International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:2, No:1, 2008

A Simple Method for Tracing PV Curve of a Radial Transmission Line

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***Abstract*—**Analytical expression for maximum power transfer through a transmission line limited by voltage stability has been formulated using exact representation of transmission line with ABCD parameters. The expression has been used for plotting PV curve at different power factors of a radial transmission line. Limiting values of reactive power have been obtained.

***Keywords*—**Power Transfer, PV Curve, Voltage Stability.

I. INTRODUCTION

TRANSMISSION systems are becoming more stressed due to increased loads and inter-utility power transfers. With growing size, along with economic and environmental pressures, the efficient operation of the system is becoming increasingly threatened due to problem of voltage instability and collapse. Voltage stability creates a serious problem in the ability of a transmission line to transfer maximum power, particularly with higher VAR demand [1].QV and PV curves are the most widely used voltage stability analysis tools today. The PV curve is formed by varying system load or transfer and plotting it against voltage. The PV curve can provide real power and voltage margins using the knee of the curve as reference point. It is used in control centers where the complexity of QV curve is impractical [2]. PV Curves at constant power factor are used to get maximum power transfer at critical voltage [3] [4]. The application of PV curve in voltage stability studies is enormous. Many proximity indicators are identified using PV Curves. Voltage and power are controllable in upper region of the PV-curve [5]. Analytical expression for complex power, real power has been obtained using exact representation of transmission line with ABCD parameters using elementary mathematics and receiving end circle diagram [6].

II. LOADABILITY OF A RADIAL TRANSMISSION LINE LIMITED BY VOLTAGE STABILITY

A radial transmission line is shown in Fig. 1 in which a generator with a constant voltage $E_S\angle\delta$ supplying complex power S_R to a load with a terminal voltage $E_R\angle 0$ through a transmission line represented by its ABCD parameters.

Complex power at the receiving end of a transmission line shown in Fig. 1 is given as:

Submitted November 27, 2007

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$$S_R = \frac{-AE_R^2}{B}\angle\beta-\alpha + \frac{E_S E_R}{B}\angle\beta-\delta \qquad (1)$$

The above equation (1) represents a circle for varying value of δ with position of centre indicated by $\frac{-AE_R^2}{B}\angle\beta-\alpha$ and radius by $\frac{E_S E_R}{B}$ where **A** = A $\angle\alpha$ and **B** = B $\angle\beta$ are the line constants and δ is power angle.

From Fig. 2:

$$OC = \frac{AE_R^2}{B} \qquad (2)$$

$$OP = S_R \qquad (3)$$

$$CP = \frac{E_S E_R}{B} \qquad (4)$$

$$\delta' = \delta-\alpha$$

$$\phi' = 180° - (\beta-\alpha)+\phi \qquad (5)$$

ϕ is the power factor angle and is positive for lagging power factor and negative for leading power factor

In ΔOCP

$$\frac{OP}{Sin\delta'} = \frac{CP}{Sin\phi'} = \frac{OC}{Sin\theta} \qquad (6)$$

From (1) to (6)

$$S_R = \frac{E_S^2 Sin\theta Sin\delta'}{ABSin^2\phi'} \qquad (7)$$

Also from ΔOCP

$$\theta = 180^0 - (\phi' + \delta') \qquad (8)$$

Therefore,

$$S_R = \frac{E_S^2 Sin(\phi'+\delta')Sin\delta'}{ABSin^2\phi'} \qquad (9)$$

For S_R to be maximum

$$\frac{dS_R}{d\delta} = \frac{dS_R}{d\delta'} = 0$$

Solution of (9) provides critical value of power angle δ, critical value of voltage and maximum value of complex power

$$\delta' cr = 90° - \frac{\phi'}{2} \text{ and } \delta cr = 90° - \frac{\phi}{2} + \alpha \qquad (10)$$

$$S_R \max = \frac{E_S^2}{4ABSin^2\phi'/2} \qquad (11)$$

$$E_R cr = \frac{E_S}{2ASin\phi'/2} \quad (12)$$

Equation (9) and (11) relate complex power with maximum complex power

$$S_R = \frac{S_{R\max}\left[Sin(\phi'+\delta')Sin\delta'\right]}{Cos^2\phi'/2} \quad (13)$$

The maximum value of active power and limiting value of reactive power Q_{Rlim}

$$P_{R\max} = S_{R\max}Cos\phi \quad (14)$$

$$Q_{R\lim} = S_{R\max}Sin\phi \quad (15)$$

Receiving end voltage is obtained as

$$E_R = \frac{E_S Sin(\phi'+\delta')}{ASin\phi'} \quad (16)$$

III. Illustrative Example

To study the loadability of a radial transmission line we consider a 275 KV three phase line with following parameters [7].

$A = 0.93\angle 1.5°$
$B = 115\angle 77.5°$
Sending end voltage is constant

Table I gives the values of ϕ, ϕ', $S_R$max, $P_R$max, δcr and $E_R$cr. Limiting value of reactive power is also listed in table I. Using equation (13) and (16) we get values of S_R, E_R, P_R, that are given in Table II, III, IV,V and VI for different power factors.

The PV curve for the above problem has been drawn for 30° lagging, 20° lagging,0°,10° leading and 20° leading power factor angle.

The algorithm used is as given below [8]:

(i) Compute $\delta' = \delta cr \pm \Delta\delta$
δcr is obtained from Table I
$\Delta\delta$ is change in value of δ from 5° to 20°
(ii) Compute S_R
(iii) Compute P_R
(iv) Compute E_R

From the curve we obtain that value of P_R increases from lagging to leading power factor. We also obtain that there are two values of E_R for a given P_R except at $P_R$max.

IV. Results

PV curve for the above problem have been drawn for 30° lagging, 20° lagging, 0°, 10° leading and 20° leading power factor angle. The curve is shown in figure 3.The limiting value of reactive power for the above power factor angles is given in Table I. $Q_{R\lim}$ is the maximum value of reactive power that can be transferred through the line at lagging power factor, whereas $Q_{R\lim}$ becomes the minimum value of reactive power at leading power factors.

V. Conclusion

Simple analytical expression for real power (P_R) critical voltage ($E_R$cr) has been formulated and had been used to draw PV curve of a radial transmission line. It is observed that real power increases from lagging to leading power factor. We also obtain that there are two values of receiving end voltage E_R for a given P_R except at $P_R$max. QV curves for fixed P_R and different E_R can also be plotted using the derived relationship and devised algorithm by varying reactive power.

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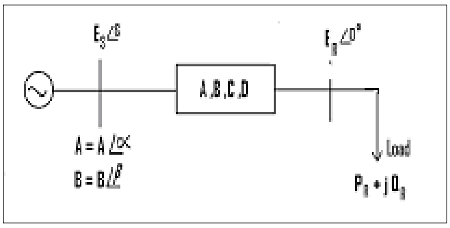

Fig. 1 Radial transmission line

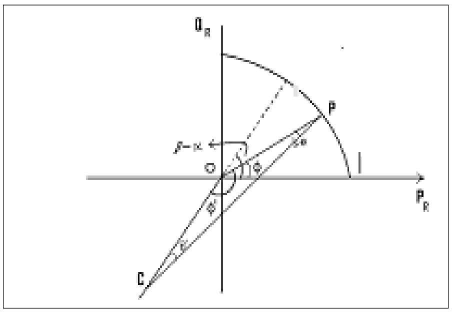
Fig. 2 Receiving end circle diagram

TABLE I
MAXIMUM COMPLEX AND REAL POWER, LIMITING REACTIVE POWER, CRITICAL ANGLE AND VOLTAGE

ϕ (Deg)	ϕ′ (Deg)	$S_R$max (MVA)	$P_R$max (MW)	$Q_R$lim (MVAR)	δcr (Deg)	E_R cr (kV)
20 Lead	84	394.82	370.97	-135.02	48	220.16
10 Lead	94	330.50	325.28	-57.37	43	202.16
0	104	284.68	284.68	0	38	187.62
20 Lag	124	226.75	213.05	77.55	28	167.45
30 Lag	134	208.63	180.67	104.31	23	160.62

TABLE II
COMPLEX AND REAL POWER, VOLTAGE AT 30° LAGGING POWER FACTOR ANGLE

δ'(Deg)	S_R(MVA)	P_R (MW)	E_R (kV)
28	198.25	171.68	127.03
18	198.25	171.68	192.98
33	144.98	125.55	92.47
13	144.98	125.55	223.88
38	90.14	78.06	57.21
08	90.14	78.06	253.08
43	37.55	32.52	21.51
03	37.55	32.52	280.35

TABLE III
COMPLEX AND REAL POWER, VOLTAGE AT 20° LAGGING POWER FACTOR ANGLE

δ'(Deg)	S_R(MVA)	P_R (MW)	E_R (kV)
33	218.92	205.71	139.36
23	218.92	205.71	194.26
38	195.71	183.90	110.21
18	195.71	183.90	219.59
43	157.82	148.30	80.23
13	157.82	148.30	243.37
48	106.39	99.97	49.63
08	106.39	99.97	265.06

TABLE IV
COMPLEX AND REAL POWER, VOLTAGE AT 0° POWER FACTOR ANGLE

δ'(Deg)	S_R(MVA)	P_R (MW)	E_R (kV)
43	278.97	278.97	165.97
33	278.97	278.97	207.83
48	262.03	262.03	143.06
28	262.03	262.03	226.46
53	234.36	234.36	119.07
23	234.36	234.36	243.38
58	196.82	196.82	94.17
18	196.82	196.82	258.43

TABLE V
COMPLEX AND REAL POWER, VOLTAGE AT 10° LEADING POWER FACTOR ANGLE

δ'(Deg)	S_R(MVA)	P_R (MW)	E_R (kV)
48	325.10	320.16	182.49
38	325.10	320.16	220.28
53	309.07	304.37	161.44
33	309.07	304.37	236.73
58	283.89	278.59	139.16
28	282.89	278.59	251.38
63	247.36	243.60	115.82
23	247.36	243.60	264.11

TABLE VI
COMPLEX AND REAL POWER, VOLTAGE AT 20° LEADING POWER FACTOR ANGLE

δ' (Deg)	S_R (MVA)	P_R (MW)	E_R (kV)
53	389.38	365.89	202.77
43	389.38	365.89	237.47
58	373.26	350.74	183.05
38	373.26	350.74	252.14
63	346.93	326.00	161.93
33	346.93	326.00	264.92
68	311.19	292.42	139.50
28	311.19	292.42	275.67

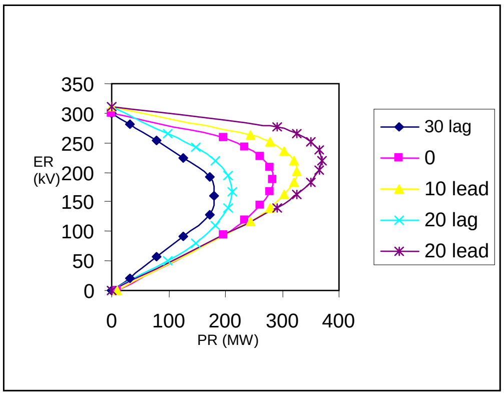

Fig. 3 PV Curve at different power factors