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A Type-2 Fuzzy Adaptive Controller of a Class of Nonlinear System

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***Abstract*—**In this paper we propose a robust adaptive fuzzy controller for a class of nonlinear system with unknown dynamic. The method is based on type-2 fuzzy logic system to approximate unknown non-linear function. The design of the on-line adaptive scheme of the proposed controller is based on Lyapunov technique. Simulation results are given to illustrate the effectiveness of the proposed approach.

***Keywords*—**Fuzzy set type-2, Adaptive fuzzy control, Nonlinear system.

I. Introduction

THE control of nonlinear systems has been an important research topic [1]-[3]. Traditionally, control system design has been tackled using mathematical models derived from physical laws. In fact, most of the parameters and structure of the system are unknown due to environment changes, modelling errors and unmodeled dynamics. To overcome the above problems in the design of control systems several techniques have been emerged in the recent years especially techniques based on the intelligent technology such as neural networks, fuzzy logic, genetic algorithms and evolutionary computation [2]-[4]-[6]. In particular, fuzzy logic systems (FLS) have been successfully applied to control complex or ill-defined processes whose mathematical models are difficult to obtain [9]-[10]. The ability of converting linguistic descriptions into automatic control strategy has made it a practical and promising alternative to the classical control scheme for achieving control of complex nonlinear systems [5]-[7]. However, fuzzy logic systems have a major drawback which is expressed in the fact that the fuzzy rules must be previously tuned by time-consuming trial-and-error procedures because of lack of adequate analysis and design techniques. To overcome this problem, researchers have focused on the Lyapunov synthesis approach to construct stable adaptive fuzzy controllers. The basic idea of most of these works is that with the universal approximation ability of fuzzy logic systems, the system uncertainties can be represented by linearly parameterized uncertainties so the standard parametric adaptive techniques can be utilized [11]-[13].

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Many researches have shown that type-1 FLS have difficulties in modeling and minimizing the effect of uncertainties [6]-[12]. One reason is that a type-1 fuzzy set is certain in the sense that the membership grade for a particular input is a crisp value. Recently, type-2 fuzzy sets, characterized by membership functions (MF) that are themselves fuzzy, have been attracting interest [6]-[12]. For such sets, each input has unity secondary membership grade defined by two type-1 MF, upper MF and lower MF.

A FLS using at least one type-2 fuzzy set is called a type-2 FLS. The wide range of applications of type-2 FLS have shown that it provide good solutions, especially in the presence of uncertainties [6]. Similar to the conventional adaptive control, adaptive fuzzy control can be categorized into direct, indirect and composite schemes according to the type of fuzzy rules [13].

In this paper, we present an adaptive fuzzy control for a class of nonlinear systems with unknown dynamic. The basic idea is that first the type-2 FLS is utilized to approximate the unknown nonlinear function, and then the fuzzy parameters are adjusted on-line by the adaptive laws with stability and convergence analysis using the Lyapunov approach in order to achieve the specified tracking performance.

This paper is organized as follows. Section 2 describes the type-2 FLS. In section 3, we propose the adaptive fuzzy control. Section 4 presents numerical results which validate the proposed approach. Concluding remarks are given in section 5.

II. Type-2 Fuzzy Logic Systems

The basic configuration of a fuzzy logic system consists of a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzifier. The structure of a type-2 FLS is similar to type-1 counterpart, the major difference being that at least one the fuzzy set in the rule base is type-2. A type-2 fuzzy set characterized by membership functions that are themselves fuzzy. The key concept is footprint of uncertainly (FOU), which models the uncertainties in the shape and position of the type-1 fuzzy set. Fig. 1 illustrates a type-2 fuzzy MF with FOU shown as shaded area. The output of inference engine for a type-2 FLS is type-2 sets. Hence a type-reducer is needed to convert them into type-1 sets before defuzzification can be carried out.

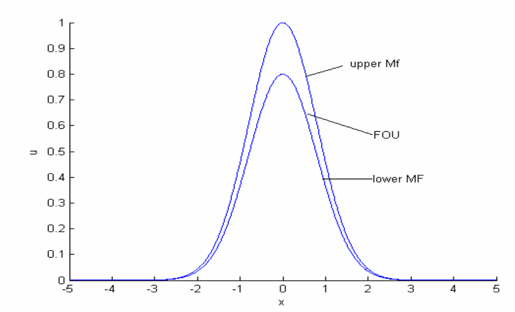

Fig. 1 Type-2 fuzzy sets

A. Inference in a type-2 FLS

The inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input vector $\underline{x} = (x_1, x_2, \ldots, x_n)^T$ to an output scalar y.

The fuzzy rule base consists of a collection of fuzzy IF-THEN rules in following form:

$$R^i : \text{If } x_1 \text{ is } \widetilde{F}_1^i \text{ and } x_2 \text{ is } \widetilde{F}_2^i \ldots \text{and } x_n \text{ is } \widetilde{F}_n^i \text{ then } y \text{ is } \widetilde{G}^i \quad (1)$$

where $\widetilde{F}_j^i$ are the antecedent sets ($i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$), $\widetilde{G}^i$ are the consequent sets and m is the number of rules.

The first step in the extended sup-star operation is to obtain the firing set

$$F^i(\underline{x}) = \prod_{j=1}^{n} \mu_{\widetilde{F}_j^i}(x_j) \quad (2)$$

by performing the input and antecedent operations. As only interval type-2 sets are used and the meet operation is implemented by the product t-norm, the firing set is the following type-1 interval set:

$$F^i(\underline{x}) = [\underline{f}^i(\underline{x}),\ \overline{f}^i(\underline{x})] \quad (3)$$

where $\underline{f}^i(\underline{x}) = \underline{\mu}_{\widetilde{F}_1^i}(x_1) * \underline{\mu}_{\widetilde{F}_2^i}(x_2) * \cdots * \underline{\mu}_{\widetilde{F}_n^i}(x_n)$

and $\overline{f}^i(\underline{x}) = \overline{\mu}_{\widetilde{F}_1^i}(x_1) * \overline{\mu}_{\widetilde{F}_2^i}(x_2) * \cdots * \overline{\mu}_{\widetilde{F}_n^i}(x_n)$.

The term $\underline{\mu}_{\widetilde{F}_j^i}(x_j)$ and $\overline{\mu}_{\widetilde{F}_j^i}(x_j)$ are the lower and upper membership grades of $\mu_{\widetilde{F}_j^i}(x_j)$, respectively.

Next, the firing set $f^i(\underline{x})$ is combined with the consequent fuzzy set of the i^{th} rule using the product t-norm to derive the fired output consequent sets.

B. Type-reduction for interval type-2 FLS and defuzzification

Since the output of the inference engine is a type-2 fuzzy set, it must be type-reduced before the defuzzifier can be used to generate a crisp output. This is the main structural difference between type-1 and type-2 FLC. The most commonly used type-reduction method is the center-of- sets type-reducer. The center-of-sets type reducer replaces each consequent set by its centroid (which itself is a type-1 set if the consequent set is type-2) and finds a weighted average of these centroids, where the weight associated with the i^{th} centroid is the degree of firing corresponding to the i^{th} rule, namely $\prod_{j=1}^{n} \mu_{\widetilde{F}_j^i}(x_j)$. The expression for the type-reduced set is given by [13]

$$\begin{aligned} &Y_{\cos}(Y^1,\ldots,Y^m,F^1,\ldots,F^m) \\ &= \int_{y^1}\cdots\int_{y^m}\int_{f^1}\cdots\int_{f^m} \tau_i^m \mu_{Y^i}(y^i) * \tau_i^m \mu_{F^i}(f^i) \Bigg/ \frac{\sum_{i=1}^m f^i y^i}{\sum_{i=1}^m f^i} \end{aligned} \quad (4)$$

where τ and $*$ denotes the chosen t-norm, $y^i \in C_i = C_{\widetilde{G}^i}$ the centroid of the i^{th} consequent set and $f^i \in F^i(\underline{x}) = \prod_{j=1}^{n} \mu_{\widetilde{F}_j^i}(x_j)$ the degree of firing associated with the i^{th} consequent set for $i = 1,2,\ldots,m$.

For an interval type-2 FLS, each Y^i and F^i is an interval type-1 set, then $\mu_{Y^i}(y^i) = \mu_{F^i}(f^i) = 1$. Equation (4) can be rewritten as

$$\begin{aligned} &Y_{\cos}(Y^1,\ldots,Y^m,F^1,\ldots,F^m) \\ &= \int_{y^1}\cdots\int_{y^m}\int_{f^1}\cdots\int_{f^m} 1 \Bigg/ \frac{\sum_{i=1}^m f^i y^i}{\sum_{i=1}^m f^i} = [y_l,\ \ y_r] \end{aligned} \quad (5)$$

The fuzzifier maps a crisp point $\underline{x} = (x_1, x_2, \ldots, x_n)^T$ into a fuzzy set. The defuzzifier maps fuzzy set in R to crisp points in R.

By using the singleton fuzzification, product inference and centre-average defuzzification, the output value of fuzzy system is:

$$y(\underline{x}) = \underline{\theta}^T \xi(\underline{x}, t) \quad (6)$$

where $\underline{\theta} = (\theta^1, \theta^2, \ldots, \theta^m)^T = (\overline{y}^1, \overline{y}^2, \ldots, \overline{y}^m)^T$ is the parameter vector, and $\xi(\underline{x}) = (\xi^1, \xi^2, \ldots, \xi^m)^T$ is the vector of fuzzy basis functions (usually we assume $\mu_{Y^i}(\overline{y}^i) = 1$).

III. Problem Statement and Design Adaptive Fuzzy Control

Consider a general class of SISO n-th order nonlinear systems described as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(\underline{x}, t) + g(\underline{x}, t)u + d(t) \\ y = x_1 \end{cases} \quad (7)$$

Or equivalently

$$\begin{cases} x^{(n)} = f(\underline{x},t) + g(\underline{x},t)u + d(t) \\ y = x \end{cases} \tag{8}$$

where $\underline{x} = (x, \dot{x}, \ldots, x^{(n-1)})^T = (x_1, x_2, \ldots, x_n)^T \in R^n$ is the state vector of the system which is assumed to be available for measurement, $u \in R$ and $y \in R$ are respectively the input and the output of the system, f, g are nonlinear system functions representing the dynamic system behaviour and control gain respectively. $d(t)$ is the unknown external disturbance. We require the system (8) to be controllable, the input gain $g(\underline{x},t) \neq 0$ is necessary.

In many real application, $f(\underline{x},t)$ and $g(\underline{x},t)$ may not be exactly known, which can be split into two parts as:

$$\begin{aligned} f(\underline{x},t) &= \bar{f}(\underline{x},t) + \Delta f(\underline{x},t), \\ g(\underline{x},t) &= \bar{g}(\underline{x},t) + \Delta g(\underline{x},t) \end{aligned} \tag{9}$$

where $\bar{f}(\underline{x},t)$ and $\bar{g}(\underline{x},t)$ denote the nominal parts and $\Delta f(\underline{x},t)$ and $\Delta g(\underline{x},t)$ represent their uncertain part respectively.

Without loss of generality, we have the following assumptions:

1) $\underline{x}$ belong to a compact set $U_x = \{x \in R^n : |\underline{x}| \leq M_x\}$, where M_x is positive constant.

2) $\Delta f(\underline{x},t)$ and $\Delta g(\underline{x},t)$ are bounded as follows:

$$|\Delta f(\underline{x},t)| \leq F(\underline{x},t),\ |\Delta g(\underline{x},t)| \leq G(\underline{x},t) \quad \forall x \in U_x \subset R^n$$

3) $|d(t)| \leq d_m$, where d_m is upper bound.

4) The gain $g(\underline{x},t)$ is strictly positive and globally bounded away from zero by a known constant g_0, i.e $g(\underline{x},t) \geq g_0 > 0$ for all $x \in U_x$

We use type-2 fuzzy logic system (6) to approximate the unknown nonlinear functions $\Delta f(\underline{x},t)$ and $\Delta g(\underline{x},t)$ respectively :

$$\begin{aligned} \Delta\hat{f}(\underline{x}/\underline{\theta}_f) &= \underline{\theta}_f^T \xi_f(\underline{x},t) \\ \Delta\hat{g}(\underline{x}/\underline{\theta}_g) &= \underline{\theta}_g^T \xi_g(\underline{x},t) \end{aligned} \tag{10}$$

Where $\underline{\theta}_f$ and $\underline{\theta}_g$ are free to be tuned adaptively and $\xi_f(\underline{x},t)$ and $\xi_g(\underline{x},t)$ are a regressive vector.

Define the optimal parameters of type-2 fuzzy systems:

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f \in \Omega_f}\left[\sup_{\underline{x} \in U_x}\left|\Delta f(\underline{x},t) - \underline{\theta}_f^T \xi_f(\underline{x},t)\right|\right] \tag{11}$$

$$\underline{\theta}_g^* = \arg\min_{\underline{\theta}_g \in \Omega_g}\left[\sup_{\underline{x} \in U_x}\left|\Delta g(\underline{x},t) - \underline{\theta}_g^T \xi_g(\underline{x},t)\right|\right] \tag{12}$$

It is assumed that the optimal vectors $\underline{\theta}_f^*$ and $\underline{\theta}_g^*$ minimize the modelling error lie in some convex region:

$$\Omega_f = \{\underline{\theta}_f \in R^m / |\underline{\theta}_f| \leq M_f\} \tag{13}$$

$$\Omega_g = \{\underline{\theta}_g \in R^m / |\underline{\theta}_g| \leq M_g\} \tag{14}$$

where the radii M_f and M_g are positive constants.

Since $\Delta\hat{f}(\underline{x}/\underline{\theta}_f)$ and $\Delta\hat{g}(\underline{x}/\underline{\theta}_g)$ are interval type-2 fuzzy systems, then the type-reduced sets will be given respectively by:

$$\begin{aligned} &\tilde{F}_{\cos}(\theta_f^1, \ldots, \theta_f^m, f^1, \ldots, f^m) \\ &= \int_{\theta_f^1} \cdots \int_{\theta_f^m} \int_{f^1} \cdots \int_{f^m} 1 \Big/ \frac{\sum_{i=1}^m f^i \theta_f^i}{\sum_{i=1}^m f^i} = [\hat{f}_l, \ \hat{f}_r] \end{aligned} \tag{15}$$

$$\begin{aligned} &\tilde{G}_{\cos}(\theta_g^1, \ldots, \theta_g^m, g^1, \ldots, g^m) \\ &= \int_{\theta_g^1} \cdots \int_{\theta_g^m} \int_{g^1} \cdots \int_{g^m} 1 \Big/ \frac{\sum_{i=1}^m g^i \theta_g^i}{\sum_{i=1}^m g^i} = [\hat{g}_l, \ \hat{g}_r] \end{aligned} \tag{16}$$

f^i and g^i are the degrees of firing associated with the i^{th} rule of the FLS $\Delta\hat{f}(\underline{x}/\underline{\theta}_f)$ and $\Delta\hat{g}(\underline{x}/\underline{\theta}_g)$, respectively.

Equation (15) and (16) may be computed using the Karnik-Mendel iterative method [13]. It has been proven that this iterative procedure can converge in at most N iterations.

Once $\hat{f}_l$, $\hat{f}_r$, $\hat{g}_l$ and $\hat{g}_r$ are obtained, they can be used to calculate the crisp output. Since the type-reduced set is an interval type-1 set, the defuzzified output is:

$$\begin{aligned} \Delta\hat{f}(\underline{x}/\underline{\theta}_f) &= \frac{\hat{f}_l + \hat{f}_r}{2} \\ \Delta\hat{g}(\underline{x}/\underline{\theta}_g) &= \frac{\hat{g}_l + \hat{g}_r}{2} \end{aligned} \tag{17}$$

$$\begin{aligned} \hat{f}_l &= \frac{\sum_{i=1}^m f_l^i \theta_f^i}{\sum_{i=1}^m f_l^i}, \quad \hat{f}_r = \frac{\sum_{i=1}^m f_r^i \theta_f^i}{\sum_{i=1}^m f_r^i} \\ \hat{g}_l &= \frac{\sum_{i=1}^m g_l^i \theta_g^i}{\sum_{i=1}^m g_l^i}, \quad \hat{g}_r = \frac{\sum_{i=1}^m g_r^i \theta_g^i}{\sum_{i=1}^m g_r^i} \end{aligned} \tag{18}$$

$\xi_{fl}(\underline{x}) = (\xi_{fl}^1, \xi_{fl}^2, \ldots, \xi_{fl}^m)^T$, $\xi_{fr}(\underline{x}) = (\xi_{fr}^1, \xi_{fr}^2, \ldots, \xi_{fr}^m)^T$, $\xi_{gl}(\underline{x}) = (\xi_{gl}^1, \xi_{gl}^2, \ldots, \xi_{gl}^m)^T$ and $\xi_{gr}(\underline{x}) = (\xi_{gr}^1, \xi_{gr}^2, \ldots, \xi_{gr}^m)^T$ are the vectors of the fuzzy basis functions with

$$\xi_{fl}^i = \frac{f_l^i}{\sum_{i=1}^m f_l^i} \text{ and } \xi_{fr}^i = \frac{f_r^i}{\sum_{i=1}^m f_r^i} \tag{19}$$

f_l^i and f_r^i denote the firing values used to compute the left point $\hat{f}_l$ and right point $\hat{f}_r$, respectively.

$$\xi_{gl}^i = \frac{g_l^i}{\sum_{i=1}^m g_l^i} \text{ and } \xi_{gr}^i = \frac{g_r^i}{\sum_{i=1}^m g_r^i} \tag{20}$$

g_l^i and g_r^i denote the firing values used to compute the left point $\hat{g}_l$ and right point $\hat{g}_r$, respectively.

$\underline{\theta}_f = (\theta_f^1, \theta_f^2, \ldots, \theta_f^m)$ and $\underline{\theta}_g = (\theta_g^1, \theta_g^2, \ldots, \theta_g^m)$ are the adjustable parameter vector of type-2 fuzzy systems $\Delta\hat{f}(\underline{x}/\underline{\theta}_f)$ and $\Delta\hat{g}(\underline{x}/\underline{\theta}_g)$.

Then

$$\begin{aligned} \hat{f}_l &= \underline{\theta}_f^T \xi_{fl}(\underline{x}) \\ \hat{f}_r &= \underline{\theta}_f^T \xi_{fr}(\underline{x}) \\ \hat{g}_l &= \underline{\theta}_g^T \xi_{gl}(\underline{x}) \\ \hat{g}_r &= \underline{\theta}_g^T \xi_{gr}(\underline{x}) \end{aligned} \tag{21}$$

Therefore

$$\begin{aligned} \Delta\hat{f}(\underline{x}, \underline{\theta}_f) &= \frac{\underline{\theta}_f^T \xi_{fl}(\underline{x}) + \underline{\theta}_f^T \xi_{fr}(\underline{x})}{2} \\ &= \underline{\theta}_f^T \left[\frac{\xi_{fl}(\underline{x}) + \xi_{fr}(\underline{x})}{2} \right] \\ &= \underline{\theta}_f^T \xi_f(\underline{x}) \end{aligned} \tag{22}$$

and

$$\begin{aligned} \Delta\hat{g}(\underline{x}, \underline{\theta}_g) &= \frac{\underline{\theta}_g^T \xi_{gl}(\underline{x}) + \underline{\theta}_g^T \xi_{gr}(\underline{x})}{2} \\ &= \underline{\theta}_g^T \left[\frac{\xi_{gl}(\underline{x}) + \xi_{gr}(\underline{x})}{2} \right] \\ &= \underline{\theta}_g^T \xi_g(\underline{x}) \end{aligned} \tag{23}$$

Based on the universal approximation theorem [11]-[13], the above fuzzy logic system is capable of uniformly approximating any well-defined nonlinear function over a compact set U_x to any degree of accuracy.

The control problem is to obtain the state $\underline{x}$ in order to track a desired state $\underline{x}_d$ in the presence of model uncertainties and external disturbance with the tracking error

$$\underline{e} = \underline{x} - \underline{x}_d = (e, \dot{e}, \ldots, e^{(n-1)})^T \in R^n \tag{24}$$

Define $\underline{C} = (c_1, c_2, \ldots, c_{n-1}, 1)^T$, the coefficients of the Hurwitz polynomial $h(\lambda) = \lambda^{n-1} + c_{n-1}\lambda^{n-2} + \cdots + c_1$, i.e. all the roots are in the open left half-plane.

If the functions $f(\underline{x},t)$ and $g(\underline{x},t)$ are completely known and $d(t) = 0$, we can solve the control problem stated above by the so-called feedback linearization method. In this method, the functions $f(\underline{x},t)$ and $g(\underline{x},t)$ are used to construct the following feedback control law:

$$u = \frac{1}{g(\underline{x},t)} (-f(\underline{x},t) + x_d^{(n)} - \sum_{i=1}^{n-1} c_i e^{(i)}) \tag{25}$$

However, $f(\underline{x},t)$, $g(\underline{x},t)$ and disturbance are actually unknown in practical systems, we cannot use them for constructing the control law (25). Therefore, to solve these problems, we replace them by their estimates with the type-2 fuzzy logic systems $\bar{f}(\underline{x},t) + \Delta\hat{f}(\underline{x}/\underline{\theta}_f)$ and $\bar{g}(\underline{x},t) + \Delta\hat{g}(\underline{x}/\underline{\theta}_g)$ to construct a self-tuning controller

$$u_c = \frac{1}{\bar{g}(\underline{x},t) + \Delta\hat{g}(\underline{x}/\underline{\theta}_g)} \begin{pmatrix} -\bar{f}(\underline{x},t) - \Delta\hat{f}(\underline{x}/\underline{\theta}_f) \\ + x_d^{(n)} - \sum_{i=1}^{n-1} c_i e^{(i)} \end{pmatrix} \tag{26}$$

However, the law u_c applied to the system (8) can not lead to a Lyapunov function $V = \frac{1}{2}\underline{e}^T P \underline{e}$ with a time derivate negative. To overcome this problem, we add a component u_s that will force the time derivate of the function of Lyapunov to be negative. u_s is called the supervision control.

$$u = u_c - u_s \tag{27}$$

The resulting control law is as follows:

$$u_s = \begin{cases} \dfrac{sign(\underline{e}^T PB)}{g_0} [F(x) + \left|\Delta\hat{f}(\underline{x}/\underline{\theta}_f)\right| + \\ G(x)|u_c| + \left|\Delta\hat{g}(\underline{x}/\underline{\theta}_g) u_c\right| + d_m] & if\, V_1 \geq \widetilde{V} \\ 0 & if\, V_1 < \widetilde{V} \end{cases} \tag{28}$$

with adaptive law is gives:

$$\begin{aligned}
&\dot{\underline{\theta}}_f = \delta_1 \underline{e}^T PB\xi_f(\underline{x},t)\\
&if(\left|\underline{\theta}_f\right| < M_f)\\
&or(\left|\underline{\theta}_f\right| = M_f \ and\ \underline{e}^T PB\underline{\theta}_f^T \xi_f(\underline{x},t) \geq 0)\\
&\dot{\underline{\theta}}_g = \delta_2 \underline{e}^T PB\xi_g(\underline{x},t)u_c\\
&if(\left|\underline{\theta}_g\right| < M_g)\\
&or\ (\left|\underline{\theta}_g\right| = M_g \ and\ \underline{e}^T PB\underline{\theta}_g^T \xi_g(\underline{x},t)u_c \geq 0)
\end{aligned} \tag{29}$$

THEOREM

For the controlled system (8) with type-2 fuzzy logic system to approximate the unknown nonlinear function (6), if assumptions (1-4) are true, then the closed-loop control system with control signal defined by (27) and adaptive law defined by (29) is globally stable in the sense that all signals involved are bounded, with the tracking error converging to zero.

PROOF

We use the Lyapunov approach in which u_s and the adaptive law are chosen such that to make a Lyapunov function decrease along the trajectories of the adaptive system.

Applying the control law (27) to the system (8), after some manipulation, results in the error dynamic equation

$$\begin{aligned}
\dot{\underline{e}} = A\underline{e} + B(\Delta f(\underline{x},t) - \Delta\hat{f}(\underline{x}/\underline{\theta}_f) + \\
(\Delta g(\underline{x},t) - \Delta\hat{g}(\underline{x}/\underline{\theta}_g))u_c + d(t) \\
-(\bar{g}(\underline{x},t) + \Delta g(\underline{x},t))u_s)
\end{aligned} \tag{30}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -0 & -c_1 & -c_2 & \cdots & \cdots & \cdots & -c_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Since A is a stable matrix, there exists a unique positive definite symmetric *nxn* matrix *P* that satisfies the Lyapunov equation

$A^T P + PA = -Q$, where Q is an arbitrary *nxn* positive - definite symmetric matrix.

We consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}\underline{e}^T P\underline{e} \tag{31}$$

The time derivative of V_1 along the system trajectory is

$$\begin{aligned}
\dot{V}_1 = -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T PB(\Delta f(\underline{x},t) - \Delta\hat{f}(\underline{x}/\underline{\theta}_f) \\
+ (\Delta g(\underline{x},t) - \Delta\hat{g}(\underline{x}/\underline{\theta}_g))u_c - (\bar{g}(\underline{x},t) \\
+ \Delta g(\underline{x},t))u_s + d(t))
\end{aligned} \tag{32}$$

$$\begin{aligned}
\dot{V}_1 \leq -\frac{1}{2}\underline{e}^T Q\underline{e} + \left|\underline{e}^T PB\right|(\left|\Delta f(\underline{x},t)\right| + \left|\Delta\hat{f}(\underline{x}/\underline{\theta}_f)\right| \\
+ \left|\Delta g(\underline{x},t)u_c\right| + \left|\Delta\hat{g}(\underline{x}/\underline{\theta}_g)u_c\right| + d) - (\bar{g}(\underline{x},t) \\
+ \Delta g(\underline{x},t))\underline{e}^T PBu_s
\end{aligned} \tag{33}$$

Substitute u_s defined in (28) into (33), then we have

$$\dot{V}_1 \leq -\frac{1}{2}\underline{e}^T Q\underline{e} \leq 0 \tag{34}$$

For adjusting the parameters of the self-tuning fuzzy controller (27), we chosen another Lyapunov function candidate

$$V_2 = \frac{1}{2}\underline{e}^T P\underline{e} + \frac{1}{2\delta_1}\underline{\Phi}_f^T \underline{\Phi}_f + \frac{1}{2\delta_2}\underline{\Phi}_g^T \underline{\Phi}_g \tag{35}$$

With

$\underline{\Phi}_f = \underline{\theta}_f^* - \underline{\theta}_f$, $\underline{\Phi}_g = \underline{\theta}_g^* - \underline{\theta}_g$

δ_1 and δ_2 are positive constants specified by the designer.

Define the minimum approximation error:

$$\begin{aligned}
\omega = \Delta f(\underline{x},t) - \underline{\theta}_f^{*T} \xi_f(\underline{x},t) \\
+ (\Delta g(\underline{x},t) - \underline{\theta}_g^{*T} \xi_g(\underline{x},t))u_c + d(t)
\end{aligned} \tag{36}$$

The error dynamic equation becomes:

$$\begin{aligned}
\dot{\underline{e}} = A\underline{e} + B\omega - B(\bar{g}(\underline{x},t) + \Delta g(\underline{x},t))u_s \\
+ B(\underline{\Phi}_f^T \xi_f(\underline{x},t) + \underline{\Phi}_g^T \xi_g(\underline{x},t)u_c)
\end{aligned} \tag{37}$$

The time derivate of V_2 along the trajectories of (37) equals

$$\begin{aligned}
\dot{V}_2 = -\frac{1}{2}\underline{e}^T Q\underline{e} - (\bar{g}(\underline{x},t) + \Delta g(\underline{x},t))\underline{e}^T PBu_s \\
+ \underline{e}^T PB\omega + \frac{\underline{\Phi}_f^T}{\delta_1}(-\dot{\underline{\theta}}_f + \delta_1 \underline{e}^T PB\xi_f(\underline{x},t)) \\
+ \frac{\underline{\Phi}_g^T}{\delta_2}(-\dot{\underline{\theta}}_g + \delta_2 \underline{e}^T PB\xi_g(\underline{x},t)u_c)
\end{aligned} \tag{38}$$

Substitute $\dot{\underline{\theta}}_f$ and $\dot{\underline{\theta}}_g$ defined by (29) into (38), then we have

$$\begin{aligned}
\dot{V}_2 \leq -\frac{1}{2}\underline{e}^T Q\underline{e} - (\bar{g}(\underline{x},t) + \Delta g(\underline{x},t)\underline{e}^T PBu_s \\
+ \underline{e}^T PB\omega
\end{aligned} \tag{39}$$

With u_s previously calculated, and given that $\underline{e}^T PB\omega$ is on the order of the minimal approximation error ω (minimal effect), $\dot{V}_1 \leq -\frac{1}{2}\underline{e}^T Q\underline{e}$ is the best result that can be obtained.

IV. Simulations Results

In this section, we test the adaptive fuzzy controller on the tracking control of a second order nonlinear servomechanism model described by the following equation [8]:

$$m\ddot{q} + \ell\dot{q} + \Delta f(q) = \tau + d \tag{40}$$

where q and $\dot{q}$ denote velocity and position, respectively. $\Delta f(q)$ is the nonlinear term depending on q. m and ℓ are the mass and damping , respectively. τ is the torque, and d is the disturbance.

The dynamic equations of the servomechanism can be described in space state as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\underline{x}, t) + u + d(t) \\ y = x_1 \end{cases} \tag{41}$$

$f(\underline{x}, t) = -\ell x_2 - \Delta f(x_1), u = \tau$
with $x_1 = q$ and $x_2 = \dot{q}$

The control objective is to maintain the system to track the desired angle trajectory $x_d = \theta_d = (\pi/3)\sin(0.01t)$.

The parameters of the model are given as $m = 1kg$, $\ell = 1$, $d = 0.045 rand(1{,}200)$, $Q = 10I$,

$$P = \begin{bmatrix} 2.6042 & -5.0000 \\ -5.0000 & 12.5000 \end{bmatrix},$$

The coefficients of the Hurwitz polynomial are set as $c_1 = 24, c_2 = 10$, $M_f = 6$ and $\widetilde{V} = 1.0267$. Set the initial condition $\underline{x} = (x_1, x_2)^T = (2, 0)^T$, the learning rate $\delta_1 = 0.5$ and step size at 0.01s.

The simulation results are shown in Fig. 3 and 4, which demonstrates the convergence that the tracking error is guaranteed with unknown nonlinear function and in the presence of disturbance. The membership function for system state x_1 is represented in Fig. 2. Then there are 3 rules to approximate the system function $\Delta f(x_1) = 0.4\sin(x_1)$.

V. Conclusion

In this paper, we presented an adaptive fuzzy control for a class of nonlinear system based on the Lyapunov synthesis approach. We introduced the type-2 fuzzy logic system to approximate the unknown nonlinear term. The main advantage of the proposed adaptive fuzzy controller is that it does not need any knowledge about the nonlinear term. The simulation results have show that the effectiveness of the adaptive controller in achieving the desired performance.

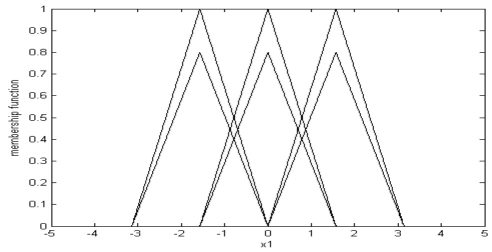

Fig. 2 - Type-2 membership functions used for the adaptive fuzzy controller.

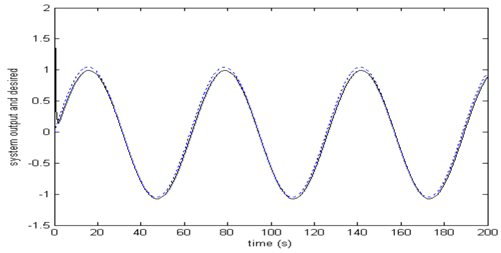

Fig. 3 - The state x_1 and its desired value x_d.

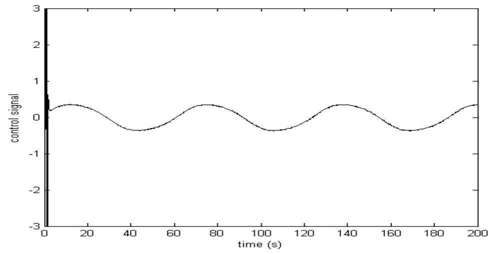

Fig. 4 - Control signal.

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