A Thermodynamic Solution for the Static and Dynamic Characteristics of a Two-Lobe Journal Bearing

B. Chetti, W. A. Crosby

Abstract—The work described in this paper is an investigation of the static and dynamic characteristics of two-lobe journal bearings taking into consideration the thermal effects. A thermo-hydrodynamic solution of a finite two-lobe journal bearing is performed by solving the generalized form Reynolds equation with the energy equation, taking into consideration viscosity variation across the film thickness. The static and dynamic characteristics were numerically obtained. The results are evaluated for different values of viscosity-temperature coefficient and Peclet number. The results show that considering the thermal effects in the solution of the two-lobe journal bearing has a marked on the study of its stability.

Keywords—Two-lobe bearing, thermal effect, static and dynamic characteristics.

I. INTRODUCTION

THE circular bearing is extensively used in engineering applications because of its manufacturing simplicity, but unfortunately it may suffer from instability problems at high rotational speeds. The instability produced by hydrodynamic forces in bearings is characterized by large amplitude vibration, which can result in bearing and therefore total machine failure. Non-circular bearings have the advantage of being more stable at higher speeds. Of the various non-circular journal bearings, the two-lobe journal bearing is considered quite attractive owing to its relative ease of manufacture.

The performance characteristics of two-lobe journal bearings have been studied by a number of investigators. Pinkus [1] and Kumar [2] theoretically investigated the effects of ellipticity on the static and dynamic characteristics, but they dealt with the laminar flow regime. This work was improved by introducing the turbulent flow regime [3]-[5]. On the other hand, Abdul-Wahed et al. [6] calculated the dynamic characteristics of 6 types of multi-lobe journal bearings in turbulent flow regime. Elasto-hydrodynamic effects were introduced in the study of two-lobe journal bearings by many researchers [7]-[9]. Tayal et al. [10] investigated the performance of two-lobe journal bearings lubricated with a non-Newtonian fluid. The forgoing analyses have been predominantly isothermal in approach.

Thermohydrodynamic solutions were first reported by Dowson and co-workers [11]. Numerous analyses of the thermo-hydrodynamic aspects in bearings followed [12]-[15].

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These analyses have been directed towards cylindrical bearings. The static performance of non-circular bearings taking into account thermal considerations was studied by a number of researchers. Basri et al. [16] investigated the thermal aspect using an adiabatic model of four-lobe, three-lobe, offset-half and orthogonally displaced journal bearings. Crosby [17] presented a thermo-hydrodynamic solution of two-lobe considering reverse flow. Hussain et al. [18] studied the prediction of temperature distribution in some type two-lobed bearings and surrounding solids. Chetti [19] investigated the static and dynamic characteristics of the three-lobe bearing taking into account thermal effects. Recently, Chauhan et al. [20] presented a comparative study based on the thermal performance of elliptical and offset-halves journal bearings.

In this study, the thermohydrodynamic analysis of the static and dynamic characteristics of the two-lobe bearings is presented.

II. PRESSURE EQUATION

If we neglect the variation in the lubricant's density, the generalized Reynolds equation could be written in the following normalized form:

$$\frac{\partial}{\partial\theta}\left(H^3 I_2 \frac{\partial P}{\partial\theta}\right) + B^2\left\{\frac{\partial}{\partial\zeta}\left(H^3 I_2 \frac{\partial P}{\partial\zeta}\right)\right\} = \frac{\partial}{\partial\theta}\left[H\left(1-\frac{I_1}{I_0}\right)\right] + \frac{\partial H}{\partial t} \quad (1)$$

The non-dimensional film thickness H is given by, Fig. 1:

$$H = 1 - \overline{\varepsilon}\left(\cos\theta\sin\phi - \sin\theta\cos\phi\right) \pm \delta\sin\theta \quad (2)$$

where

$$\overline{\varepsilon} = \varepsilon(1-\delta) \quad \text{and} \quad P = p\left(c/R\right)^2 / \mu_i'\omega \quad (3)$$

The negative and positive signs in (2) are used to determine the film thickness in upper lobe and lower lobe respectively. And the integrals

$$I_0 = \int_0^1 \frac{1}{\mu} d\eta \; ; \; I_1 = \int_0^1 \frac{\eta}{\mu} d\eta \; ; \; I_2 = \int_0^1 \frac{\eta}{\mu}\left(\eta - \frac{I_1}{I_0}\right) d\eta \quad (4)$$

This equation is then set into a finite difference form by using central difference scheme.

A. Pressure Boundary Conditions

The boundary conditions are;

$$P(\theta, 0) = P(\theta, 1) = 0. \tag{5}$$

$$P\left(\theta_{s1}, \zeta\right) = 0. \; ; \; P\left(\theta_{s2}, \zeta\right) = 0. \tag{6}$$

$$\frac{\partial P\left(\theta_{t1}, \zeta\right)}{\partial \theta} = 0 \; ; \; \frac{\partial P\left(\theta_{t2}, \zeta\right)}{\partial \theta} = 0. \tag{7}$$

and

$$P\left(\theta, \zeta\right) = 0 \text{ for } \theta_{e1} \ge \theta \ge \theta_{t1} \text{ and } \theta_{e2} \ge \theta \ge \theta_{t2} \tag{8}$$

III. TEMPERATURE EQUATION

The temperature distribution within the oil film is determined by the following form of non-dimensional energy equation [17]

$$\frac{1}{Pe}\left(\frac{\partial^2 T}{\partial \eta^2}\right) - H^2 u \left(\frac{\partial T}{\partial \theta}\right) + \mu \left(\frac{\partial u}{\partial \eta}\right)^2 = 0 \tag{9}$$

where

$$u = -H^2 \left(\frac{\partial P}{\partial \theta}\right) \left\{ \frac{I_1}{I_0} \int_0^\eta \frac{1}{\mu} d\eta - \int_0^\eta \frac{\eta}{\mu} d\eta \right\} + 1 - \frac{1}{I_0} \int_0^\eta \frac{1}{\mu} d\eta \tag{10}$$

$$\frac{\partial u}{\partial \eta} = \frac{1}{\mu}\left\{ H^2 \left(\eta - \frac{I_1}{I_0}\right)\left(\frac{\partial P}{\partial \theta}\right) - \frac{1}{I_0} \right\} \tag{11}$$

$Pe = \dfrac{\rho C_p \omega C^2}{k}$ is Peclet number

The non-dimensional viscosity μ is assumed to be defined by the relationship

$$\mu = \exp(\alpha T) \tag{12}$$

where α is a viscosity-temperature coefficient.

Equation (9) is of parabolic type and the Crank-Nicolson method is used to obtain the finite difference scheme.

A. Temperature Boundary Conditions

The boundary conditions for temperature are;

$$T\left(\theta_{s1}, \eta\right) = T\left(\theta_{s2}, \eta\right) = T_i \tag{13}$$

$$T\left(\theta, 0\right) = T_j \tag{14}$$

$$T_j = T_i \tag{15}$$

$$\frac{\partial T\left(\theta, 1\right)}{\partial \eta} = 0 \tag{16}$$
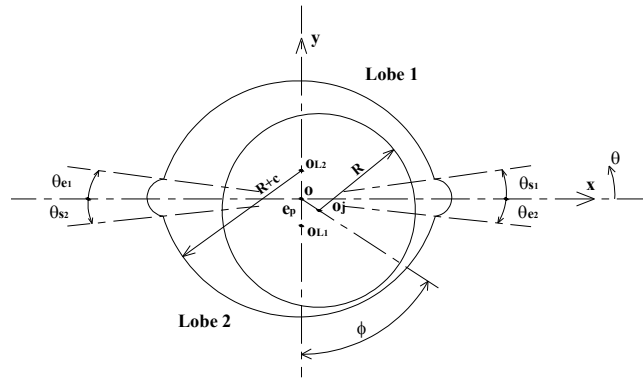

Fig. 1 Journal Bearing Configurations

IV. STATIC CHARACTERISTICS

The static characteristics of the bearing are obtained by solving the Reynolds equation (1) for static loading ($\dot{x} = \dot{y} = 0$) and the energy equation (9).

The numerical procedure adopted for obtaining the pressure and temperature fields are as follows:

An initial value for attitude angle ϕ is assumed in order to calculate the film thickness by (2). Initial values of pressure are assumed to be zero at all points, and a suitable initial temperature is assumed, so that the viscosity at the nodal points could be calculated. The generalized Reynolds equation (1) is solved using Gauss-Seidel method with over-relaxation. Negative pressures are set to zero to account for Swift-Stieber boundary conditions. The velocity u and its gradient edge $\dfrac{\partial u}{\partial \eta}$ are then computed at every nodal point. The energy equation is solved; thus a new temperature distribution is obtained. Convergence is assumed when at each point the relative difference in pressure between successive iterations does not exceed 0.1%. The horizontal and vertical load components

$$W_x = \int_0^1 \int_0^{2\pi} p \sin \theta \, d\theta \, d\zeta \; ; \; W_y = \int_0^1 \int_0^{2\pi} p \cos \theta \, d\theta \, d\zeta \tag{17}$$

are computed. The equilibrium position of the journal is achieved when W_x is zero. A satisfactory value for ϕ is considered to be attained if $|W_x / W_y| \le 0.001$. Otherwise, another value of ϕ is assumed.

V. DYNAMIC CHARACTERISTICS

The total stiffness coefficients for a multi-lobe bearing are given by:

$$\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} = -\begin{Bmatrix} \partial / \partial x \\ \partial / \partial y \end{Bmatrix} \begin{bmatrix} W_x & W_y \end{bmatrix} \tag{18}$$

In these equations the partial derivatives of x and y are found by changing x or y by a small value about the static equilibrium position of the bearing.

The expressions for damping coefficients are given by:

$$\begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} = - \begin{Bmatrix} \partial / \partial \dot{x} \\ \partial / \partial \dot{y} \end{Bmatrix} \begin{bmatrix} W_x & W_y \end{bmatrix} \qquad (19)$$

The partial derivatives of $\dot{x}$ and $\dot{y}$ are obtained by giving a small value to $\dot{x}$ and $\dot{y}$ respectively corresponding to the equilibrium position of the bearing.

VI. STABILITY ANALYSIS

The linearized equations of the disturbed motion of the journal centre are

$$M\,\ddot{x} + K_{xx}\,x + C_{xx}\,\dot{x} + K_{xy}\,y + C_{xy}\,\dot{y} = 0$$
$$M\,\ddot{y} + K_{yx}\,x + C_{yx}\,\dot{x} + K_{yy}\,y + C_{yy}\,\dot{y} = 0 \qquad (20)$$

Equations (20) are used to study the stability of the bearing system. Harmonic solution of the type:

$$x = x e^{\lambda t} \;,\; y = y e^{\lambda t} \qquad (21)$$

Thus (20) can be written as:

$$\begin{bmatrix} K_{xx} - M\nu^2 + i\nu C_{xx} & K_{xy} + i\nu C_{xy} \\ K_{yx} + i\nu C_{yx} & K_{yy} - M\nu^2 + i\nu C_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad (22)$$

For a nontrivial solution the determinant must vanish and equating the real and imaginary parts to zero gives:

$$\overline{M}\gamma^2 = \frac{\overline{C}_{xx}\,\overline{K}_{yy} + \overline{C}_{yy}\,\overline{K}_{xx} - \overline{C}_{xy}\,\overline{K}_{yx} - \overline{C}_{yx}\,\overline{K}_{xy}}{\overline{C}_{xx} + \overline{C}_{yy}} \qquad (23)$$

$$\gamma^2 = \frac{\left(\overline{K}_{xx} - \overline{M}\gamma^2\right)\left(\overline{K}_{yy} - \overline{M}\gamma^2\right) - \overline{K}_{yx}\,\overline{K}_{xy}}{\overline{C}_{xx}\,\overline{C}_{yy} - \overline{C}_{xy}\,\overline{C}_{yx}} \qquad (24)$$

where

$$\overline{C}_{ij} = C_{ij}\frac{c\omega}{W} \;,\; \overline{K}_{ij} = K_{ij}\frac{c}{W} \;,\; \overline{M} = \frac{Mc\omega^2}{W} \;,\; \gamma = \frac{\nu}{\omega}$$

From (23) and (24), the critical mass and the whirl ratio γ are calculated. $\overline{M}_c$ is the critical mass parameter above which the bearing is unstable.

VII. RESULTS AND DISCUSSION

Computer solutions for a two-lobe journal bearing with two supply grooves; each one has an angle of 20 deg, have been evaluated for one aspect ratio $B = 1$, three values of Peclet number (5, 50, 200) and three values of viscosity-temperature coefficient (0.01, 0.1, 0.3).The ellipticity ratio used here is 0.5.

Fig. 2 shows the variation of the maximum oil temperature Tmax with the eccentricity ratio for different values of the viscosity-temperature coefficient and Peclet number. The value of Tmax decreases with the increase of the viscosity-temperature coefficient (Fig. 2 (a)).This is due to the decreases of the oil viscosity. Tmax increases with the increase of Peclet number and the eccentricity ratio (Fig. 2 (b)). The Peclet number is related to the velocity of journal, so if the velocity of journal increases the Peclect number increases and as a result, the temperature increases.

Fig. 3 gives the variation of the attitude angle with the eccentricity ratio corresponding to different values of viscosity-temperature coefficient and Peclet number. The curves show that the attitude angle increases with increasing of both viscosity-temperature coefficient and Peclet number.
The effect of these two parameters on the load carrying capacity of bearing is shown in Fig. 4, an increase of α and Pe produces a decrease of the load carrying capacity.
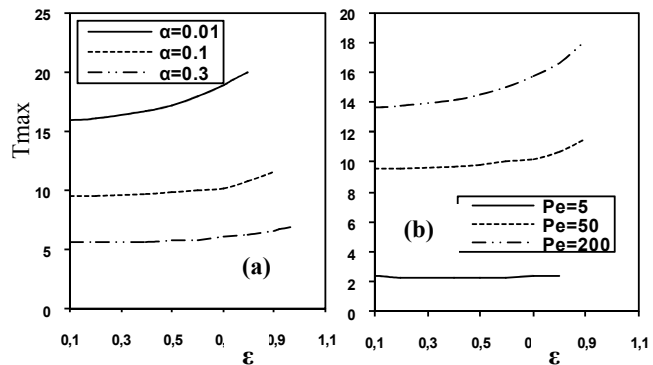

Fig. 2 Tmax versus ε. (a) for various values of α (b) for various values of Pe

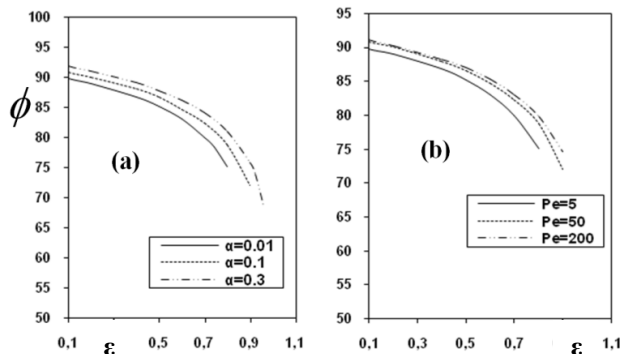

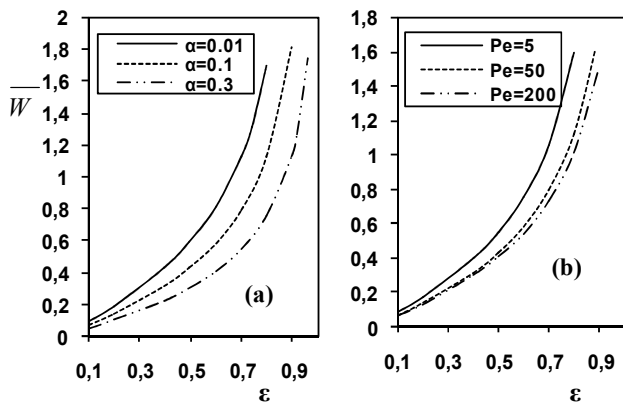Fig. 3 ϕ versus ε. (a) for various values of α (b) for various values of Pe

Fig. 4 $\overline{W}$ versus ε. (a) for various values of α (b) for various values of Pe

The stability charts are shown in Fig. 5 for various values of viscosity-temperature coefficient and Peclet number. The lower and upper sides of each curve corresponding to stable and unstable regions respectively. The critical mass decreases as the viscosity-temperature coefficient and Peclet number increase and the reduction being greater at higher eccentricity ratios. Fig. 6 exhibits the variation of whirl ratio with eccentricity ratio for different values of viscosity-temperature coefficient and Peclet number. From the figures, it can be seen that the whirl ratio increases with an increases of both α and Pe. The effect of these two parameters on whirl ratio is significant at higher eccentricity ratio.
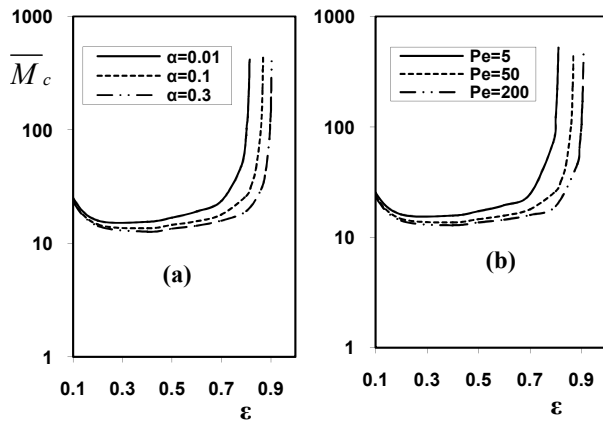

Fig. 5 $\overline{M}_c$ versus ε. (a) for various values of α (b) for various values of Pe
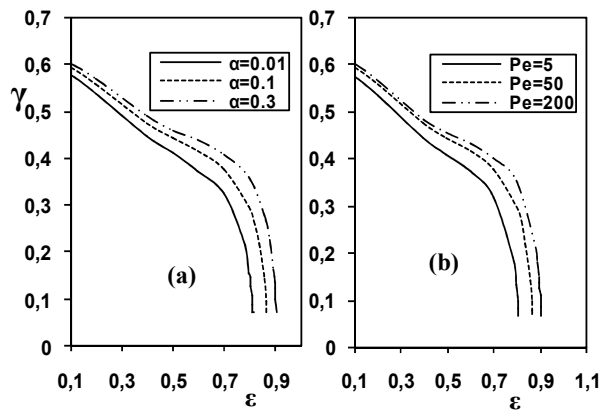

Fig. 6 γ versus ε. (a)for various values of α (b)for various values of Pe

VI. CONCLUSIONS

On the basis of the results and discussions presented in the previous section, the following conclusions are drawn:

(1) The maximum temperature the decreases with increases of the viscosity-temperature coefficient and with the decrease of Peclet number.

(2) The attitude angle increases with the increase of the viscosity-temperature coefficient and with the increase of Peclet number.

(3) For a given bearing geometry and eccentricity ratio, both the load carrying capacity and the critical mass decrease as the viscosity-temperature coefficient and Peclet number increase.

(4) The whirl ratio increases with the increases of the viscosity-temperature coefficient and with the increase of Peclet number.

(5) Considering the thermal effects in the solution of the two-lobe journal bearing, the thermohydrodynamic solution decreases the stability region and increases the whirl ratio.

NOMENCLATURE

B	Aspect ratio
c	The radial clearance
C_P	Specific heat of lubricant
C_{ij}	Damping coefficients
$\overline{C}_{ij}$	Dimensionless damping coefficients , $C_{ij}c\omega / W$
e	Eccentricity
h	Oil film thickness
H	Dimensionless oil film thickness, h / c
I_0, I_1, I_2	Integrals given by equation (4)
K_{ij}	Stiffness coefficients
$\overline{K}_{ij}$	Dimensionless stiffness coefficients, $K_{ij}c / W$
L	Bearing length

M	Mass of journal
M_c	Critical mass of journal
$\overline{M}_c$	Dimensionless critical mass of journal, $M_c c\omega^2 / W$
p	Pressure
P	Dimensionless pressure, $p(c/R^2)/\mu\omega$
R	Journal radius
S	Sommerfeld number, $1/(2\pi\overline{W})$
T	Non-dimensional temperature
T_i	Non-dimensional inlet oil temperature
T_j	Non-dimensional journal temperature
u,v,w	Velocity components
U	Velocity of the journal
W	Bearing load
$\overline{W}$	Dimensionless bearing load, $W(c/R)^2/\mu\omega RL$
x,y,z	Circumferential, radial and axial coordinates respectively
$\overline{x},\overline{y},\overline{z}$	Dimensionless circumferential, radial and axial coordinates respectively, $x/R, y/h, z/L$
α	Viscosity-temperature coefficient
δ	Ellipticity ratio
ε	Eccentricity ratio, e/c
θ	Angular coordinate
θ_{e1}, θ_{e2}	Angular coordinates at the end of the bearing
θ_{s1}, θ_{s2}	Angular coordinates at the start of the bearing
μ	Lubricant Viscosity
ρ	Lubricant density
ν	Whirl frequency
γ	Whirl ratio
ϕ	Attitude angle
ω	Angular velocity of the journal

REFERENCES

[1] O. Pinkus, " Analysis of Elliptical Bearings, " *ASME Trans.*, Vol. 78, pp.965,1956

[2] A. Kumar, R.Sinhasan and D.v.Singh, "Performance Characteristics of Two-lobe Hydrodynamic Journal Bearings," J. Lubr. Technol., Vol. 102, pp.425, 1980.

[3] S. C. Soni, R. Sinhasan and D. V. Singh , " Performance Characteristics of Non-circular Bearings in Laminar and Turbulent Flow Regimes, " ASLE. Trans., Vol. 24 No 1, pp.29-41, 1981.

[4] H. Hashimoto, S. Wada and H. Tsunada , "Performance Characteristics of Elliptical Journal Bearings in Turbulent Flow Regime" Bulletin of JSME, Vol. 27 No 232 , pp.2265, 1984.

[5] S. C. Soni, R. Sinhasan and D. V. Singh, "Nonlinear analysis of Two-lobe Bearings in Turbulent Flow Regimes," Wear, Vol. 103, pp.11-27, 1985.

[6] N. Abdul-Wahed, D. Nicolas and M. T. Pascal, "Stability and Unbalance Response of Large Turbine Bearings" J. Lubr. Technol ASME. Trans, Vol. 104, pp.66-75, 1982.

[7] A. Singh and B. K. Gupta, "Stability Limits of Elliptical Journal Bearings Supporting Flexible Rotors" Wear, Vol. 77, pp. 159-170, 1982.

[8] H. N. Chandrawat and R. Sinhasan, "A study of a Two-lobe Journal Bearing Considering Elastohydrodynamic effects" Wear, Vol. 127, pp.161-177, 1988.

[9] D. V. Singh, R. Sinhasan and K. Prabhkaran Nair, "Elastohydrodynamic Effects in Elliptical Bearings" Tribology International, Vol. 22 No 1, pp.43, 1989.

[10] S. P. Tayal, R. Sinhasan and D. V. Singh, "Finite Element Analysis of Elliptical Bearing Lubricated by Non-Newtonian Fluid" Wear, Vol. 80, pp.71-81, 1982.

[11] D. Dowson and C. N. March, "The Thermo-hydrodynamic analysis of Journal Bearings" Pro. Inst. Mech. Eng London, Vol. 181 No 30, pp.117-126, 1966.

[12] A. Seireg and H. Ezzet, "Thermohydrodynamic Phenomena in Fluid Film Lubrication" J. Lubr. Technol ASME. Trans., pp.187-194, 1973.

[13] H. McCallion, F. Youcif and T. Lloyd, "The Analysis of Thermal Effects in a Full Journal Bearing" J. Lubr. Technol ASME. Trans., pp.578-587, 1976.

[14] T. Suganani and A. Z. Szeri, "A Thermohydrodynamic Analysis of Journal Bearings" J. Lubr. Technol ASME.Trans., Vol. 101, pp.21-27, 1979.

[15] R. Boncompain, M. Fillon and J. Frene, "Analysis of Thermal effects in hydrodynamic Bearings" J. Tribol ASME. Trans., Vol. 108, pp.219-224, 1986.

[16] S. Basri and D. T. Gethin, "A Comparative Study of the Thermal Behavior of Profile Bore Bearings" Tribology International, Vol. 23 No 4, pp.265-276, 1990.

[17] W. A. Crosby, "A Thermohydrodynamic Solution of the Two-lobe Bearing Considering Reverse Flow at the Leading and Trailing Edges, " Wear, Vol. 143 , pp.159-173, 1991 .

[18] A. Hussain, K. Mistry and S. Biswas, "Thermal Analysis of Non-circular Bearing" J. Tribol ASME.Trans., Vol. 118, pp. 246-254, 1996.

[19] B. Chetti, "A static and dynamic characteristics of three-lobe bearing including the thermal effects" Proceedings of the International Conference on Modeling and Simulation (MS'07), Algeria, pp.1-6, 2007.

[20] A. Chauhan, R. Sehgal and R. K. Sharma, "Investigation on the thermal effects in non-circular journal bearings" Tribology International Vol. 44, pp. 1765-1773, 2011.