS-DYNA is developed by Livermore software technology corporation (LSTC), Livermore, CA., USA. It's find the applications in automobile, aerospace, construction, manufacturing and bioengineering industries. The code origins lie in highly nonlinear, transient dynamic finite element analysis using explicit time integration. In this work nonlinear finite element model of a FLM is developed in LS-DYNA platform and simulated using explicit solver. The results are compared against the derived AMM model and linear finite element models.

Organization of the paper is as follows. The dynamic model of a FLM using AMM is presented in section II. Finite element formulation using LS-DYNA presented in section III. Section IV gives the simulation results with discussions. A parametric study is presented in section V followed by conclusion.

II. The Dynamic Model of a FLM Using AMM

The schematic diagram of a FLM is shown in Fig. 1. Let, *OXY* be the inertial coordinate frame and *Oxy* be the moving frame. The clamped end of the flexible arm (length l, uniform flexural rigidity EI, mass m and the mass moment of inertia I_b) is attached to the hub (inertia I_h) and connected to the rotor (inertia I_r and gear ratio G). An input torque (*u*) is applied to move the link and the end-effecter carries a payload mass (m_p).

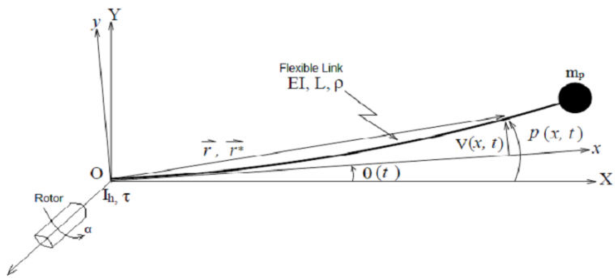

Fig. 1 Schematic diagram of flexible link manipulator

Theoretically the elastic deflection of beam consists of infinite number of separable modes. In AMM modeling, the link deflection is described by a truncated modal expansion i.e. only finite number of modes is considered. The deflection of the manipulator is expressed as a summation of modes. Each mode is assumed to be a product of two functions, one as a function of the distance along the length of the manipulator (mode function) and the other, as a generalized co-ordinate (modal variable) as a function of time. The Euler–Bernoulli beam theory and the assumed modes method can be used to express the deflection $v(x,t)$ of a point located at a distance x along the arm as:

$$v(x,t) = \sum_{i=0}^{n} \phi_i(x).\delta_i(t) \tag{1}$$

where $\phi_i(x)$ is the mode shape function and $\delta_i(t)$ is the time varying modal variable and *n* is the number of finite modes. The overall motion of the flexible link manipulator consists of the rigid body motion, which is defined by the hub angle *(θ(t))*, and the elastic motion, which is defined by the first two modal coordinates $\delta_1(t)$ and $\delta_2(t)$.To derive the dynamic equations of the motion of the FLM, the total energy associated with the system need to be computed. The mode functions and mode frequencies are obtained by solving the boundary value problem (or Eigen value problem) for a beam in flexure.

A. Kinetic Energy

The total kinetic energy of the system is due to the motion of the link, hub and rotor and due to the kinetic energy associated with the payload. Kinetic energy due to the motion of the hub can be written as:

$$T_h = I_h \dot{\theta}^2 \tag{2}$$

Kinetic energy of the rotor is given by:

$$T_r = J\dot{\alpha}^2 \tag{3}$$

Kinetic energy due to the motion of the link can be expressed as:

$$T_l = \frac{1}{2}\rho\int_0^l [(x^2 + v^2(x,t))\dot{\theta}^2 + \dot{v}^2(x,t) + 2x\dot{\theta}\dot{v}(x,t)]dx \tag{4}$$

Similarly, the kinetic energy associated with the payload can be written as

$$T_p = \frac{1}{2}m_p[(l^2 + v^2(l,t))\dot{\theta}^2 + \dot{v}^2(l,t) + 2x\dot{\theta}\dot{v}(l,t)] \tag{5}$$

B. Potential Energy

The total potential energy P of the system is due to the elastic deformation (P_{el}) of the link and due to the joint deflection (P_j). The potential energy due to the elastic deflection of the link is given by:

$$P_{el} = \frac{1}{2}EI\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)dx \tag{6}$$

Let K_s be the spring constant of the joint. Then potential energy of the joint can be written as:

$$P_j = \frac{1}{2}K_s(\alpha - \theta)^2 \tag{7}$$

C. Internal Structural Damping of Arm

With the assumption that the internal structural damping of the arms is viscoelastic, its dissipation function can be defined as [15]

$$V_D = \frac{1}{2}\int_0^l CI\left(\frac{\partial}{\partial t}\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)\right)^2 dx \tag{8}$$

$$C = \varepsilon l^2\sqrt{\frac{AE}{l}}$$

where ε and l are the damping ratio and length of the arm respectively

D. Friction at joint

By assuming dry friction between the rotor and joint, the friction torque T_f can be defined by following equation:

$$T_f = \begin{cases} T_s & \dot{\theta} < 0.0 \\ -T_s < T_f < T_s & \dot{\theta} = 0.0 \\ -T_s & \dot{\theta} > 0.0 \end{cases} \tag{9}$$

where T_s is the static friction torque at the joint

E. Equations of Motion

In order to obtain the differential equations of motion to adequately describe the dynamics of flexible link manipulator, the Lagrange's approach can be used.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial V_D}{\partial \dot{q}_i} = Q_i \tag{10}$$

$$i = 1,2,......., n$$

where $q_i = [\alpha(t) \quad \theta(t) \quad \delta_1(t) \quad \delta_2(t) \quad ..... \delta_n(t)]$

$Q_i = [T_f \quad u \quad 0 \quad 0 \quad ..... \quad 0]^T$

u - Torque applied at the hub

By substituting the kinetic energy, potential energy and its dissipation energy in (10), the dynamic equations of the motion of the system for first two modes can be derived in the form:

$$D(\theta,\delta)\begin{bmatrix}\ddot{\alpha}\\ \ddot{\theta}\\ \ddot{\delta}\end{bmatrix} + \begin{bmatrix}0\\ g_1(\theta,\dot{\theta},\delta,\dot{\delta})\\ g_2(\theta,\dot{\theta},\delta,\dot{\delta})\end{bmatrix} + \begin{bmatrix}h_1(\theta,\alpha)\\ h_2(\theta,\alpha)\\ h_3(\theta,\alpha)\end{bmatrix} = \begin{bmatrix}T_f\\ u\\ 0\end{bmatrix} \tag{11}$$

where D is the inertia matrix, ***g***1*(·)* and ***g***2*(·)* are the vectors containing terms due to coriolis, centrifugal forces, and due to the interactions of the link angles and their rates with the modal displacements and their rates. ***h****(.)* is the vectors containing joint flexibility and damping. Equation (11) gives the complete dynamic model of a flexible link robot manipulator with all significant dynamics.

III. LS-DYNA Finite Element Model

In general LS-DYNA is designed for dynamic analysis of highly nonlinear problems. It has the capabilities to simulate a wide range of different physical phenomena using analysis techniques such as: explicit and implicit time integration schemes, nonlinear dynamics, large deformations and Eigen value analysis. The solution of nonlinear system of equations using this time integration scheme only requires the inversion of mass matrix within each step. The computational effort is mainly influenced by the formation of the internal forces via the elements and contact surfaces. Many LS-DYNA FE discretizations of real industrial problems are dominated by thin shell elements including some beam and solid elements. Various types of element formulations are offering the choice between the computational efficiency and improved accuracy. More than 100 different models are available to represent many types of highly nonlinear material properties.

The flexible link is represented by shell elements with a uniform thickness of 3.2mm. The length and width of the beam are 900mm and 32mm respectively. Element size of 5 mm is chosen to represent the flexible link as shown in Fig. 2 Belytschko-Tsay formulation with 5 through thickness integration points is chosen for the shell elements The Material properties for Flexible-Link corresponding to Aluminum is defined in *MAT_ELASTIC**.** One end of the beam is rigidly attached to a revolute joint, which represents the hub motor shaft. A bang-bang type of torque input is applied at this revolute joint using *LOAD_RIGID_BODY.

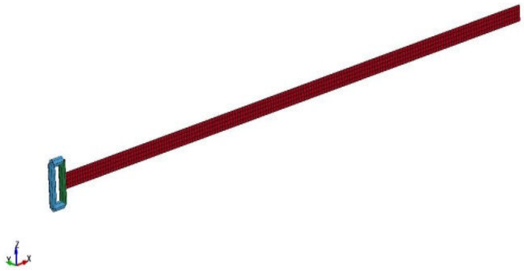

Fig. 2 Flexible-link manipulator model (LS-DYNA)

IV. Simulation Results

The dynamic equations characterizing the behavior of the Flexible-Link manipulator is simulated using the fourth-order Runge-Kutta integration method. The physical parameters of the manipulator are given in Table I. A bang-bang type torque input with amplitude of 0.3Nm as shown in Fig. 3, is applied at revolute joint.

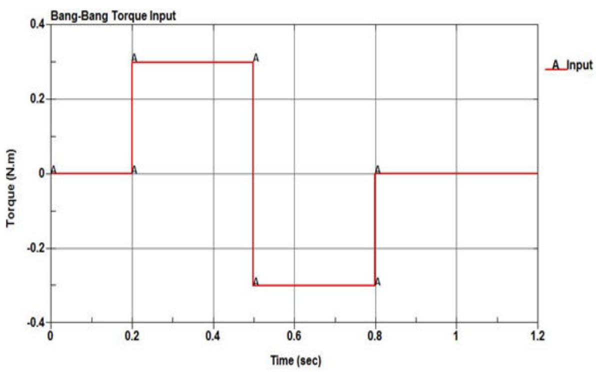

Fig. 3 Bang-bang torque input

TABLE I
Parameters Of Flexible-Link Manipulator System

parameter	meaning	value
L	Beam length	0.9 m
W	Beam width	0.0032 m
H	Beam height	0.019 m
E	Beam young's modulus	71×10^{9} Pa
I	Cross-section moment of inertia	5.253×10^{-11} m^4
$\mathbf{I_h}$	Hub inertia	5.8598×10^{-4} $kg\ m^2$
P	Mass density	2710 kg/ m^3
$\mathbf{K_s}$	Stiffness constant	100 Nm/rad
G	Gear ratio	1

The LS-DYNA finite element model of a FLM is simulated using explicit time integration for a time period of one second. The hub angle response of the LS-DYNA model is compared with that of AMM and linear FEM [12] models as shown in Fig. 4. The maximum hub angle is 0.68 radians in the AMM and FEM methods while it is 0.42 radians in LS-DYNA model. It seems that the elastic behavior of the beam is significantly overestimated in the AMM method.

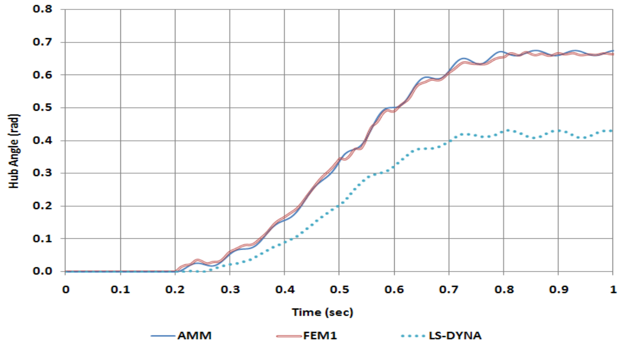

Fig. 4 Comparison between AMM, FEM and LS-DYNA

Fig. 5 shows the kinetic and internal energy change in the system as a function of time. Initially the kinetic energy is zero and the kinetic energy starts to increase at 0.2 sec (at the time when the torque input is applied). The kinetic energy steadily increases up to 0.5 sec during the positive phase of the torque input. A reverse torque is applied from 0.5 sec to 0.8 sec to limit any further motion of the flexible link and during this phase the kinetic energy of the system drops down and approaches to zero. However, the elastic deflection induced in the flexible link continues for some period, the kinetic and internal energy oscillations can be observed thereafter.

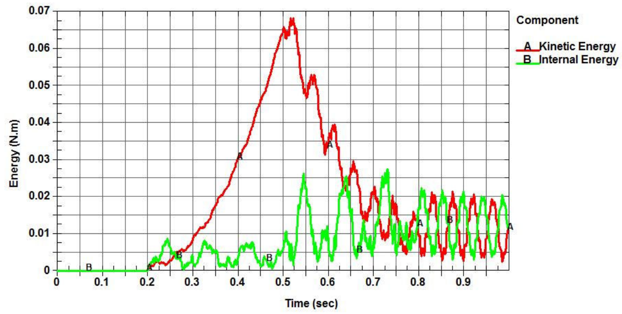

Fig. 5 Kinetic energy and internal energy change in the system

V. Parametric Study

A parametric study is conducted with a beam length of 0.9m and by varying the payload. Three conditions are simulated, one with no-payload, one with 0.05 kg payload and the other with 0.2 kg payload. Fig. 6 shows the hub angle response for the three different payloads.

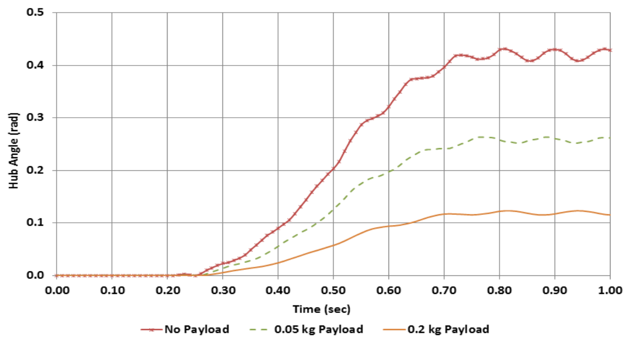

Fig. 6 Hub angle responses for 0.9m length beam with different payloads

As the payload increases, the angular position start reduces. This is similar to the observation in the previous parametric study based on length of the beam. The heavier the beam, the harder it rotates around the motor shaft and oscillations is reduced while moving. Figs. 7 and 8 show the variations in energy levels for 0.05kg and 0.2kg payloads.

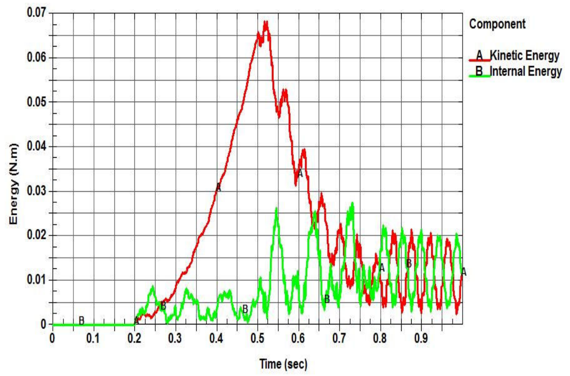

Fig. 7 Kinetic energy and internal energy change the system (0.05kg payload)

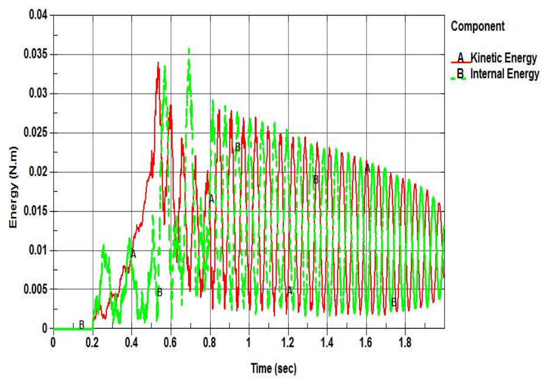

Fig. 8 Kinetic energy and internal energy in change in the system (0.2kg payload)

Another parametric study also conducted to estimate the hub angle response for various lengths of the flexible link manipulator. Three lengths are considered in this study. The hub-angle response for the applied bang-bang torque input is shown in Fig. 9.

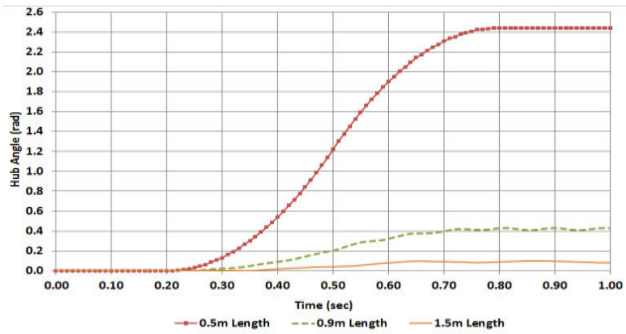

Fig. 9 Hub angle responses for different lengths of the beam

As expected, the hub angle response is more for the shorter length (0.5m) beam. The 1.5m length beam barely moved with the applied torque. As the longer beam is heavier and hence energy required is also more. So, choosing the beam length also plays an important role while design.

VI. Conclusion

By incorporating all the significant dynamics associated with the system, the nonlinear dynamic model of a flexible link robot manipulator is derived based on Euler-Lagrange and assumed modes approaches. Also a nonlinear finite element model is established in LS-DYNA platform to compare the results against derived AMM model. A simulation result shows the LS-DYNA model gives the smooth hub-angle response for the applied bang-bang torque input, which is sluggish in AMM and linear FEM models. Small changes in the elastic behavior of the link is due to the over estimation of stiffness matrix. Impact of payload mass on the dynamics of a FLM is addressed by plotting the changes in kinetic and internal energy levels. LS-DYNA analysis has the advantages over standard ANSYS and CAD/CAE procedures. It is more economical and easier to use the commercial finite element codes in the case of systems containing highly nonlinear effects.

References

[1] W. J. Book, Recursive Lagrangian dynamics of flexible manipulator arms, *The International journal of Robotics Research,* vol. 3, no. 3, pp. 87-101, Fall 1984.

[2] Chang-Jin Li and T. S., Systematic Methods for Efficient Modeling and Dynamics Computation of Flexible Robot Manipulators, *IEEE trans. on Syst., Man, Cybern.,* vol. 23, no. 1, pp. 77-95, Jan 1993.

[3] M. Martins, Z. Mohamed, M.O. Tokhi, Approaches for dynamic modeling of Flexible Manipulator systems, *IEEE Proc.-Control Theory Appl.,* Vol. 150, No. 4, July 2003.

[4] Dwivedy SK, Eberhard P., Dynamic analysis of flexible manipulators, a literature review, *Mechanism and Machine Theory,* 2006; 41:749-777.

[5] Hasting, G.G., and Book W.J., A linear dynamic model for flexible robot manipulators, *IEEE Control Syst. Mag., Vol. 7, pp. 61–64, 1987*

[6] M. A. Ahmad, Z. Mohamed and N. Hambali, Dynamic Modeling of a Two link Flexible Manipulator System Incorporating Payload, IEEE *Proceedings Control Theory and Application*, 2008;pg 96-101
[7] Yang, G. B., and Donath, M., 1988, Dynamic modeling of a one link robot manipulator with both structural and joint flexibility, *In Proceedings of the IEEE Conference on Robotics and Automation,* pp. 476–48
[8] B. Subudhi, A.S. Morris, Dynamic modeling, simulation and control of a manipulator with flexible links and joints, *Robotics and Autonomous Systems* 41 (2002) 257–270
[9] Baker, W.E., Woolam, W.E., and Young, D., Air and Internal Damping of Thin Cantilever Beams, *Int.J. Mech.Sci. Pergamon Press Ltd*, Vol.9, 1967, pp.743-766.
[10] Jin Fu Zhang, Dynamic Modeling of Planar Flexible Multi-Link Manipulators with Accounting for both Link Foreshortening and Link Material Damping, *International journal of Advanced Materials Research*, Vols, 199-200(2011) pp 19-24
[11] Tokhi. M. O. Mohamed, Z. and Azad, A. K. M, Finite difference and finite element approaches to dynamic modelling of a flexible manipulator", *Proceedings of IMechE-I: Journal of Systems and Control Engineering*, Vol.211(1997), pp. 145-156.
[12] J. Chung and H. H. Yoo, Dynamic analysis of a rotating cantilever beam by using the finite element method, *Journal of Sound and Vibration*(2002) 249(1), 147-164.
[13] R. Fotouhi, Dynamic analysis of very flexible beams, *Journal of Sound and Vibration* 305(2007), 512-533.
[14] H. Karagulle, L. Malagaca, Analysis of end-point vibrations of a two-link manipulator by integrated CAD/CAE procedures, *Finite Elements in Analysis and Design* 40(2004) 2049-2061.
[15] A. Nemov et al. /, Dynamic structural analysis of a fast shutter with a pneumatic actuator**,** *Fusion Engineering and Design* (2013).
[16] Wu Cai, Zefeng Wen , Xuesong Jin, Wanming Zhai, Dynamic stress analysis of rail joint with height difference defect using finite element method, *Engineering Failure Analysis* 14(2007) 1488-1499.
[17] Wilhelm Rust and Karl Schweizerhof, Finite element limit load analysis of thin-walled structures by ANSYS (implicit) , LS-DYNA (explicit) and in combination, *Thin-Walled Structures (Elsevier) 41 (2003) 227-244.*
[18] *Himanshu v. Gajjar, Anish H. Gandhi, and Harit k. Raval, Finite Element of Sheet Metal Air bending using Hyperform LS-DYNA, World Acadamy of science,Engineering and Technology 8 2007.*