

# A Simplified Analytical Approach for Coupled Injection Method of Colloidal Silica with Time Dependent Properties

M. A. Nozari, R. Ziaie Moayed

**Abstract**—Electro-osmosis in clayey soils and sediments, for purposes of clay consolidation, dewatering, or cleanup, and electro injection in porous media is widespread recent decades. It is experimentally found that the chemical properties of porous media especially PH change the characteristics of media. Electro-osmotic conductivity is a function of soil and grout material chemistry, altering with time. Many numerical approaches exist to simulate the of electro kinetic flow rate considering chemical changes. This paper presents a simplified analytical solution for constant flow rate based on varying electro osmotic conductivity and time dependent viscosity for injection of colloidal silica.

**Keywords**—Colloidal silica, electro-osmosis, pH, viscosity, zeta potential.

## I. INTRODUCTION

**G**ROUTING has a wide application in civil engineering, this includes: to reduce permeability of soil mass to control seepage and loss of water, to increase the strength of material below the foundation of heavy structures, or to reduce the deformability of the material in the foundation; to stabilize the foundation soil to aid the construction; to connect distinct structural elements into a homogeneous structure by injecting the seams between them with grout compounds; and to improve cohesionless soils to decrease their liquefaction potentials [1]-[4]. Generally, soils such as clay and silt possess negatively charged surfaces due to isomorphous substitutions and broken bonds. The cations in the pore water are attracted to the soil particle surface, forming a diffusive double layer around the soil particles. If a D. C. voltage is applied to the soil specimen, the hydrated cations in the diffusive double layer, primarily, and the cations in free water subsequently, move toward the cathode. This advective movement drags the molecules of water with it, which is called electro-osmotic flow [5]. Theoretical investigation on electro-osmosis started nearly 200 years ago. The first general observations on electro-osmosis phenomenon were made by Russ in 1808 on thin clay diaphragms. Helmholtz presented theory based on a mathematical model in 1879 and later refined by Smoluchowski in 1914 [6]. This theory is one of the earliest and widely used theoretical descriptions of electro-osmosis. In this theory, soil pores are treated as a liquid-filled capillary.

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From this theory, the electro-osmotic flow through a cross sectional area  $A$  normal to the flow direction is:

$$q_e = k_e \frac{\partial v}{\partial x} = \frac{\zeta n D \mu_w}{\mu} \frac{\partial V}{\partial x} \quad (1)$$

where,  $V$  is the electric potential,  $k_e$  is the coefficient of electro-osmotic permeability,  $\zeta$  is zeta potential,  $\mu$  is viscosity ( $\mu_w$  is the viscosity of water),  $n$  is the porosity,  $D$  is dielectric constant, and  $x$  is the direction of D.C. current induction [7].

In view of better control of civil, geotechnical, and environmental engineering applications of electro-kinetic phenomena, fundamental research on transport phenomena in electrically charged porous materials has been. In such media the mechanical and physicochemical mechanisms implied in electro-osmotic flow are fully coupled the liquid flow is accompanied by pore water pressure, chemical composition, electrical, and thermal gradients as well as deformation and/or sample cracking [8]-[10]. In particular, soil pH is recognized as the key physicochemical parameter for heavy metal ions extraction from contaminated soils under an applied electric field as well as for electro-osmotic flow [11]. In electro-osmotic injection, acid produced at anode by electrolysis will cause drop of pH around anode. This low pH situation will accelerate the gelation of grout materials introduced at anode. The gel may clog the soil pores and hinder further electro-osmotic flow. When metal anodes are used, anodes will be corroded and soil adjacent to anode will be impregnated with metal oxide. The metal ions can also act as reactant and accelerate the gel formation [4]. It is experimentally found that the electro-osmotic permeability is pH-dependent. The value of  $k_e$  decreases with time as the acid front generated at the anode or ions injected at electrodes sweeps across the specimen. This decrease of  $k_e$  may due to the formation and introduction of the electrolysis products at the electrodes and the changing chemistry across the soil [5] or due to the nonuniform changes in conductivity coupled with internal changes in pore pressures during the processing [4]. The decrease of  $k_e$  will cause drop of coefficient of water transport efficiency (flow volume per ampere-hour), and in turn will cause an increase in cost and power consumption. Reference [11] conducted electro-osmosis experiments on rigid cylindrical samples containing 0.01M NaCl-water saturated Speswhite kaolinite and proposed an empirical relation to correlate the PH to electro-osmotic permeability [12]. The increase of electro-osmosis conductivity change versus PH has been shown in Fig. 1.

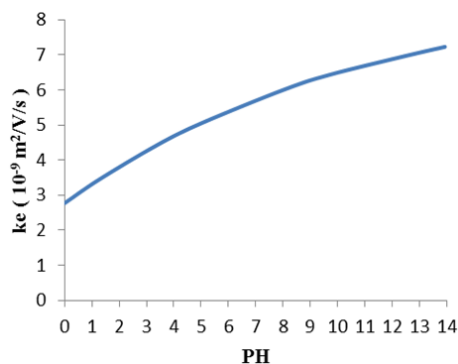


Fig. 1 Electro-osmotic permeability  $k_e$  as a function of kaolinite sample pH, The fitted values from experimental data have been found in [12]

Increasing the pH will increase the repulsion potential and hence enhance the release of deposited colloidal particles from the medium solid surfaces and limit further deposition. In colloidal systems, Zeta potential is the electric potential at the slipping plane in the double layer surrounding a particle suspended in a fluid, in reference to a point in the bulk fluid far away from the particle. In other words, zeta potential is the potential difference between the dispersion medium and the stationary layer of fluid attached to the dispersed particle. The double layer is composed of the stationary Stern layer and the diffuse layer. The Zeta potential, measured in mV at the slipping plane or the interface between the Stern layer and the diffuse layer is widely used as a measure of the particle stability in colloids. The higher the Zeta potentials absolute values are the more stable the particles in the colloids. Zeta potential is not measurable directly but it can be calculated using theoretical models and an experimentally determined electrophoretic mobility. When an electric field is applied to charged particles in the colloids, particles move toward an electrode opposite to their surface charge. Since the velocity is proportional to the amount of charge of the particles, zeta potential can be estimated by measuring the velocity of the particles. Electrophoretic light scattering is the method most popularly used to determine the velocity of the particles suspended in a liquid medium under an applied electric field. In order to determine the speed of the particles, the particles are irradiated with a laser light and the scattered light emitted from the particles is detected. Since the frequency of the scattered light is shifted from the incident light in proportion to the speed of the particles movement, the electrophoretic mobility of the particles can be measured from the frequency shift of the scattered light. The zeta potential dependency on PH value was clearly studied in previous researches. Fig. 2 shows the PH dependent value of zeta potential for a silica based suspensions reported by [13]. The silica sols, and indeed all oxide sols, show an increasing negative zeta potential with increasing pH as the pH is raised above the isoelectric point. The magnitude of the zeta potential decreases uniformly at each PH as the salt concentration is increased [13].

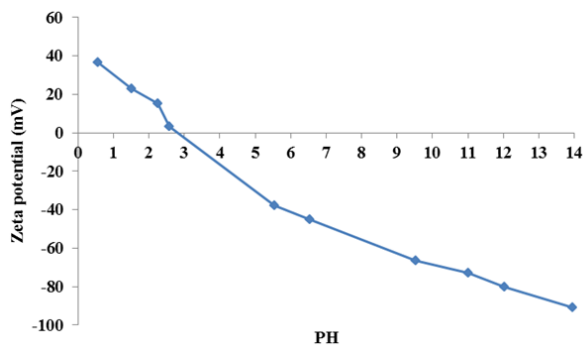


Fig. 2 Variation of Zeta potential for samples of quartz #325 and vitreous silica #325 [13]

Colloidal silica is an aqueous dispersion of silica nanoparticles that its gelation time could be adapted by adjusting the ionic strength or pH of the solution. The flow rate of the colloidal silica is integrally governed by hydraulic conductivity due to changing the gel viscosity [1]. Owing to the fact that colloidal silica viscosity increases with time many researchers have studied the influence of this characteristic to model the flow regime in one and three-dimensional space and developed numerical approaches [14]. When particles move with groundwater in porous media, they may undergo filtration (surface and straining filtration), gravitational settling, or physicochemical filtration. For colloid-size particles, physicochemical filtration becomes important. Colloid transport in saturated porous media can generally be described by mechanisms of advection, hydrodynamic dispersion, and mass transfer between the aqueous and solid phases within porous media [1]. Temperature fluctuations in the laboratory are small enough that they do not influence the gel time. With a constant colloid size and percentage of colloids, the primary variables influencing gel times are ionic strength and pH. Higher ionic strengths will accelerate the gelation of colloidal silica or shorten gel time because the addition of salt to the colloidal silica solution shrinks the double layer around the particles, increasing the probability of inter particle collisions. The presence of cations in the groundwater or exchangeable cations in the soil can influence gel time by this mechanism [1]. Reference [14] developed a simple analytical solution for predicting the required grout injection pressures with time as the gelling polymer solution is injected into a one-dimensional column from one end at a constant flow rate. It is assumed that the porosity remains constant during the grout injection. To simplify the governing equation, [14] considered only the advection component and assumed that dispersion/diffusion has no effect of grout gelation and its injection. Similarly, capillary and adsorption are supposed to be negligible. The viscosity time dependent model, proposed by [15] has been applied.

There have been some numerical modeling studies performed to analyze the electro-osmotic injection in soil porous media. Numerical models often need a proof that they represent the physical problem with an acceptable accuracy. Testing of numerical solutions against an analytical solution is

an important phase of code verification and forms a critical step in the code development process [16]. The process of verification provides the confidence that the numerical code approximates the set of governing differential equations subjected to given boundary conditions and initial conditions. An acceptable matching between the results obtained from analytical and numerical solutions does not always guarantee an accurate solution to the problem. However, it provides a confidence measure that the governing equations thought to be representing the processes are solved with a reasonable accuracy. This paper presents an analytical approach to achieve how the constant flow rate could be analytically acquired.

## II. GOVERNING EQUATION AND ANALYTICAL SOLUTION

### A. Review Stage

A one-dimensional saturated sand column of length ( $L$ ) is being injected with colloidal silica at a constant flux ( $q_T$ ). Total flow rate  $q_T$  is the summation of electro-osmotic and hydraulic ones as (2):

$$q_T = q_h + q_e = \frac{k_0 \mu_w}{\mu} \frac{\partial h}{\partial x} + \frac{\zeta n D_0 \mu_w}{\mu} \frac{\partial v}{\partial x} \quad (2)$$

$k_0$  is the intrinsic permeability coefficient of porous media. Reference [15] proposed (3) to simulate time dependent viscosity by nonlinear regression approach

$$\mu(t) = a_1 + a_2 e^{a_3 t} \quad (3)$$

$a_1$ ,  $a_2$ , and  $a_3$  are fitting parameters. For a gel time of one hour, the parameter values for  $a_1$ ,  $a_2$  and  $a_3$  are reported as  $3.22 \times 10^{-3}$  Pa.s,  $2.82 \times 10^{-3}$  Pa.s and  $1.35 \times 10^{-3}$  Pa.s, respectively [15]. These numerical values are provided to give a feel for the estimates of the coefficients. Fig. 3 shows a typical trend of viscosity changing with time.

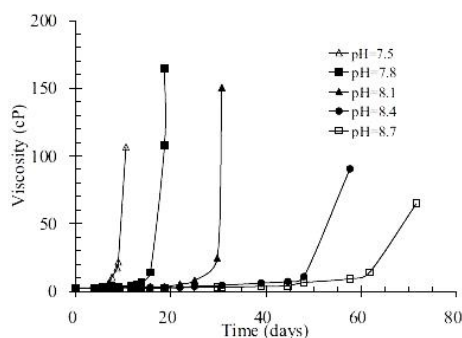


Fig. 3 Typical colloidal silica viscosity curves for 5 weight percent solutions at anionic strength of 0.1 N [1]

Now the time travel of grout from start to point  $x$  is calculated by

$$t = \frac{x}{q_T/n} \quad (4)$$

Reference [14] calculated the viscosity as a function of  $x$  by

using (3):

$$\mu(x) = a_1 + a_2 e^{\left(\frac{a_3 n x}{q_T}\right)} \quad (5)$$

As previously mentioned, the zeta potential is a function of  $x$  and  $t$  and differs in initial value for grout and water:

$$\zeta = \zeta(PH(x, t)) \quad (6)$$

$t$  is the time that grout reaches  $x$ . Indices of  $\zeta$  indicates zeta potential relates to water or grout.

### B. Analytical Solution

At this paper, applying variation of zeta potential and viscosity the mass conversion rule resolved. It assumed that the grout (colloidal silica) has the same density as water. In addition, the initial electro-chemical properties of colloidal silica are constant. Additionally, we assume that dispersion/diffusion does not effect of grout gelation and its injection. Similarly, the processes, such as capillary and adsorption are negligible. The compressibility effects are also considered not to be affecting the grout injection because the injection pressures are only slightly above the normal injection pressures. Considering constant voltage gradient, equal dielectric constants for grout and water, and integrating between  $x=0$  and  $x=x_l$  for grouted region, (2) could be rewritten as:

$$\frac{q_T}{\mu_w A} \int_0^{x_1} \mu(x) dx + k_0 \int_0^{x_1} \frac{\partial h}{\partial x} dx + nDE \int_0^{x_1} \zeta_{gr}(x, t) dx = 0 \quad (7)$$

$E$  is obtained by

$$E = \frac{\partial v}{\partial x} \quad (8)$$

Solving (7),

$$\frac{q_T}{k_0 \mu_w A} \left[ a_1 x_1 + \frac{q_T a_2}{n a_3} \left( e^{\left(\frac{a_3 n x_1}{q_T}\right)} - 1 \right) \right] + \frac{nDE}{k_0} \int_0^{x_1} \zeta_{gr}(x, t) dx + h(x_1, t) - h(0, t) = 0 \quad (9)$$

For ungrouted region, the viscosity of water is constant and hydraulic conductivity does not change. So (7) becomes:

$$\frac{q_T}{k_0 A} \int_{x_1}^L dx + \int_{x_1}^L \frac{\partial h}{\partial x} dx + \frac{nDE}{k_0} \int_{x_1}^L \zeta_w(x, t) dx = 0 \quad (10)$$

Similar to (9), solving (10)

$$\frac{q_T}{k_0 A} (L - x_1) + h(L, t) - h(x_1, t) + \frac{nDE}{k_0} \int_{x_1}^L \zeta_w(x, t) dx = 0 \quad (11)$$

By using (4) in both (9) and (11) and applying the boundary hydraulic, condition at  $x_l$ , for entire the injection volume:

$$\frac{q_T}{k_0 \mu_w A} \left[ a_1 \frac{q_T}{n} t + \frac{q_T a_2}{n a_3} (e^{(a_3 t)} - 1) \right] + \frac{q_T}{k_0 A} \left( L - \frac{q_T}{n} t \right) + \frac{nDE}{k_0} \left[ \int_0^{x_1} \zeta_{gr}(x, t) dx + \int_{x_1}^L \zeta_w(x, t) dx \right] + h(L, t) = h(0, t) \quad (12)$$

Comparing (12) with analytical solution in [14], obviously, the terms include zeta potential changes the needing hydraulic head at  $x=0$ . Additionally  $q_h$  in [14] is substituted by  $q_T$  in other terms. Having the data base to generate an empirical relation for zeta potential following the  $PH(x, t)$ , analytical solution could be validated. It should be noted, the trend of PH alteration depends on  $E$ , chemical properties of colloidal silica, and circumstances of porous media [12], [13].

### III. CONCLUSION

Considering simultaneous hydraulic and electro-osmotic flow, an analytical solution is developed based on relations derived before for time dependent viscosity and zeta potential changing with time. This analytical method is useful for verification of numeral methods especially electro kinetic injection of colloidal silica. Electrochemical properties alteration has been applied to incorporation of initial and time dependent chemistry characteristics of both porous media and silica dispersion.

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