A Study of the Effectiveness of the Routing Decision Support Algorithm

Wayne Goodridge, Alexander Nikov, and Ashok Sahai

Abstract—Multi criteria decision making (MCDM) methods like analytic hierarchy process, ELECTRE and multi-attribute utility theory are critically studied. They have irregularities in terms of the reliability of ranking of the best alternatives. The Routing Decision Support (RDS) algorithm is trying to improve some of their deficiencies. This paper gives a mathematical verification that the RDS algorithm conforms to the test criteria for an effective MCDM method when a linear preference function is considered.

Keywords—Decision support systems, linear preference function, multi-criteria decision-making algorithm, analytic hierarchy process.

I. INTRODUCTION

MULTI-CRITERIA decision-making (MCDM) methods evolved as a response to the observed inability of people to effectively analyze multiple streams of dissimilar information. They can help to improve quality of decisions by making decision-making process more explicit, rational, and efficient [1]. As a consequence, MCDM methods are used widely in engineering [1], [3], manufacturing [4] and business [5]. However, most of these techniques have irregularities in terms of the reliability of ranking of the best alternatives. Because many of the criteria are subjective in nature and have uncertainty associated with them, it is difficult to determine in a multiple criteria problem how the alternatives should be ranked. Hence different MCDM techniques used for a multi-criteria problem could produce different ranked alternatives.

The Analytic Hierarchy Process (AHP) [7] technique is widely used. However, there are a number of arbitrary aspects of the AHP procedure. For example, the scale that is used is ad-hoc. Why the 1-9 range? In addition, the ranking scale can be manipulated to get a preferred outcome. Furthermore, there is the hidden assumption that if i is weakly preferred to j and j is weakly preferred to k, then a consistent decision maker must have i absolutely preferred to k. However, the use of a word such as weakly does not necessary make this transitive

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relationship hold. Consequently, this can lead to a decision maker forgetting prior assessments as he goes, which in turn can result in inconsistencies in the decision process. Another problem with the AHP approach is that when asked to rank a list of things according to some criterion, such as preference value, risk or cost, numbers are assigned to the relative objects position in the list. The main problem with this approach is that the measurement scale chosen is ordinal at best. A ranking of 10 does not mean that the preference for an item is twice that of an item rated 5. The AHP approach provides insight into the tradeoffs embedded in a decision making process. However, it is clear that preferences are extracted by many pair-wise comparisons. This can make large decision problems very difficult to analyze. More importantly, it complicates group decision making since new pair-wise comparisons must be done for each decision maker.

Apart from the widely used AHP technique there is another family of MCDM models that uses "outranking relations" to rank a set of alternatives - the ELECTRE method and its derivatives. Most of the ELECTRE methods were shown to have a rank reversal problem [1]. This occurs when a non-optimal alternative is replaced by a worse alternative and the decision method changes the ranking of the alternative that was the best before the change was made. This means that most of the ELECTRE methods cannot be trusted.

Value functions [12] are considered to be part of another family of MCDM models. Multi-Attribute Utility Theory (MAUT) [13] uses value functions to map changes of values of performance of the alternatives in terms of a given criterion into a dimensionless value. However, MAUT is very difficult to use because it utilizes non-intuitive concepts of subjective probabilities which are difficult to measure. For example, what is the probability that Australia will beat England in Ashes 2009? This probability cannot be obtained by dividing the number of favorable outcomes by the number of possible outcomes. Rather, assigning probability of say, 0.7 to this event by a cricketing expert is more feasible. Such an approach to probability seems to be a mere opinion.

The Routing Decision Support (RDS) algorithm [6] is a MCDM technique that was originally used to find the best paths to route packets in a computer network based on a set of network constraints and user goals. The algorithm was designed primarily to find a Pareto optimal path [6] from a set of paths in a computer network in such a way that the quality of service (QoS) goals of the user and network can be satisfied. The basic rational behind the RDS was to design a

fast simple decision process for QoS criteria such as bandwidth, cost, delay and jitter. The input required for the RDS to make a "good" decision was significantly less than other decision techniques like the analytic hierarchy process [7] and the ELECTRE methods [8]-[11]. In addition, the RDS was designed with measurement concepts in mind and with the goal of improving on the mathematical deficiencies of other techniques.

This paper is organized as follows. The next section describes the RDS algorithm and gives examples of how it works. The third section discusses the three test criteria that have been used in [1] to test the performance of MCDM models and describes mathematical proofs that the RDS method meets the three test criteria. Some concluding comments are presented in the last section.

II. THE RDS ALGORITHM

In this section the RDS algorithm will be explained. Because the RDS has its origins in a networking environment, an example will be given of how the RDS can be used to assist IP traffic engineers with finding suitable routes for a given set of user requirements. To make the RDS algorithm applicable for business decision-making, a key modification to the algorithm will be presented. Finally, another example will be given to illustrate how the RDS can be used with a regular type of decision problem.

A. Introduction to the RDS Stage

The RDS algorithm uses the structure of the decision matrix (cf. Fig. 1).

$$\begin{array}{cc} \begin{bmatrix} C_1 & C_2 & .... & C_m \end{bmatrix} & \text{Criteria} \\ \text{Alternatives} \begin{bmatrix} w_1 & w_2 & .... & w_m \end{bmatrix} & \text{Weights} \end{array}$$

$$\begin{array}{c} A_1 \\ A_2 \\ \\ A_n \end{array} \begin{Vmatrix} a_{11} & a_{12} & .... & a_{1m} \\ a_{21} & a_{22} & .... & a_{2m} \\ - & - & - & - \\ a_{n1} & a_{n2} & ... & a_{nm} \end{Vmatrix}$$

Fig. 1 Structure of decision matrix

The algorithm uses a scale s to convert values of a_{ij} for each criterion $j \in [1,m]$ into a dimensionless value in the domain $[-1,1]$. The domain of $[-1,1]$ rather than $[0,1]$ as in the case of value functions is the first difference that the RDS method presents. The rationale for the $[-1,1]$ domain will be explained later. The scale s can be any monotone continuous increasing or decreasing function whose domain exists in $[-1,1]$. In addition, s behaves differently for benefit and cost criteria [14], [15] (A benefit criterion means that the higher an alternative scores in terms of it, the better the alternative is. The opposite is considered true for the cost criteria). This is because for a given benefit criterion the highest value of criterion amongst the alternatives is assigned to a_j and the lowest value is assigned to b_j. For example, if a person wishes to buy bandwidth of 250, 300, 400 Mbps from Internet service providers A, B and C respectively, each selling at the same price then $a = 400$ and $b = 250$. Similarly, if a person wishes to buy a car from a set of three cars costing $2000, $3000 and $4000, then $a = 2000$ and $= 4000$.

$$s_j(x) = \begin{cases} 2\dfrac{(x - b_j)}{(a_j - b_j)} - 1 & a_j \neq b_j \\ 1 & a_j = b_j \\ 1 & x > a_j \\ -1 & x < b_j \end{cases}$$

Fig. 2 Linear mapping function for s

The RDS algorithm takes four major concepts into consideration in the decision process. The first concept is the formulation of a decision matrix and the scaling of each element in this matrix based on scale s_j. The second concept is that the RDS algorithm allows a given decision maker to express a set of preferences d_j for each criterion. The third concept is the weightings w_j for each criterion such that $\sum w_j = 1$. Note that a decision makers set of preferences is different from the weightings of the criterion for a given decision problem. For example, in a computer network, the Internet service provider (ISP) in a given decision problem may set the weightings of bandwidth to 0.6 and delay to 0.4. However, a given network user may wish to indicate that delay is more important for them than bandwidth. The RDS algorithm allows for this type of flexibility in the decision making process. Finally, the fourth concept is an output vector $\bar{y}$ which is a $1 x n$ vector of real points. Each position in this vector is associated with an alternative and the position that carries the largest value is the best alternative. Similarly, an ordering of the $\bar{y}$ will result in the decision alternatives being ranked. Fig. 3 shows how these four concepts work together.
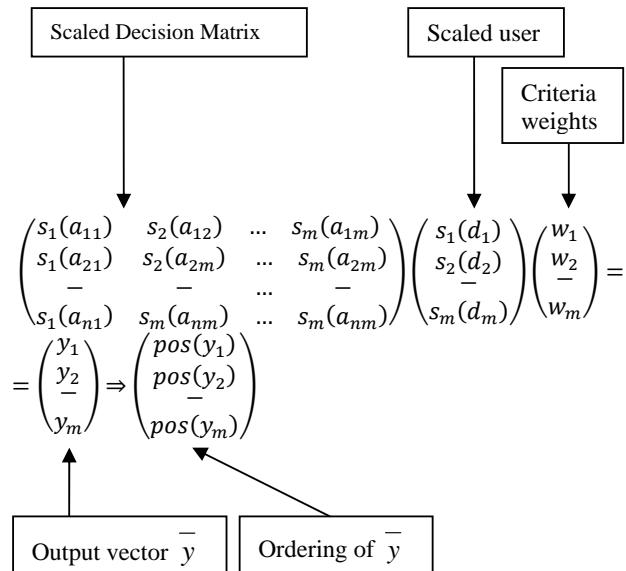
Scaled Decision Matrix   Scaled user   Criteria weights

$$\begin{pmatrix} s_1(a_{11}) & s_2(a_{12}) & ... & s_m(a_{1m}) \\ s_1(a_{21}) & s_2(a_{2m}) & ... & s_m(a_{2m}) \\ - & - & ... & - \\ s_1(a_{n1}) & s_m(a_{nm}) & ... & s_m(a_{nm}) \end{pmatrix} \begin{pmatrix} s_1(d_1) \\ s_2(d_2) \\ \\ s_m(d_m) \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \\ w_m \end{pmatrix} =$$

$$= \begin{pmatrix} y_1 \\ y_2 \\ \\ y_m \end{pmatrix} \Rightarrow \begin{pmatrix} pos(y_1) \\ pos(y_2) \\ \\ pos(y_m) \end{pmatrix}$$

Output vector $\bar{y}$ Ordering of $\bar{y}$

Fig. 3 Pictorial view of concepts that make up the RDS decision process

B. IP Traffic Engineering Example of the RDS

Fig. 4 shows the topology of the network used to demonstrate the RDS algorithm. Each link is characterized by three weights that correspond to bandwidth, delay and preferred customers (PC) respectively; PC is a metric that determines the degree that a given customer traffic is welcomed on the link (3 = highly welcomed, 2 = can moderately use, 1 = can use but not desired). In the network node A is used as the ingress router and node E is used as the egress router. In addition, the requests that network users could make on the network are summarized in Table 1. The route server collects routing protocol updates by peering with a link state protocol. In this way the entire topology of the network can be constructed at the route server. By this topology are calculated all the possible routes between ingress and egress nodes. The weightings for all criteria are assumed to be equal.
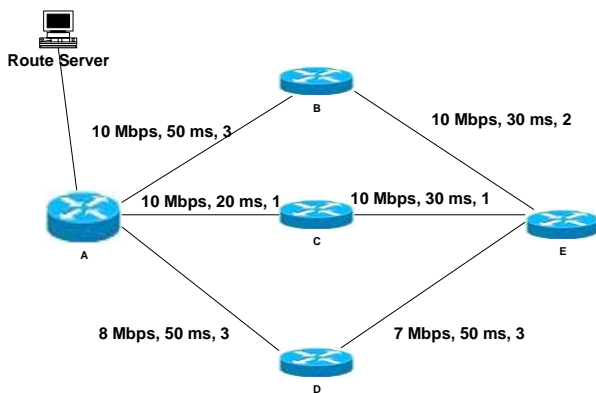

Fig. 4 Example network that illustrates how the RDS works. Each link is has bandwidth, delay and PC metrics

TABLE I
USER GOALS FOR TRAFFIC

Goal	User Request
1	6 Mbps, 50 ms, PC = 1
2	6 Mbps, 50 ms, PC = 1
3	6 Mbps, 100 ms, PC = 1

Since no links are less than 6 Mbps the feasible paths connecting A and B are A-B-E, A-C-E, and A-D-E. These paths are passed to the RDS algorithm, which addresses goals 1 to 3 given in Table I. For the given paths, the matrix in Fig. 5 shows the value of each metric on each of the three routes that connect A and E. Fig. 6 shows the matrix after preference function scaling.

$$M = \begin{pmatrix} 10 & 80 & 2 \\ 10 & 50 & 1 \\ 7 & 100 & 3 \end{pmatrix} \begin{matrix} \leftarrow A\text{-}B\text{-}E \\ \leftarrow A\text{-}C\text{-}E \\ \leftarrow A\text{-}D\text{-}E \end{matrix}$$
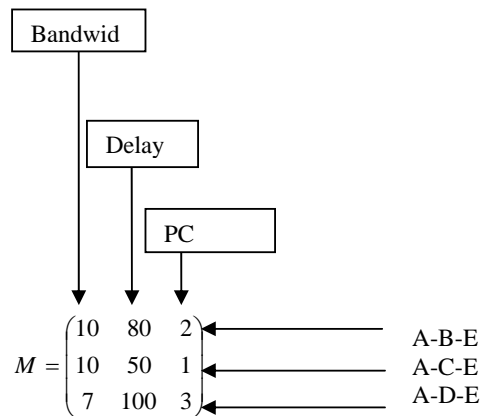
Fig. 5 Matrix containing columns of metric vales for each network path

$$M' = \begin{pmatrix} 1 & 0.2 & 0 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

Fig. 6 Matrix *M* after preference function scaling has been performed

Table II shows a summary of the scaled values user traffic goals vector v, the product of Mv and the path that the algorithm selected. It can be seen from the results that the RDS algorithm selected paths that match the user goals.

TABLE II
PATHS SELECTED BY RDS ALGORITHM BASED ON USER GOALS IN TABLE I

Goal	V	Mv	Path Selected
1	$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0.8 \\ 3 \\ -3 \end{pmatrix}$	A-C-E
2	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0.8 \\ 1 \\ -1 \end{pmatrix}$	A-C-E
3	$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1.2 \\ 1 \\ -1 \end{pmatrix}$	A-B-E

C. Generalization of the RDS Algorithm

The RDS algorithm in its original format may not meet the needs of a typical business problem. This is because the way the RDS handles tradeoffs is by using a set of criterion preferences (v is the example above) which is a part of the matrix multiplication process Mv and measures what a user desires for a given criteria against an alternative. However, in most decision problems the user is concerned with the

optimization of a particular criterion rather than finding an alternative closest to the value specified for that criterion by the user. Since the maximum value for a criterion is always mapped into 1 and the minimum value is always mapped into -1, the possible range of values that the set of user preferences for a given set of criteria can take is [1, 0,-1]. The value of 0 for a given criterion means that the user does not wish this criterion to participate in the decision making process. The value of -1 means that the user wishes an alternative that has the worst possible value for the given criterion – this is why -1 multiplied by $a_{ij} = -1$ would lead to a positive value and thus increase the chances of that alternative having the highest weighting in $\overline{y}$. Furthermore, it is important to note that the set [1, 0,-1] makes it easy for the user to perform 'what-if' analysis for a given decision problem.

D. Buying a Car (Example of the RDS)

Suppose a woman by the name of Cheryl wishes to buy a used car from the set of used cars in Table III and considers low mileage and low price to be very important criteria. However, since she has little experience in buying cars, she does not wish to put any weighting on the criteria. To find the best alternative from the set of cars for Cheryl, Fig. 7 shows how the RDS procedure is carried out. The best car for Cheryl to buy is car 1, followed by car 3 and car 2.

It is important to note that car 3 seems to be the best option. However, considering the fact that Cheryl is interested in low price and low mileage makes the problem interested. Car 1 has a lower price than car 3 whilst car 3 has a better mileage than car 1. However, when the "global picture" is taken into account $3000 difference in price between car 1 and car 3 is more valuable than a 2 mileage difference (note: mileage and price have the same weights). Hence car 1 is the better option.

TABLE III
SET OF USED CARS AND THEIR PROPERTIES (CRITERIA)

	Price	MPG	Mileage	Age
Car 1	15000	10	100000	10
Car 2	12000	6	200000	7
Car 3	18000	12	50000	8
Car 4	23000	7	100500	11
Car 5	13000	5	300000	7

$$\begin{pmatrix} 0.454 & 0.429 & 0.600 & -0.5000 \\ 1.000 & -0.714 & -0.200 & 1.000 \\ -0.091 & 1.000 & 1.000 & 0.500 \\ -1.000 & -0.429 & 0.596 & -1.000 \\ 0.818 & -1.000 & -1.000 & 1.000 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix} =$$

$$= \begin{pmatrix} 0.2635 \\ 0.2000 \\ 0.2273 \\ -1.1010 \\ -0.0455 \end{pmatrix} \Rightarrow \begin{pmatrix} 0.2635 \\ 0.2273 \\ 0.2000 \\ -0.0455 \\ -0.1010 \end{pmatrix}$$

Fig. 7 Pictorial view of RDS algorithm for finding the best used car

III. TESTING THE RDS ALGORITHM

Three test criteria were proposed in [1] to test the relative performance of various MCDM methods. These tests are as follows:

TEST CRITERION #1
*An **effective MCDM** method should not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative (given that the relative importance of each decision criterion remains unchanged*

Proof that the RDS satisfies Criterion #1
Benefit criteria are assumed to simplify the proof. However, because the RDS algorithm uses a function to map user values to $[-1,1]$ the result would also be true for cost criteria or a combination of benefit and cost criteria.

Suppose that the RDS (Routing Decision System) method produces the following ranking for a set of alternations:

$$A_1 \geq A_2 \geq A_3 \geq \ldots\ldots \geq A_n$$

Now suppose alternative $A_p = E, p \in [2, n]$ is selected. If E is replaced by a dominant (non-optimal) alternative, say F, then by definition of "dominance":

$$E_1 \geq F_j, \forall j \in [1, m] \qquad (1)$$

For the new set of alternatives let

$$p_j(x) = 2\frac{(x-b_j)}{(a_j-b_j)} - 1 = 1 + 2\frac{(x-a_j)}{(a_j-b_j)} \qquad (2)$$

Now, as b_j decreases (because of (1)), $a_j - b_j$ increases. Therefore, $2\frac{(x-a_j)}{(a_j-b_j)}$ decreases. Hence $p_j(x)$ decreases. So that $s_j(x) \geq p_j(x)$ and hence
$\sum_{j=1}^{m} s_j(x_{ij}) s_j(d_j) w_j \geq \sum_{j=1}^{m} p_j(x_{ij}) p_j(d_j) w_j$
due to the fact that the user preferences $s_j(d_j)$ and w_j are the same for both problems.

Since A_1 had the highest value of $\sum_{j=1}^{m} s_j(x_{ij}) s_j(d_j) w_j$ and was not replaced, and every real point in the new problem $\sum_{j=1}^{m} p_j(x_{ij}) p_j(d_j) w_j$ is less than the original problem, it follows that A_1 would retain the same position in the output vector of the RDS, with the highest value. Hence the RDS satisfies test criterion #1.

Remark 1: It must be emphasized here that the function *s* does not have to be linear. Any function which is continuous and increasing or decreasing in the domain $[-1,1]$ will work.

TEST CRITERION #2
The rankings of alternatives by an effective MCDM method should follow the transitivity property.

Proof that the RDS satisfies Criterion #2
Suppose that the RDS algorithm ranked a set of alternatives

of a decision problem in some way. Next, suppose that this problem is decomposed into a set of smaller problems, each defined on two alternatives at a time and the same number of criteria as in the original problem. Then, according to this test criterion all the rankings which are derived from the smaller problems should satisfy the transitivity property. That is, if alternative A_1 is better than alternative A_2, and alternative A_2 is better than alternative A_3, then one should also expect alternative A_1 is better than alternative A_3. This will now be shown as follows:

Given the decision problem A, defined on a set of n (assumed to be odd and divisible by 3) alternatives ranked as follows:

$$A_1 \geq A_2 \geq A_3 \geq ...... \geq A_n \qquad (3)$$

Suppose that A is decomposed into sets of smaller problems, each consisting of pairs of alternatives as follows:

$$\{A_1, A_n\}, \{A_2, A_{n-1}\}, \{A_3, A_{n-2}\} .... \qquad (4)$$

Let $\{A_p, A_q\}$ and $\{A_q, A_z\}$ represent the pairs where $p = 1..\dfrac{n}{3}; q = \dfrac{n}{3}+1..\dfrac{2n}{3}; z = \dfrac{2n}{3}+1..n;$

Assume that $A_p < A_z$ in system $\{A_p, A_z\}$, when $A_q \geq A_z$ and $A_p \geq A_q$ in systems $\{A_q, A_z\}$ and $\{A_p, A_q\}$ respectively. Now $A_p < A_z$

$$\Rightarrow \sum_{j=1}^{m} s_j(x_{pj})s_j(d_j)w_j < \sum_{j=1}^{m} s_j(x_{zj})s_j(d_j)w_j$$

$$\Rightarrow \sum_{j=1}^{m} s_j(x_{pj}) < \sum_{j=1}^{m} s_j(x_{zj})$$

Now $x_{ij} = \dfrac{s_j(x_{ij})(a_j - b_j) + (a_j + b_j)}{2}$

$$\sum_{j=1}^{m} (x_{pj} - x_{zj}) < \dfrac{a_j - b_j}{2} \sum_{j=1}^{m} \big(s_j(x_{pj}) - s_j(x_{zj})\big) <$$

$$< \dfrac{a_j - b_j}{2}\left[\sum_{j=1}^{m} s_j(x_{pj}) - \sum_{j=1}^{m} s_j(x_{zj})\right] < 0$$

(Assumption that $A_p < A_q$)

Since, $A_p \geq A_q$,

$$\sum_{j=1}^{m} (x_{pj} - x_{qj}) = \dfrac{a_j - b_j}{2} \sum_{j=1}^{m} \big(s_j(x_{pj}) - s_j(x_{qj})\big) \geq 0$$

$$\Rightarrow \sum_{j=1}^{m} s_j(x_{pj}) \geq \sum_{j=1}^{m} s_j(x_{qj})$$

And $A_q \geq A_z$,

$$\sum_{j=1}^{m} (x_{qj} - x_{zj}) = \dfrac{a_j - b_j}{2} \sum_{j=1}^{m} \big(s_j(x_{qj}) - s_j(x_{zj})\big) \geq 0$$

$$\Rightarrow \sum_{j=1}^{m} s_j(x_{qj}) \geq \sum_{j=1}^{m} s_j(x_{zj})$$

Based on this, $\sum_{j=1}^{m} s_j(x_{pj}) \geq \sum_{j=1}^{m} s_j(x_{zj})$ which contradicts

the assumption that $A_p < A_z$. Hence given that $A_q \geq A_z$ and $A_p \geq A_q$ in systems $\{A_q, A_z\}$ and $\{A_p, A_q\}$ respectively then in system $A_p < A_z$ then $A_p \geq A_z$.

Remark 2: $\sum_{j=1}^{m} (x_{pj} - x_{zj}) \geq 0$ is true for benefit

objectives. If the objective is a cost-metric, then

$\sum_{j=1}^{m} (x_{pj} - x_{zj}) \leq 0$.

For simplicity, benefit-objectives were assumed. However, for a problem having a mixture of benefit and cost metrics the following rule has to be applied: If metric j is a benefit metric then $(x_{pj} - x_{zj})$ is used and if j is a cost metric then $(x_{zj} - x_{pj})$ is used. Using this simple decision step means that the above proof still holds when cost and benefit objectives are considered in the same system.

TEST CRITERION #3
For the same decision problem and when using the same MCDM method, after combining the rankings of the smaller problems that an MCDM problem is decomposed into, the new overall ranking of the alternatives should be identical to the original overall ranking of the un-decomposed problem.

Explanation: As before, suppose that an MCDM problem is decomposed into a set of smaller problems, each defined on two alternatives and the original decision criteria. Next suppose that the rankings of the smaller problems follow the transitivity property. Then, according to this test criterion when the rankings of the smaller problems are all combined together, the new overall ranking of the alternatives should be identical to the original overall ranking before the problem decomposition.

Proof that the RDS satisfies Criterion #3
Given the decision problem *A*, defined on a set of n alternatives ranked as follows:

$$A_1 \geq A_2 \geq A_3 \geq ...... \geq A_n$$

Suppose that A is decomposed into sets of smaller problems, each consisting of pairs of alternatives as follows:

$$\{A_1, A_2\}, \{A_3, A_4\}, ... \{A_p, A_{p+1}\}\{A_{p+}, A_{p+3}\}...A_n \quad (5)$$

Combining $\{A_p, A_{p+1}\}\{A_{p+2}, A_{p+3}\}$ gives

$$\{A_p, A_{p+1}\} \cup \{A_{p+2}, A_{p+3}\} = \{A_p, A_{p+1}, A_{p+2}, A_{p+3}\}$$

We wish to show that $A_p \geq A_{p+1} \geq A_{p+2} \geq A_{p+3}$

The interesting system is $\{A_{p+1}, A_{p+2}\}$

In the un-decompose problem we know that

$$\sum_{j=1}^{m} (x_{(p+1)j} - x_{(p+2)j}) > 0$$

Hence $\dfrac{a_j - b_j}{2} \sum_{j=1}^{m} \left(s_j(x_{(p+1)j}) - s_j(x_{(p+2)j}) \right) \geq 0$

$$\Rightarrow \sum_{j=1}^{m} s_j(x_{(p+1)j}) \geq \sum_{j=1}^{m} s_j(x_{(p+2)j})$$

$$\Rightarrow A_{p+1} \geq A_{p+2}$$

By the same reasoning we have that $\{A_p, A_{p+1}\}$ and $\{A_{p+2}, A_{p+3}\}$ implies that $A_p \geq A_{p+1}$ and $A_{p+2} \geq A_{p+3}$. From criterion #2, the transitive property would imply that: $A_p \geq A_{p+1} \geq A_{p+2} \geq A_{p+3}$. Hence criterion #3 is satisfied.

IV. Conclusion

Most papers related to this work use simulation to show that established MCDM models meet the three test criteria outlined in this paper. However, no research to the best of the author's knowledge has used Mathematics to show the effectiveness of the AHP or variants of the ELECTRE against the outlined test criteria in [1], [15]. And even when simulation was used the results were not encouraging. Hence this paper demonstrates that the RDS algorithm has a strong basis and could provide the reliability and consistency that is lacking from many of the other MCDM models. Furthermore, these advantages of RDS algorithm can be tested by case studies with real data applications.

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