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A Study on Metal Hexagonal Honeycomb Crushing Under Quasi-Static Loading

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***Abstract*—In the study of honeycomb crushing under quasi-static loading, two parameters are important, the mean crushing stress and the wavelength of the folding mode. The previous theoretical models did not consider the true cylindrical curvature effects and the flow stress in the folding mode of honeycomb material. The present paper introduces a modification on Wierzbicki's model based on considering two above mentioned parameters in estimating the mean crushing stress and the wavelength through implementation of the energy method. Comparison of the results obtained by the new model and Wierzbicki's model with existing experimental data shows better prediction by the model presented in this paper.

***Keywords*—**Crush strength, Flow stress, Honeycomb, Quasi-static load.

I. Introduction

HONEYCOMB cellular structures, due to their light weight and high energy-absorbing capability, have been used extensively as energy absorbers or cushions to resist external loads. Previous studies on the crushing behavior of honeycomb structures included the early work reported by McFarland, who developed a semi-empirical model ,in which the failure modes was local buckling, to predict the crushing stress of hexagonal cell structures subjected to axial loading [1].

A subsequent paper by Wierzbicki gave an important analysis for the out-of-plane crushing resistance of hexagonal-cell structures, and presented results in a form convenient for design purposes. Comparisons of results from the analysis were made with experimental results. The analysis did not consider the effects of curvature and flow stress and material assumed to be rigid-perfect plastic [2].It has been discussed in [3] that considering the flow stress in the analysis of thin walled structures, leads to a better description of the material behavior. Subsequently mechanical properties of honeycomb structures in the lateral directions were investigated both analytically and experimentally by Gibson *et al.* [4], and Gibson and Ashby [5]. Various experimental and numerical studies on quasi-static and dynamic crush behaviors of honeycombs under out-of-plane compressive, multiaxial or combined loads have been reported [6-11]. For example Wu and Jiang [6] focused on the investigation of the crushing phenomena of honeycomb structures under both quasi-static and dynamic loading conditions considering the effects of cell dimension, material strength and number of cells under loading. In [7] it is shown that the theoretical values in [6] must be corrected and their right values are computed.

In this article, through implementation of energy method based on Wierzbicki's model and considering curvature effects and flow stress, crushing strength and wavelength have been determined and evaluated by experimental results of [6].

II. Folding Element Definition

Because of the regular and symmetrical structure of the hexagonal honeycomb, it can be assembled from one folding element consisting of two angle elements joined together forming an angle of 120° (Fig. 1). These two angle elements are bonded by means of an adhesive whose strength is smaller than that of the material itself, so during the crushing process, a part of the bond adjacent to the vertical edge is broken and the two angle elements are partially torn off.

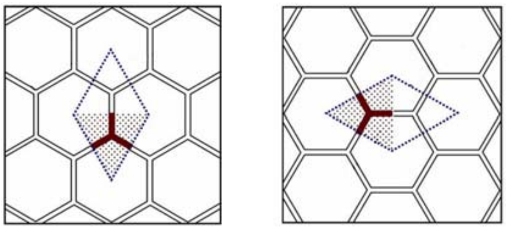

Fig. 1 Folding element of hexagonal honeycomb

The global collapse mode of an angle element is shown in Fig. 2 that consists of (a) four plane trapezoidal elements moving as rigid bodies, (b) two sections of cylindrical surfaces that have an inextensional mode and only absorb the energy that is required for forming the two plastic hinges above and below them, (c) two sections of conical surfaces bonded by two propagating straight hinge lines, and (d) a section of a toroidal shell which undergoes an extension.

The folding mode for the angle element shown in Fig. 2 is a one degree of freedom system whose geometry can be described either by the crushing distance δ, or the angle of rotation of the common walls in a folding element α, or the horizontal displacement of point D, defined as s (see Fig. 3).

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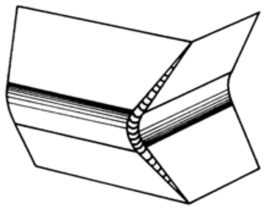


Fig. 2 The folding mode of an angle element [2]

III. Wierzbicki's Model

For hexagonal honeycomb analysis under out of plane quasi static force, Wierzbicki modeled the folding element shown in Fig. 2 by the element presented in Fig. 3. In fact, he neglected the cylindrical sections' curvatures. In this paper these curvatures will be explained.

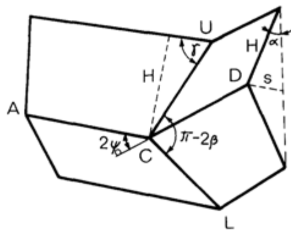


Fig. 3 Global geometry of the folding mode [2]

Three dissipated energy terms are considered in [2] consisting of (a) dissipated energy in the toroidal surface, E_1, (b) dissipated energy in the plastic hinges of the cylindrical surfaces, E_2 and (c) dissipated energy in the plastic hinges of the conical sections, E_3.

Energy dissipation in the toroidal shell, E_1, is determined according to (1) in which the first term in the integrant denotes the bending energy and the second term denotes the extension energy.

$$\dot{E}_1 = \int_S \left(M_\varphi \dot{K}_\phi + N_\varphi \dot{\lambda}_\phi \right) ds \tag{1}$$

where M_ϕ is the circumferential bending moment of buckling of thin plate; $\dot{K}_\phi$ is the rate of curvature of the toroidal element; N_ϕ is the circumferential membrane force during the buckling of the plate; $\dot{\lambda}_\phi$ is the rate of extension of the toroidal element; and S denotes the surface of the toroidal section. The symbol $\dot{(\ )}$ demonstrates the derivative with respect to time.

It is established while the plate thickness is very smaller than the toroidal shell curvature radius, we can substitute $M_\phi = 0$ and $N_\phi = N_0$ in (1) where N_0 denotes the fully plastic membrane force and can be written as $4M_0/h$ where h is the plate thickness and M_0 denotes the fully plastic bending moment and can be calculated as

$$M_0 = \frac{1}{4}\sigma_0 h^2 \tag{2}$$

where σ_0 denotes the flow stress. Finally (1) leads to

$$E_1 = 33.6 M_0 \frac{Hb}{h} \tag{3}$$

where b is the smaller radius of the toroidal shell and H is the half-wavelength of the folding mode.

Energy dissipation due to formation of horizontal hinge lines in the cylindrical sections of a folding element is given as below

$$E_2 = 6M_0 \pi D \tag{4}$$

where D is the width of the cell wall.

Energy dissipation due to buckling of the inclined hinge lines is given as

$$E_3 = \int_L M_0 \dot{\theta}\, dl \tag{5}$$

where L is the total length of the inclined hinge lines and $\dot{\theta}$ is the rate of rotation at the plastic hinge lines. Each of these lines consists of two segments: L_1 of a joint line between a conical surface and the trapezoidal element and an arc of a circle L_2 which is the joint line between cylindrical and toroidal surfaces and can be written as

$$L = L_1 + L_2, \tag{6}$$

finally E_3 is

$$E_3 = 19.12 M_0 \frac{H^2}{b} \tag{7}$$

The mean crushing force is defined from the requirement that the total internal dissipated energy is equal to the external work of external force P_m. Wierzbicki assumed that the crushing distance is equal to the wavelength of the folding mode $2H$, so

$$P_m.2H = E_1 + E_2 + E_3 \tag{8}$$

Substituting (3), (4), and (7) in (8) leads to

$$\frac{P_m}{M_0} = 16.8\frac{b}{h} + 3\pi\frac{D}{H} + 9.56\frac{H}{b} \tag{9}$$

The unknown parameters in (9) for calculation of P_m are H and b which denote the half-wavelength and the small radius of the toroidal shell respectively. The least possible value of the crushing force that collapse is occurred can be obtained by

$$\frac{\partial P_m}{\partial H} = 0, \frac{\partial P_m}{\partial b} = 0 \tag{10}$$

So H, b and P_m are

$$H = 0.821\sqrt[3]{hD^2} \tag{11}$$

$$b = 0.683\sqrt[3]{h^2 D} \tag{12}$$

$$P_m = 8.61\sigma_0 h^{5/3} D^{1/3} \tag{13}$$

IV. The New Model

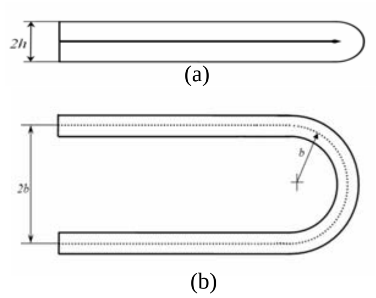
In this paper, the Wierzbicki's model is improved by considering the complete effects of the cylindrical sections' curvature and flow stress. Sec. A explains the curvature effects and Sec. B improves the model by flow stress correction.

A. Curvature Effects

As pointed in the section III, Wierzbicki modeled the folding element in Fig. 2 by the element of Fig. 3 and he did- not consider the cylindrical curvature effects completely. In this section, we take all the curvature effects into account. For this purpose, first the external work will be corrected and subsequently the internal dissipated energy terms will be modified.

It is assumed in (8) that the two trapezoidal surfaces meet after complete collapse as shown in Fig. 4(a), but in practice, a distance of $2b$ between the two trapezoidal surfaces is kept as shown in fig. 4(b). In fact, the actual crushing distance is $2(H-b)$. So (8) changes into

$$P_m.(2H-2b)=E_1+E_2+E_3 \tag{14}$$



(b)

Fig. 4 Schematic of crushing of a cell wall, (a) in the Wierzbicki's model, (b) in the new model

now we consider the effects of curvature on the internal dissipated energy terms.

In (1), ds denotes the toroidal section element and can be written as

$$ds = rd\phi\, bd\theta \tag{15}$$

where r is the larger radius of toroidal shell, b is the smaller radius of toroidal shell, $d\phi$ is the circumferential coordinate element and $d\theta$ is the meridional coordinate element as shown in Fig. 5.

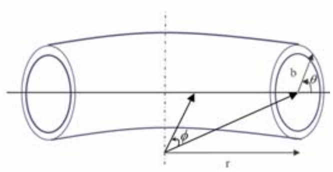


Fig. 5 Toroidal coordinates

Model presented in [2] has considered the curvature effects in derivation of dissipated energy in toroidal surface, E_1.

The total length of horizontal hinge lines is equal to the width of the basic panel element, C, that is approximately equal to the length of ACD in Fig. 3. In fact, because these hinges are horizontal, whether considering the curvature or not does not have meaningful effect on E_2.

L in (6) is the inclined hinge lines length that has been correctly written as L_1+L_2, so E_3 is consequently true.

As explained above, the total internal dissipated energy in (8) is correct.

Substituting (3), (4) and (7) into (14) and implementing (10) leads to

$$-\frac{1}{(H-b)^2}\left[16.8\frac{b}{h}H+3\pi D \quad 9.56\frac{H^2}{b}\right] + \frac{1}{(H-b)}\left[16.8\frac{b}{h}+19.12\frac{H}{b}\right] \quad 0 \tag{16}$$

$$-\frac{1}{(H-b)^2}\left[16.8\frac{b}{h}H+3\pi D \quad 9.56\frac{H^2}{b}\right] + \frac{1}{(H-b)}\left[16.8\frac{H}{h}-9.56\frac{H^2}{b^2}\right]=0 \tag{17}$$

Mixing (16) with (17) leads to

$$H^2-\left[2b+1.756\frac{b^2}{h}\right]H-1.756\frac{b^3}{h} \quad 0 \tag{18}$$

Therefore

$$H=b+0.878\frac{b^2}{h} \quad \left[\left(b+ \quad 0.878\frac{b^2}{h}\right)^2 \quad 1.756\frac{b^3}{h}\right]^{1/2} \tag{19}$$

Substituting (19) into (16) or (17), gives a single equation in terms of b which is obtained using an appropriate numerical approach. After b is calculated, H is found by substituting b into (19) and finally P_m is obtained from (14).

B. Flow Stress

The stress-strain diagram for many metals consists of two regions, an elastic region and a plastic region. In collapse analysis, it is assumed that the material is rigid-perfect plastic. In fact, stress-strain diagram is approximated with a horizontal line, which meets the stress axis at σ_0 as shown in Fig. 6.

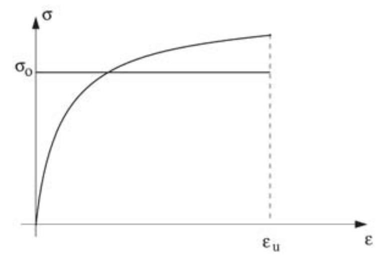


Fig. 6 Energy equivalent flow stress

In [2], σ_0 is approximated with the yield stress σ_y, but [3] shows σ_0 is the flow stress and can be calculated as

$$\sigma_0=\sqrt{\frac{\sigma_y\,\sigma_u}{1+n}} \tag{20}$$

where σ_y and σ_u denote the yield strength and the ultimate strength of the material respectively and n is the exponent of power law. So M_0 is

$$M_0=\frac{1}{4}\sigma_0 h^2=\frac{1}{4}\sqrt{\frac{\sigma_y\,\sigma_u}{1+n}}\,h^2 \tag{21}$$

V. Results and Discussion

The two important parameters which should be calculated in the study of metal honeycomb crushing under quasi-static load are the mean crushing stress σ_m and the half-wavelength of the folding mode H. The Wierzbicki's model estimates H by (11), but the new model suggests that H can be calculated by (19).

σ_m is defined as the ratio of the crushing force P_m to the contributing area over which the force P_m is acting. P_m is obtained by (13) for Wierzbicki's model and by (14) for the new model. The contributing area of the basic element shown in Fig. 1 is

$$A = \frac{\sqrt{3}}{4} S^2 \quad (22)$$

where S -that is the cell size- can be defined as the distance of two parallel walls in a cell and is equal to

$$S = \sqrt{3}\, D \quad (23)$$

Therefore, for Wierzbicki's model

$$\sigma_m = \frac{P_m}{A} = 16.56\,\sigma_y \left(\frac{h}{S}\right)^{5/3} \quad (24)$$

and for the new model

$$\sigma_m = \frac{P_m}{A} = \frac{\sqrt{\frac{\sigma_y\,\sigma_u}{1+n}}\,h^2}{\sqrt{3}\left(H-b\right)S^2}\left[16.8\frac{b}{h}H \quad \sqrt{3}\,\pi S + 9.56\frac{H^2}{b}\right] \quad (25)$$

where H and b can be calculated from (19) and either (16) or (17).

E. Wu and W. Sh. Jiang in [6] have reported experimental results for a few types of the honeycomb cellular structures under quasi-static and impact loading in the axial direction. Table I shows the specimens' properties. The wall thickness is 0.0254mm for all six types of specimens. For 5052-H38 and 5056-H38 aluminum, the tensile yield strengths are 255 and 345 MPa and the ultimate tensile strengths are 290 and 415 MPa respectively. n is approximately 0.057 for the first 4 specimens and 0.078 for the last 2 specimens. It is pointed out in [7] that the Wierzbicki's values in [6] are incorrect and must be corrected, so in this paper the Wierzbicki's values that have been reported in [7] are used.

TABLE I
HONEYCOMB SPECIMEN'S SPECIFICATIONS

Type No.	Cell size S (mm)	Alloy	Density (kg/m3)	Specimen height (mm)
1	4.763	5052	49.656	35.7
2	4.763	5052	49.656	12.7
3	3.175	5052	72.081	25.4
4	3.175	5052	72.081	14.3
5	3.175	5056	72.081	20.7
6	3.175	5056	72.081	10.0

Table II compares the mean crushing stress and the half-wavelength of folding mode obtained from experiment, Wierzbicki's model and the new model. Unfortunately, the half-wavelength value isn't calculated for the type number 2 in [6], so H is compared for 5 specimens.

TABLE II
SPECIMENS' CRUSH STRENGTH PER AREAL DENSITY ($\sigma_m/\rho\left(10\text{^}3\,Nm/kg\right)$) AND HALF-WAVELENGTH OF FOLDING MODE (H$\left(mm\right)$)

Type No.	1	2	3	4	5	6
σ_m/ρ (Experimental)	22.07	22.37	31.2	34.72	37.64	39.04
σ_m/ρ (Wierzbicki)	13.80	13.80	18.69	18.69	25.29	25.29
σ_m/ρ (new model)	16.86	16.86	23.51	23.51	32.44	32.44
H (Experimental) (mm)	0.916	—	0.665	0.669	0.696	0.701
H (Wierzbicki) (mm)	0.475	—	0.363	0.363	0.363	0.363
H (New model) (mm)	0.542	—	0.480	0.480	0.480	0.480

Fig. 7 shows the half-wavelength values H, obtained from experiment, Wierzbicki's model and the new model. It is clear that the estimation of H by the new model is more accurate than that of Wierzbicki's model. Using the least squares error method shows that the Wierzbicki's model and the new model errors are 47.13 and 32.26 respectively.

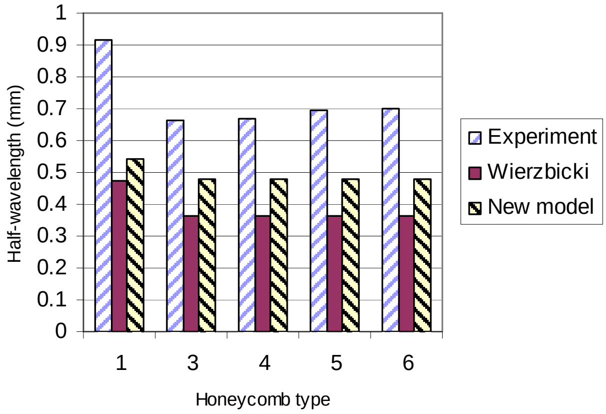


Fig. 7 Comparison of the measured and predicted values of half-wavelength of folding mode

The value of σ_0 has no effect on H, so the improvement of estimations of H is only a result of considering the curvature effects.

Fig. 8 shows the mean crushing stress obtained from experiment and theoretical models. Using the least squares error method shows that the new model has an error equal to 23.42, while the Wierzbicki's model error is 38.56. it can be shown that 79.59 percent of this error reduction is the result of considering curvature effects, while the rest of it owes to flow stress modification.

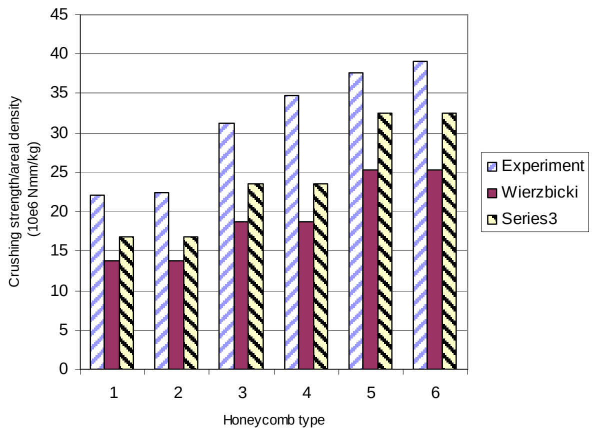


Fig. 8 Comparison of the crush strength per areal density between the experimental data and the theoretical predictions

VI. Conclusion

In this work, an analytical study on crushing behavior of metal hexagonal honeycombs under out of plane quasi-static loading has been presented. The previous theoretical models have been modified by considering curvature effects and flow stress. Comparison of the obtained results with the experimental values presented in literature states that the proposed model has decreased the mean crushing stress calculation error from 38.5% down to 23.5% and the wavelength calculation error from 47% down to 32%.

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