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Acceleration Analysis of a Rotating Body

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Abstract**—**The velocity of a moving point in a general path is the vector quantity, which has both magnitude and direction. The magnitude or the direction of the velocity vector can change over time as a result of acceleration that the time rate of velocity changes. Acceleration analysis is important because inertial forces and inertial torques are proportional to rectilinear and angular accelerations accordingly. The loads must be determined in advance to ensure that a machine is adequately designed to handle these dynamic loads. For planar motion, the vector direction of acceleration is commonly separated into two elements: tangential and centripetal or radial components of a point on a rotating body. All textbooks in physics, kinematics and dynamics of machinery consider the magnitude of a radial acceleration at condition when a point rotates with a constant angular velocity and it means without acceleration. The magnitude of the tangential acceleration considered on a basis of acceleration for a rotating point. Such condition of presentation of magnitudes for two components of acceleration logically and mathematically is not correct and may cause further confusion in calculation. This paper presents new analytical expressions of the radial and absolute accelerations of a rotating point with acceleration and covers the gap in theoretical study of acceleration analysis.

Keywords**—**acceleration analysis, kinematics of mechanisms.

I. INTRODUCTION

Machines and mechanisms running at a high speed are considered by a kinematic analysis of their velocities and accelerations, and subsequently analyzed as a dynamic system in which forces due to accelerations are analyzed using the principles of kinetics. Force and stress analysis is commonly based on the acceleration analysis of links and points of interest in the mechanism or machine.

Many methods and approaches exist to find accelerations in mechanisms [1]–[8]. Curvilinear motion of a body with a variable speed is going on with a linear acceleration, which is commonly separated into two elements: tangential and centripetal or radial components of acceleration. The tangential component is formed as a result of a change in the magnitude of the velocity vector and its direction is perpendicular to the line that connects the point with the center rotation. The radial component is created as result of a change in the direction of the velocity vector and its direction is always toward the center of a point rotation. The expression of an absolute acceleration, α of a point is the vector sum of the tangential, α_t and radial, α_r components as shown by equation [1]–[8]:

$$\overline{\alpha} = \overline{\alpha_t} + \overline{\alpha_r} \tag{1}$$

where α – absolute acceleration, α_t – tangential acceleration, and α_r – radial acceleration.

The tangential acceleration of a rotating point has the following expression [1]–[8].:

$$\alpha_t = \varepsilon r \tag{2}$$

where ε – an angular acceleration of rotating point, r – a radius of rotation of a point.

The radial acceleration of a rotating point has the following expression [1]–[8]:

$$\alpha_r = r\omega^2 \tag{3}$$

where ω – an angular velocity of rotation of a point considered, other component presented above.

The expression of an absolute acceleration of a rotating point has the following equation [3], [4]:

$$\alpha = r\sqrt{\varepsilon^2 + \omega^4} \tag{4}$$

where components presented above.

Known expressions of a tangential acceleration (Eq. 2) and a radial one (Eq. 3) is presented in textbooks and handbooks [1]–[8]. In physics, an angular velocity of a rotating point with acceleration is variable at the time**.** Expression of a tangential acceleration (Eq. 2) produced at condition when a point rotates with an angular acceleration, ε. Eq. (3) of a radial acceleration produced at condition when an angular velocity, ω of a rotating point is constant [1]–[8], however the angular velocity of a rotating point is variable due to its angular acceleration. At condition when an angular velocity is constant, the rotating point does not have a tangential acceleration.

This situation confuses users of textbooks and contains logical and mathematical error that results on the magnitude of centrifugal and inertial forces and on the magnitude of an absolute acceleration of a rotating point. Some textbooks remark that for calculation of acceleration is accepted an instant angular velocity of a rotating point [2], [4]**,** However, this explanation does not give more clarity.

It is known that a rotation of a point around a center with an angular acceleration gives constantly increasing angular velocity of rotation one [1]–[8]. The magnitude of a centrifugal force and other forces will increase with increasing of an angular velocity. Incorrect calculation of magnitudes of

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acting forces at the extreme regimes of a machine work can be expressed as unpredictable failure of mechanisms with further consequences. This is an on-going phenomenon in reality but experts cannot prove the failure of mechanisms and machines due to lack of correct formula of the radial and absolute accelerations in textbooks, handbooks and standard methods for calculating forces acting on rotating components of machines.

The aforementioned imposes the need to produce correct equation of the radial and absolute accelerations of a point of a rotating body.

II. ANALYTICAL SOLUTION FOR ACCELERATION ANALYSIS

An angular acceleration deals with a motion of a rigid body and it is the time rate of change of an angular velocity one. Figure 1 shows the link *OP* with the radius *r* in pure rotation, pivoted at point *O* in the *xy* plane. To solve any acceleration analysis problem of a rotating point graphically, it is necessary to have only three equations: $V = \omega r$ – a linear velocity, α_t - a tangential acceleration, and α_r - a radial acceleration of a rotating point [1]–[8].

To solve the acceleration analysis problem of the rotating point, which is subjected to a variable angular velocity, it is necessary to develop a new acceleration analysis of a rotation of a point *P*. Figure 1 shows a link *OP*, whose positions are defined by the radius *r*, and by locations of point *P* at spot *a* and spot *b*. Two positions of point *P* are separated by a small angle θ. Point *P* has pure rotation around a fixed pivot *O*. The velocity vector of point *P* will change along the curvature of motion and due to rotating with variable speed, the magnitudes of linear velocities V_a and V_b of a point *P* are different. Hence the velocity polygon of rotating point *P* is vectorally solved for these changes in velocity, V_{ab} (Fig. 1).

$$\overline{V_{ab}} = \overline{V_a} + \overline{V_b} \quad (5)$$

The vector, V_{ab} of change the linear velocity can be presented by two vectors V_r and V_{cb}.

$$\overline{V_{ab}} = \overline{V_r} + \overline{V_{cb}} \quad (6)$$

The radial velocity vector V_r to this curvature is directed toward the center of rotation and crosses the vector velocity V_b at the point *d*. At such condition the length of the segment *Od* is equal to the length of segment *Oa***.**

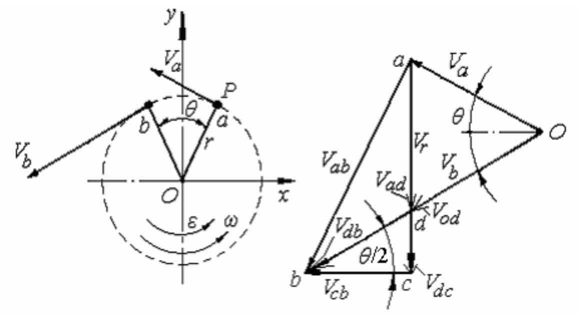

Fig. 1. Acceleration of a rotating point

The changed velocity vector V_b can be presented as sum of two vectors V_{od} and V_{db} (Fig. 1).

$$\overline{V_b} = \overline{V_{od}} + \overline{V_{db}} \quad (7)$$

Because the magnitude of vector V_{od} is equal to the magnitude vector V_a, hence the vector V_{od} represents the change of the velocity of the point *P* (location *b*, Fig. 1) when the angular velocity of rotation, ω is constant. The vector V_{db} presents the change of the velocity vector due to the acceleration of the rotating point.

From physics, the known expression of the variable linear velocity of a point is given by the following equation [1]–[8]:

$$V = V_{in} + \alpha t \quad (8)$$

where the first component V_{in} - an initial velocity of a moving point, the second component $\alpha t = V_e$ – the extra velocity of a moving point due to acceleration one, α – a linear acceleration, t – a time consideration.

The changed linear velocity of the rotating point *P* according to Eq. (8), is expressed by the following equation:

$$V_b = V_{od} + \alpha t \quad (9)$$

where V_{od} presents the initial velocity of the moving point *P*, $V_{in,}$ according to Eq. (8).

Eq. (9) is expressed at angular motion parameters by the following equation [1]–[8]:

$$V_b = r\omega_{in} + \varepsilon r t \quad (10)$$

where $V_{od} = r\omega_{in}$, ω_{in} - an initial angular velocity of a rotating point, and $\alpha t = \varepsilon r t$, all other parameters of equation are given above.

The second component of the linear velocity, $\varepsilon r t$ from Eq. (9), is equal to the second component of the velocity, V_{db} from Eq. (6), hence

$$V_{db} = \varepsilon r t \quad (11)$$

The radial velocity vector V_r also can be presented as the sum of two vectors V_{ad} and V_{dc}.

$$\overline{V_r} = \overline{V_{ad}} + \overline{V_{dc}} \quad . \quad (12)$$

For this case, the radial acceleration of the rotating point expressed as first derivative of Eq. (12) with respect to time [9] is

$$\frac{dV_r}{dt} = \frac{dV_{ad}}{dt} + \frac{dV_{dc}}{dt}, \quad (13)$$

giving expression of the radial acceleration as two components

$$\alpha_r = \alpha_{r(ad)} + \alpha_{r(dc)} \tag{14}$$

The first component of Eq. (13) as derivative of the radial vector velocity with respect to time is the expression of the first component of radial acceleration of a rotating point. In Fig. 1, because the angle θ is small, the following relationship can be stated

$$\frac{dV_{ad}}{dt} = V_a \frac{d\theta}{dt} \tag{15}$$

The second component of Eq. (13) one is additional expression of the radial acceleration of the rotating point, due to acceleration one. The equation for the second component of the radial acceleration (Eq. 13) is expressed in the following computation.
The second component of the radial velocity vector V_{dc} expressed from the vector triangle, *dbc* (Fig. 1).

$$V_{dc} = V_{db}\sin(\theta/2), \tag{16}$$

where $V_{db} = \varepsilon rt$ from Eq. (11), and $\theta/2$ is infinitely small.

After substituting and transformation, the second component of the radial velocity vector V_{dc} will have the following expression with two variables t and θ.

$$V_{dc} = \varepsilon rt\theta/2 \tag{17}$$

The first derivative of Eq. (17), with respect to time, gives expression of the second component of the radial acceleration

$$\frac{dV_{dc}}{dt} = \frac{d(\varepsilon rt\theta/2)}{dt} \tag{18}$$

After substituting Eq. (15), Eq. (18) to Eq. (13) and transformation, the expression of the change of radial velocity vector V_r will have the following expression with two variables t and θ.

$$\frac{dV_r}{dt} = V_a \frac{d\theta}{dt} + \frac{d(\varepsilon r\theta/2)}{dt} \tag{19}$$

Using the relationships between linear velocity and angular velocity, the following equation for the radial acceleration of a rotating point can be derived

$$\alpha_r = r\omega_{in}^2 + \varepsilon r(\theta + \omega_e t)/2 \tag{20}$$

where ω_{in} - an initial angular velocity of a rotating point, $\omega_e = d\theta/dt$ – an extra angular velocity of a rotating point due to acceleration one, and $\theta = \omega_e t$

The first component of Eq. (20) is known expression of the radial acceleration of a rotating point at condition when an angular velocity is constant $\alpha_{r(ad)} = r\omega_{in}^2$ [1]–[8].

The second component of Eq. (20), after substituting and transformation will have the following expression: $\alpha_{r(dc)} = r\omega_e\varepsilon t$, where $\omega_e = \varepsilon t$, then

$$a_{r(dc)} = r(\varepsilon t)^2 \tag{21}$$

The total expression of the radial acceleration of a rotating point is obtained after substituting Eq. (21) into Eq. (20), and following transformation gives the equation:

$$a_r = r[\omega_{in}^2 + (\varepsilon t)^2] \tag{22}$$

Obtained Eq. (22) of the total radial acceleration involves the initial angular velocity, ω_{in} of a rotating point, the angular acceleration one, ε, a time t of rotation, and radius r of rotation of a point P (Fig. 1). A new formula will not confuse users in the calculation of the radial acceleration of a rotating point with a variable angular velocity. When an angular acceleration of a rotating point, $\varepsilon = 0$, the equation of a radial acceleration has expression of Eq. (3).

The equation of the absolute acceleration of a rotating point, with variable angular velocity, obtained after substituting Eq. (2) of the tangential acceleration and Eq. (22) of the radial acceleration into Eq. (1). Converting this equation into algebraic form and transformation gives the following expression:

$$a = r\sqrt{\varepsilon^2 + [\omega_{in}^2 + (\varepsilon t)^2]^2} \tag{23}$$

The new equation of an absolute acceleration of a rotating point is quite different from the known expression that is presented in textbooks, (Eq. 4[1]–[8].

III. CASE STUDY

The rotating point shown in Fig.1 has the initial angular velocity of 10 rad/s, accelerates at the rate of 3 rad/s^2 and has the radius of rotation of 0.1 m. It is necessary to determine the magnitudes of the tangential component and the radial component of accelerations and absolute one of the point P after 5 seconds of rotation.

Eq. (2) used to determine the tangential component of acceleration is:

$$\alpha_t = \varepsilon r = (3)(0.1) = 0.3 \text{ m/s}^2$$

The new Eq. (22) used to determine the radial component of acceleration is:

$$a_r = r[\omega_{in}^2 + (\varepsilon t)^2] = 0.1[10^2 + \{(3)(5)\}^2] = 32.5\text{m/s}^2$$

The absolute acceleration of the rotating point according to new Eq. (23) is:

$$a = r\sqrt{\varepsilon^2 + [\omega_{in}^2 + (\varepsilon t)^2]^2} = 0.1\sqrt{3^2 + [10^2 + \{(3)(5)\}^2]^2} = 32.501\text{m/s}^2$$

The result of the radial acceleration obtained by the new Eq. (22) is based on real conditions of mechanism work and include changed an angular velocity of the rotating point. Known Eq. (3) for calculation of a radial acceleration, which is presented in any textbooks, includes initial value of the angular velocity of rotating point. This is difference in approach for calculation of the radial acceleration.

IV. RESULT AND DISCUSSION

Relevant equations presented for radial and absolute accelerations of a rotating point will enable the calculation of more accurate results as function of a radius of a rotating point, angular velocity, and angular acceleration one. These equations are different from known equations of radial and absolute accelerations presented in textbooks of physics, kinematics and dynamics of machinery. In engineering, all machines work with a variable velocity of rotating elements that is real condition of functioning of any mechanism. It is very important to calculate an exact result for rotating components with acceleration for a machine. New equations of radial and absolute accelerations give correct result compared to those presented in the textbooks [1]–[8].

V. SUMMARY

Analytical solution of acceleration analysis of a rotation point that is presented in textbooks is not perfect and cannot give reliable equations of radial and absolute accelerations. The main contribution of this research paper is in the development of analytical equations that give a correct result for calculation of the radial and absolute accelerations of a rotating point as function of a radius, angular velocity, and angular acceleration.

New equation of the radial acceleration of a rotating point and hence new equation of an absolute acceleration have been obtained. These equations will be useful in calculating the correct magnitude of the normal acceleration and absolute one for a rotating point. New expressions of the radial and absolute accelerations of a rotating point developed in this paper can be used by designers and manufacturers for the creation of reliable machines that are guaranteed to perform better and will not have unpredictable failure. New acceleration analysis of a rotating point should be used in textbooks of physics, kinematics and dynamics of machinery to give accurate mathematical method for producing equation of the radial and absolute accelerations and accurate results in calculation.

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