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A Study on the Quality of Hexapod Machine Tool's Workspace

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Abstract**—**One of the main concerns about parallel mechanisms is the presence of singular points within their workspaces. In singular positions the mechanism gains or loses one or several degrees of freedom. It is impossible to control the mechanism in singular positions. Therefore, these positions have to be avoided. This is a vital need especially in computer controlled machine tools designed and manufactured on the basis of parallel mechanisms. This need has to be taken into consideration when selecting design parameters. A prerequisite to this is a thorough knowledge about the effect of design parameters and constraints on singularity. In this paper, quality condition index was introduced as a criterion for evaluating singularities of different configurations of a hexapod mechanism obtainable by different design parameters. It was illustrated that this method can effectively be employed to obtain the optimum configuration of hexapod mechanism with the aim of avoiding singularity within the workspace. This method was then employed to design the hexapod table of a CNC milling machine.

Keywords**—** Hexapod, Machine Tool, Singularity, Workspace.

I. INTRODUCTION

THE table or spindle of a hexapod machine tool (HMT) consists of a lower (fixed for table and moving for spindle) platform, and an upper (moving for table and fixed for spindle) platform, six actuators and twelve spherical and universal joints connecting the actuators to the upper and lower platforms. Such configuration provides the upper platform with six degrees of freedom in space. In this research this mechanism is utilized as the table of a CNC milling machines. One shortcoming to parallel mechanisms is the presence of singular points within their workspace. Singularity causes the mechanism to become unstable. In other words, in singular positions the mechanism gains or loses one or several degrees of freedom and thus is not controllable. From the qualitative aspect of the issue it can be stated that the farther the mechanism is from the singularity the more stable and thus more controllable it is.

Angeles J. [1] studied a 2-DOF robot consisting of a 5-bar mechanism with closed chain and obtained graphs of identical condition index number. In this way they considered the isotropy of the robot. They defined the isotropy of the robot as a concept which indicates the maximum stability of the robot. They suggest that isotropy stands against singularity. Danialy [2] worked on the isotropic design of two special kinds of parallel robots.

Depending upon the length of the pods, the upper platform assumes a specific position in space. The relationship between the length of the actuators and the position of the upper platform, which are obtained form the direct (forward) and inverse kinematics of the robot is extensively presented in the literature.

To solve the direct kinematics of the robot, Nanua P. and Waldron K.J. [3], Innocenti C. and Castelli P. [4] presented an analytical method, Patel A. J. and Ehman K. F. [5], Zhao Z. and Peng S. [6] employed a geometrical method, Shi X. and Fenton R.G. [7] utilized sensors.

Spong M. W. [8], and Asada H. [9] worked on Jacobian of hexapod mechanisms. They proposed a definition which is employed in this paper.

To the knowledge of authors the effects of various design parameters on the quality of HMT's workspace have not been sufficiently studied. Such an investigation can provide a practical means to optimal design of the HMT's mechanism. This was undertaken by the authors of this paper. For this purpose, a quantitative index was introduced as a quality condition index for evaluating singularities of different configurations of a hexapod mechanism obtainable by different design parameters. This method was then employed to design the hexapod table of FP4MB CNC milling machine.

II. WORKSPACE

The workspace of HMT is a region in space which the central point of HMT's end effecter (the central point of the upper platform) can reach and rotate around its coordinate axes. The workspace of a typical conventional milling machine with the same capability of machining is regarded as a reference for calculating the volume of HMT's workspace. The workspace of a conventional milling machine is a cubic

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region in space whose dimensions are determined by its X, Y, and Z-axis strokes. Considering such a cubic region as a reference, the definition of HMT's workspace is slightly modified, so that the portion of the reference workspace which the central point of the end effecter of HMT can reach and rotate symmetrically around its axes is regarded as the HMT's workspace. This region can be represented by a set of discrete points. These points are termed valid points. Valid points can be obtained by checking whether the mechanical limitations of HMT mechanism permit the end effecter to reach those points and rotate around its axes through a predefined limit angle.

The mechanical limits of a HMT mechanism are listed as follows: angular limit of spherical joints as specified by the manufacturer (the angle between the mating parts of the joints cannot exceed a certain value), motion limit to avoid collision between the pods and also between the hexapod structure and the structure of the machine tool (column, headstock, spindle, and ram), and the length of pod as design requirement.

The schema of a HMT table is illustrated in Fig. 1, where $\mathbf{X_i}$ is a vector representing the ith position assumed by the end effecter's centre point expressed in a global coordinate system with its origin attached to the centre point of the lower platform; $\mathbf{S_k}$, the position vector of the kth (k=1 to 6) spherical joint's centre point expressed in a local coordinate system with its origin attached to the centre point of the upper platform when the latter is located at its ith position; $\mathbf{L_{ijk}}$, the length vector of the kth pod acting as an actuator; and $\mathbf{U_k}$, the position vector of the kth universal joint in the global coordinate system. The subscript j represents the jth orientation of the end effecter denoted by $\boldsymbol{\theta_j} = [a_j\ b_j\ c_j]^T$ where a_j, b_j and c_j are the rotation angles of the end effecter around X, Y and Z axes, respectively.

Numerical method was employed to calculate the volume of the workspace. A computer program was developed to obtain the set of valid points. The algorithm developed for this program is based on the following analysis:

Considering an appropriate steps S, the workspace of HMT is discretized into a finite number of points. These points are represented by their position vectors, as follows:

$$\mathbf{X} = \{\mathbf{X_1},...,\mathbf{X_n}\} \quad (1)$$

The pod's length vector can be determined from inverse kinematics, as follows:

$$\mathbf{L}_{ijk} = \mathbf{X}_i + R_j\mathbf{S}_k - \mathbf{U}_k$$

or in its general form:

$$\mathbf{L} = \mathbf{X} + R\mathbf{S} - \mathbf{U} \quad (2)$$

where $\mathbf{R_j}$ is the rotation matrix which transforms the local coordinates to the global coordinates when the end effecter assumes its jth orientation, $\boldsymbol{\theta_j}$.

The length of pod, l_{ijk}, can be found as follows:

$$l_{ijk} = |\mathbf{L}_{ijk}| \quad (3)$$

Those $\mathbf{X_i}$s at which either $\boldsymbol{\theta_j}$ or $\mathbf{L_{ijk}}$ violates any of the mechanical limits are excluded from **X** yielding valid position vector field denoted by Q. The mechanical limits of the mechanism are obtained by taking the geometrical relations of the HMT's structure into consideration. This is discussed in the following section

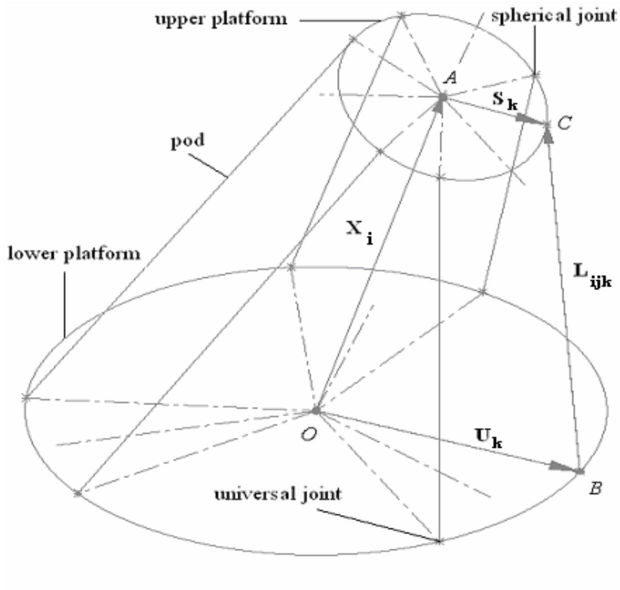

Fig. 1. Schematic illustration of HMT's mechanism

III. MECHANICAL LIMITATIONS OF HMT'S WORKSPACE

As mentioned earlier, there are four mechanical limits to the HMT's mechanism, as follows:

A. Angular Limit by spherical joints

The limit angle of the spherical joint is assumed to be denoted by φ_{all}. Then the angle between a pod and the upper platform, φ, should satisfy the following relation:

$$\varphi \le \varphi_{all} + \pi/2 \quad (4)$$

where φ can be obtained by the dot product of the vectors of respective pod and spherical joint. It is obvious that the above constraint should be checked for each pod.

B. Collision between Pods

Second mechanical limitation is that actuators (pods) should not collide with each other. This limitation is mathematically expressed as follows.

$$D < d \quad (5)$$

where D is the diameter of actuator and d is the perpendicular distance between two pods.

C. Collision between the Structure of Hexapod and the Structure of Machine Tool

Violation of this limitation is precluded by the design procedure. The boundary of the workspace is already determined as was mentioned earlier. The design of the hexapod table should of course be adapted to the structural design of the machine tool in order to avoid any possible collision.

D. Length of Each Pod

Due to the design consideration of the actuators, each pod can become neither larger nor smaller than a certain value. This limitation is described mathematical, as follows:

$$l_{min} \le l_{ijk} \le l_{max} \quad (6)$$

IV. STABILITY ANALYSIS

A. Stability Analysis

Stability analysis of a parallel mechanism is one of the most essential concerns of their design. Stability analysis of HMT's mechanism is conducted by considering its singularity. In singular positions the mechanism is not stable or in other words it is not controllable. In these positions, when pods are actuated, the upper platform dos not move (losing degrees of freedom) or cannot hold a unique position (gaining degrees of freedom). This implies that for a set of pods' velocity vectors there would be no movement in the upper platform or the upper platform would have nonzero velocity vectors with no movement in pods. The mathematical model of this fact is developed as follows.

The velocity vectors of the end effecter and pods have been worked out in several papers, for example [10]. In inverse kinematics, the velocity vector of the end effecter, **V**, is transformed into the velocity vector of pods, **q**, by the inverse Jacobian matrix, as follows:

$$\mathbf{q} = \mathbf{J}^{-1}\mathbf{V} \quad (7)$$

where $\mathbf{q} = \left[\dot{\mathbf{L}}_1\ \dot{\mathbf{L}}_2\ \dot{\mathbf{L}}_3\ \dot{\mathbf{L}}_4\ \dot{\mathbf{L}}_5\ \dot{\mathbf{L}}_6\right]^T$ and $\mathbf{V} = \left[\dot{\mathbf{X}}\ \dot{\boldsymbol{\theta}}\right]^T$.

If the position of a spherical joint being determined by its centre point is denoted by **C**, it can be concluded from Fig. 1 that $\mathbf{C}_{ijk}=\mathbf{X}_i+R_jS_k$ and thus the velocity of spherical joint can be expressed in its general form as follows:

$$\dot{\mathbf{C}} = \dot{\mathbf{X}} + \boldsymbol{\psi} \times (\mathbf{RS}) \quad (8)$$

where $\dot{\mathbf{X}}$ and ψ are the linear and angular velocities of the upper platform.

The scalar velocity component of pod along its length, $\dot{l}$, can be found as follows:

$$\dot{l} = \dot{\mathbf{C}} \cdot \mathbf{n} \quad (9)$$

where **n** is the unit vector of pod. From (8-9):

$$\dot{l} = \left[\dot{\mathbf{X}} + \boldsymbol{\psi} \times (\mathbf{RS})\right] \cdot \mathbf{n} = \dot{\mathbf{X}} \cdot \mathbf{n} + \boldsymbol{\psi} \cdot \left[(\mathbf{RS}) \times \mathbf{n}\right] \quad (10)$$

or

$$\dot{\mathbf{l}} = \mathbf{n}^T\dot{\mathbf{X}} + \left[(\mathbf{RS}) \times \mathbf{n}\right]^T \boldsymbol{\psi} = \left[\mathbf{n}^T\ \left[(\mathbf{RS}) \times \mathbf{n}\right]^T\right]\left[\dot{\mathbf{X}}\ \ \boldsymbol{\psi}\right]^T = \mathbf{J}^{-1}\mathbf{V} \quad (11)$$

where

$$\mathbf{J}^{-1} = \left[\mathbf{n}^T\ \left[(\mathbf{RS}) \times \mathbf{n}\right]^T\right] \text{ and } \mathbf{V} = \left[\dot{\mathbf{X}}\ \ \boldsymbol{\psi}\right]^T.$$

In direct kinematics, (11) changes to

$$\mathbf{V} = \mathbf{J}\,\dot{\mathbf{l}} \quad (12)$$

For the existence of unique solution det**J** should be nontrivial or singularity would develop and the system gets unstable. In order to find singular points within the workspace, the system of equations det **J** = 0 should be solved. This is too complicated.

B. Singular Value Decomposition

A simpler solution can be obtained by singular value decomposition (SVD) of the Jacobian matrix. The SVD of **J** is written as follows:

$$\mathbf{J} = \boldsymbol{\Gamma} \sum \boldsymbol{\Xi}^T \quad (13)$$

where **Γ** and **Ξ** are two orthogonal matrices and **Σ** is a diagonal matrix with the same dimension as **J**. The diagonal entries of **Σ** are in fact the singular values of **J** and are obtained by

$$\sigma_i = \sqrt{\lambda_i} \quad (14)$$

where λ_i is the ith eigenvalue of $\mathbf{J}^T\mathbf{J}$ [11].

C. Evaluation of Workspace

For trivial **J**, σ_is approach zero and an ill condition would prevail. This is more obvious when the decomposed inverse Jacobian matrix is considered:

$$\mathbf{J}^{-1} = \boldsymbol{\Xi}\left[\operatorname{diag}\left(\frac{1}{\sigma_i}\right)\right]\boldsymbol{\Gamma}^T \quad (15)$$

For existence of $\mathbf{J}^{-1}$, none of σ_is should be zero. In other words large $\frac{1}{\sigma_i}$s lead to ill conditioned inverse Jacobian matrix and thus small σ_is result in ill conditioned Jacobian matrix. This lays the foundation for introducing the condition number, *K,* as an index of the quality of workspace, as follows:

$$K = \frac{\sigma_{max}}{\sigma_{min}} \tag{16}$$

where σ_{max} and σ_{min} are maximum and the minimum singular values. A matrix is singular if its condition number is infinite, and it is ill-conditioned if its condition number is too large. In other words, a large value for the condition number of a matrix implies that the matrix is near its singularity. An ill-conditioned matrix is sensitive to the changes made to its entries and can become singular by small changes of its entries. In case of HMT's mechanism it can be said that when Jacobian matrix of the mechanism is ill-conditioned the mechanism is near its singular position since small changes made to J's entries, that is small changes of the mechanism's position, will make it become singular and thus will make the mechanism become unstable.

The smaller the *K*, the farther the mechanism is to the instability. It is evident that K ranges from unity to infinity. For the sake of convenience, it is decided to work with the reciprocal of *K*, as follows:

$$K' = \frac{1}{K} = \frac{\sigma_{min}}{\sigma_{max}} \tag{17}$$

where K' is a new index named here as condition index. The nearer the K' to unity, the more stable would be the mechanism and the more desirable would be the position.

D. Average Condition Index

The concept of the condition index is to be extended to the whole workspace of HMT. Within the HMT's workspace some points are nearer to instability and some are nearer to stability. An index called average condition index ACI is developed to evaluate the whole workspace of HMT in terms of stability. ACI is defined as follows.

$$AC = \frac{1}{n}\sum_{i=1}^{n} K'_i \tag{18}$$

It can be stated that the nearer the ACI is to unity, the more stable and thus desirable is the workspace. Such procedure provides a means to evaluate (in terms of stability) the workspace of HMT, which is affected by design parameters.

V. EVALUATION OF DIFFERENT HMT LAYOUTS

The design parameters of a hexapod are shown in Fig 2. These parameters include Ru, radius of the upper platform; R_l, radius of the lower platform; H, vertical distance between the upper and the lower platforms in the initial position, Initial position is referred to as a position on the Z axis at half way the vertical stroke; α, angular distance between the spherical joints; β, angular distance between the universal joints; γ, conic angle of the mechanism in the initial position; λ, apex angle of each adjoining pair of pods in the initial position.

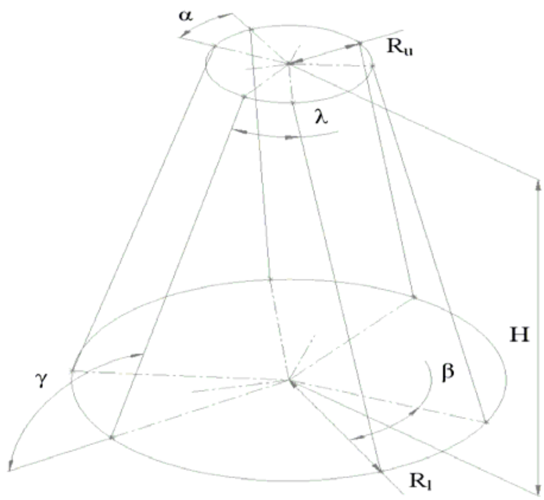

Fig. 2. Typical design parameters of a hexapod

Prior to the comprehensive and detailed mechanical design of the HMT's structure, layout design is to be conducted. The most suitable layout is that delivering the maximum workspace. Among various design parameters, there exist several independent parameters that can be varied to achieve the optimum layout. These parameters are Ru, α and β which are termed principal design parameters. The problem of layout design thus reduces to finding the principal design parameters for maximum ACI of HMT's workspace. The effect of these design parameters on ACI of HMT's workspace is discussed in the following sections.

Due to the complexity of the kinematics of HMT's mechanism, the effects of design parameters on ACI are considered experimentally. A full factorial design of experiment is conducted with each design parameters at 10 levels. For each experiment, the ACI is calculated from (18).

Fig. 3 illustrates the effect of Ru on ACI. It can be concluded from this figure that Ru is reciprocally related to ACI; that is by increasing Ru, ACI decreases. Since R_l is fixed, the increase of Ru can be readily realized from Fig 2 to result in the increase of γ. Therefore, the conic angle of the mechanism, γ, is directly related to the stability of the mechanism. By increasing the value of γ, ACI increases and the mechanism becomes more stable.

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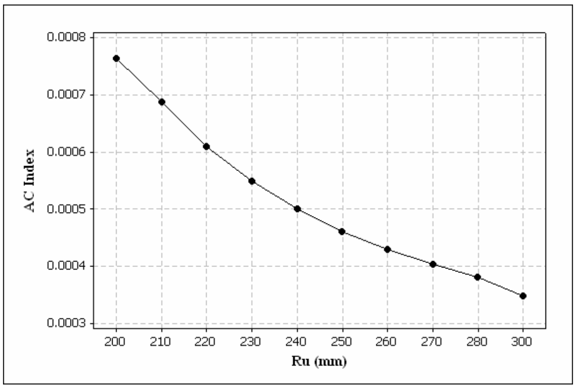

Fig. 3. The effect of Ru on ACI

Fig. 4 shows the effect α on ACI. It is observed from this figure that ACI follows falling and rising trends as α increases. ACI is minimum at α=55 degrees and is maximum at the smallest value of α.

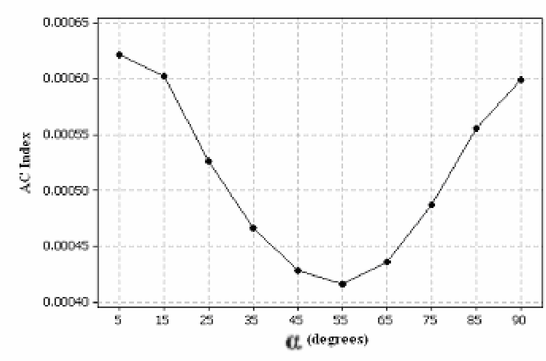

Fig. 4. The effect of α on ACI

The effect of β on ACI is depicted in Fig 5. This figure shows that β like α follows falling and rising trends. ACI is minimum around β=55 degrees, and is maximum at the smallest value of β.

The coupled or interactive effect of α and β on ACI is illustrated in Fig. 6. It can be seen from Fig. 6 that for small values of α, ACI undergoes a rising trend as β increases. Therefore, it can be said that β has a direct effect on ACI for small values of α. For large values of α, however, ACI decreases as β increases; that is β has an inverse effect on AC index for large values of α. The same phenomenon is visualized for α from Fig 6. For small values of β, α has a direct effect on ACI while for large values of β, α has an inverse effect on ACI. The most important observation made form Fig 6 is that ACI is minimum when α and β are equal, and maximum when α is small and β is large. It has also another maximum value at a region in which α is large and β is small.

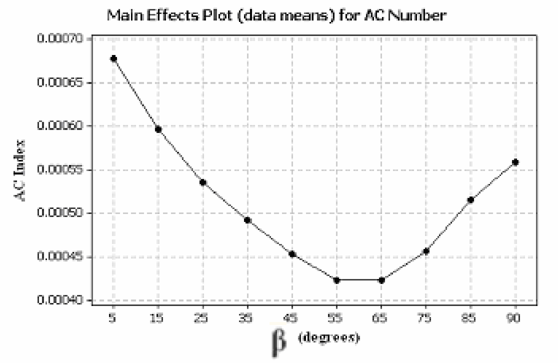

Fig. 5. The effect of β on ACI

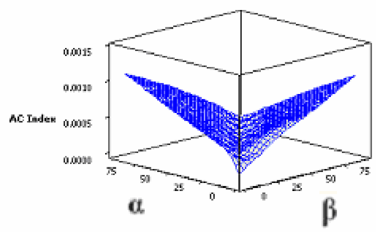

Fig. 6. The coupled effect of α and β on ACI

Fig. 7 shows the coupled effect of α and β on λ. It can be seen from Fig 7 that λ becomes maximum for small values of α and large values of β. Small values of β and large values of α also yield maximum λ . This fact in conjunction with what was concluded above can be used to conclude that λ has a direct effect on ACI. In other words ACI increases and thus the workspace becomes more stable as λ increases. This implies that the so called "saw tooth" configuration results in the most stable workspace.

VI. CONCLUSION

New concepts consisting of condition index and average condition index were proposed in this paper for optimum design of hexapod table of machine tools. It was illustrated that the average condition index can be used to evaluate the quality of the workspace. Layout and design parameters were explained and the effects of design parameters on the quality of HMT's workspace were investigated. The following conclusions can be drawn from this study:

The radius of the upper platform has an inverse impact on the volume of the workspace.

The angular distance between spherical joints (α) and universal joints (β) have a coupled effect on the quality of HMT's workspace.

The most stable HMT is obtained at large values of β and small values of α, or vice versa.

The coupled effects of α and β on the apex angle of adjoining pods (λ) were also studied and it was shown that for large values of β and small values of α, or vice versa, λ becomes maximum.

The latter two conclusions can be summed up to state that the so called "saw tooth" configuration provides HMT with the most stable layout or in other words a layout of the highest quality.

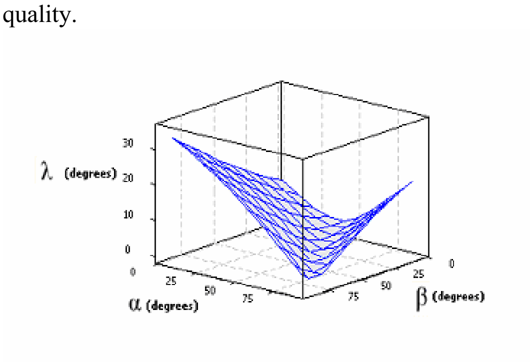

Fig. 7. The coupled effect of α and β on λ

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