

# A self-Consistent Scheme for Elastic-Plastic Asperity Contact

Xu Jianguo

**Abstract**—In this paper, a generalized self-consistent scheme, or “three phase model”, is used to set up a micro-mechanics model for rough surface contact with randomly distributed asperities. The dimensionless average real pressure  $\bar{p}$  is obtained as function of the ratio of the real contact area to the apparent contact area,  $A_r/A_0$ . Both elastic and plastic materials are considered, and the influence of the plasticity of material on  $\bar{p}$  is discussed. Both two-dimensional and three-dimensional rough surface contact problems are considered.

**Keywords**—contact mechanics, plastic deformation, self-consistent scheme.

## I. INTRODUCTION

CONTACT problems are central to Solid Mechanics, because contact is the principal method of applying loads to a deformable body [1]. All the real surfaces are rough with a large number of micro-scale asperities. Contact actually happens between these asperities. Generally the real contact area is very small [2], for instance, in the order 1% or less. Real contact is concentrated on a cluster of microscopic actual contact areas. So real contact pressure and contact area are important for the contact properties. Furthermore, the configuration of wafer bonding in the MEMS packaging industry generates large real contact area, even full contact area when complete bonding [3]. Great amount of research work has been carried out on rough surfaces, from the measurement and characterization of the surface roughness, to the simulation of their contact. A recent review by Majumdar and Bhushan provided a good survey on the recent works related to the rough surfaces [4].

Usually the heights and cap radii of these asperities are distributed randomly. Statistical law is often used in the research of contact mechanics of rough surfaces, but it needs analytical relations between the quantities related to single asperities in order to obtain the contact property of a whole surface [5]. Such analytical relations often cannot be obtained, especially when the material properties are nonlinear. Some other model can be searched to solve this problem.

It is well known that there are a few well-developed analytical models in composite mechanics for estimating the effective properties, such as modulus, thermal conductivity and

average stresses in the composites with the second phase particles [6]–[10]. Hereinto, the “three phase model”, or “generalized self-consistent method (scheme)” by its own author, is now considered having certain advantages and merits and therefore survived critics of the researchers. Recently, based on the “self-consistent scheme”, Fan and Sze proposed a micro-mechanics model for imperfect interface in dielectric materials [11].

In the present study, we also adopted this analytical model to look for the relation between average pressure  $\bar{p}$  and the ratio of the real contact area to the apparent contact area,  $A_r/A_0$ , which is

$$\bar{p} = f(A_r/A_0) \quad (1)$$

Here, we treated  $\bar{p}$  between two contact surfaces as effective property if the dimension of the asperity is much smaller than the bulk contact dimensions.

It is realized that the asperity contact causes plastic deformation near the contact region because of high stress level [2], [4], [5]. We included the plastic deformation in our self-consistent model to investigate the influence of the plastic deformation on the contact properties, such as the average contact pressure and the contact area.

The FEM will be used to obtain numerical results for this problem, which is shown to be a more reliable and better numerical scheme than the singular integral equation approach [12].

## II. MULTIPLE ASPERITY CONTACT OF ROUGH SURFACES AND ITS SELF-CONSISTENT MODEL

We analyze the contact problem between a half-space with a rough surface and a rigid half-space with a flat surface as shown in Fig. 1. The problem may be two-dimensional or three-dimensional. When the profile of the rough surface does not change along the direction perpendicular to the figure, it is a two-dimensional contact problem; otherwise, it is a three-dimensional one. We will analyze two-dimensional plane strain problem and three-dimensional isotropic rough surfaces that has no preference in any direction in terms of its characteristic parameters describing the surface roughness behavior. The upper half-space is pushed downward with a pressure  $\bar{p}$  applying on it far from the interface. Because the asperities of the half-space has randomly-distributed density, heights and peak radii, the nominal or average contact pressure is same anywhere on the interface. Both elastic and plastic

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material will be considered.

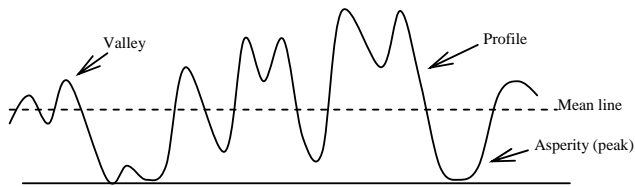


Fig. 1 Multiple asperity contact of rough surface

Here, we assume the asperity dimensions are much smaller than the bulk contact dimension. Then based on “self-consistent scheme”, the average contact pressure  $\bar{p}$  can be treated as effective property. The interaction among the asperities is modeled by Fig. 2. Only one asperity with average characterization parameters describing the surface roughness is considered, and the effect of the rest of the asperities is replaced by the average pressure  $\bar{p}$ . This model has three “phases” (or three regions to be exact), namely

- (i)  $x \in (-L, -l)$  and  $x \in (l, L)$  are the effective contact region, prescribed with the average contact pressure  $\bar{p}$ ;
- (ii)  $x \in (-l, -c)$  and  $x \in (c, l)$  are the non-contact regions;
- (iii)  $x \in (-c, c)$  is the asperity contact region.

Here, the three-dimensional contact problem now is simplified to be axisymmetric.

The average pressure applied on the interface and far from the interface is defined as

$$\bar{p} = \frac{1}{A_0} \iint_{A_0} p(x, y) dx dy \quad (2)$$

By applying the “self-consistent scheme”, now it is

$$\bar{p} = \frac{1}{2l} \int_{-c}^c p(x) dx \quad (3)$$

for the plane strain contact, and

$$\bar{p} = \frac{1}{\pi l^2} \int_{-c}^c p(r) 2\pi r dr \quad (4)$$

for three-dimensional contact with isotropic surface.

Now the interaction between randomly-distributed asperities is modeled by the interaction between a single asperity and the region with the average pressure acting on. But the average pressure  $\bar{p}$  can only be obtained after the pressure distribution  $p(x)$  (or  $p(r)$  for the axisymmetric contact) in the single asperity contact region is obtained, while the pressure distribution in this single asperity contact region is generated from the average pressure. So, they are coupled. Solving this pair of pressures ( $\bar{p}$  and  $p(x)$  (or  $p(r)$  for the axisymmetric contact)) simultaneously based on the “self-consistent model” is the essence of our micro-mechanics model set up here.

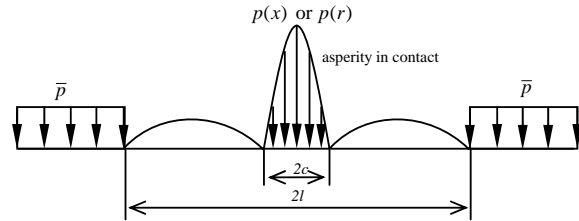


Fig. 2 Three-phase model for asperity contact

### III. FINITE ELEMENT NUMERICAL MODELS

For the models in Fig. 2 set up based on the “self-consistent scheme”, finite element method can be used to solve this elastic-plastic problem. We note that if we consider only elastic deformation in this figuration, singular integral equation method can also be used [12]. But here due to the existing plastic deformation, we adopted the finite element method [13].

Here, for an easy comparison with the existing close form analytical solution for the elastic contact between a periodic sinusoidal profile and a flat surface [14], [15], the asperity shapes are chosen as

$$H(x) = 2\Delta \left[ \sin^2\left(\frac{\pi x}{2l}\right) + \alpha \sin^2\left(\frac{\pi x}{l}\right) \right] \quad (5)$$

for the plane strain contact, and

$$H(x) = 2\Delta \left[ \sin^2\left(\frac{\pi r}{2l}\right) + \alpha \sin^2\left(\frac{\pi r}{l}\right) \right] \quad (6)$$

for three-dimensional contact with isotropic surface.

As in Fig. 3. Here we add a term with a changeable coefficient  $\alpha$  to the equation of the asperity profile to obtain different “flatness” of the asperity. We obtained numerical results by finite element simulation for  $\alpha$  with values -0.1, 0 and 0.1.

In Fig. 3, the finite element model has a large effective region enough for the boundary effect can be ignored. Equations (2) to (4) indicate that the average pressure acting on the contact interface between the single asperity and the flat surface is equal to the pressure applied on top of the upper half-space, having a value of  $\bar{p}$ .

Because we will consider the effect of plasticity on the contact property, for this problem, the independent physical quantities include:

- (i) The geometrical dimensions:  $l, \Delta, \alpha$  ( $L$  can be ignored because of the large effective region);
- (ii) The material properties: the elastic modulus  $E$ , Poisson's ratio  $\nu$ , and the yielding stress  $\sigma_s$  of the upper half-space.

Dimensional analysis showed that for this problem, all configurations with same dimensionless quantities  $\frac{\Delta}{l}, \frac{\sigma_s}{E}, \alpha$

have similarity, so the parametric study for this problem will be fulfilled by changing these three dimensionless quantities. Here, the elastic modulus  $E$  has a value of 50GPa, and the Poisson's ratio of 0.3. The rest of the quantities are chosen as follows:

$$\frac{\Delta}{l} = 0.0025, 0.005$$

$\sigma_s = 10\text{MPa}, 50\text{MPa}, \infty$  (i.e., elastic material).

The commercial finite element package ANSYS is adopted to simulate the contact of the model in Fig. 3 numerically [13]. The edge of the contact area of the single asperity is extracted from the contact state of the nodes at interface.

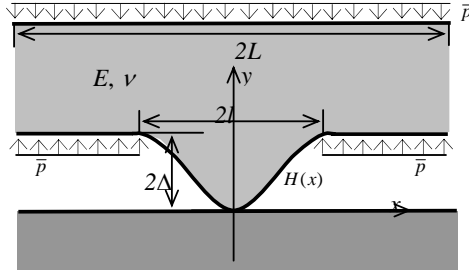


Fig. 3 The finite element model

#### IV. FINITE ELEMENT NUMERICAL RESULTS

In this section, we present the numerical results from our FEM simulations. In order to examine whether our self-consistent model produces correct results, the FEM numerical results for the periodic sinusoidal plane strain cases with  $\alpha = 0$  are compared with the existing closed form analytical solutions [14], [15]. It is shown that they coincide well with each other.

The average pressures calculated from our model when approaching full contact are listed in Tables 1 to 4, here the dimension  $l = 1\text{mm}$ .

TABLE I

AVERAGE PRESSURES (MPa) FOR PLANE STRAIN CASES WITH  $\Delta = 0.0025\text{mm}$

$\alpha \backslash \sigma_s$	10MPa	50MPa	$\infty$ (elastic contact)
-0.1	11.95	57.3	256.1
0	11.88	56.99	213.8
0.1	11.79	56.41	171.4

TABLE II

AVERAGE PRESSURES (MPa) FOR AXISYMMETRIC CASES WITH  $\Delta = 0.0025\text{mm}$

$\alpha \backslash \sigma_s$	10MPa	50MPa	$\infty$ (elastic contact)
-0.1	10.3	49.71	200
0	10.23	49.43	159.9
0.1	10.15	48.75	119.7

TABLE III

AVERAGE PRESSURES (MPa) FOR PLANE STRAIN CASES WITH  $\Delta = 0.005\text{mm}$

$\alpha \backslash \sigma_s$	10MPa	50MPa	$\infty$ (elastic contact)
-0.1	12.5	58.17	509.3
0	12.33	57.9	425.9
0.1	12.18	57.55	341.6

TABLE IV

AVERAGE PRESSURES (MPa) FOR AXISYMMETRIC CASES WITH  $\Delta = 0.005\text{mm}$

$\alpha \backslash \sigma_s$	10MPa	50MPa	$\infty$ (elastic contact)
-0.1	10.66	50.35	398.3
0	10.53	50.1	318.3
0.1	10.39	49.73	238.5

The above-mentioned analytical solution gave the average pressure as  $\bar{p} = \pi \frac{E}{1-\nu^2} \frac{\Delta}{2l}$  for the plane strain elastic contact

between two periodic sinusoidal surfaces. So the analytical solution of full contact for the average pressures of the plane strain contact is

$$\bar{p} = \pi \frac{E}{1-\nu^2} \frac{\Delta}{2l} = \pi \frac{50}{1-0.3^2} \frac{0.0025}{2 \times 1} \text{GPa} \approx 215.8\text{MPa}$$

for  $\Delta = 0.0025\text{mm}$ , and

$$\bar{p} = \pi \frac{E}{1-\nu^2} \frac{\Delta}{2l} = \pi \frac{50}{1-0.3^2} \frac{0.005}{2 \times 1} \text{GPa} \approx 431.5\text{MPa}$$

for  $\Delta = 0.005\text{mm}$ .

Our corresponding values of the average pressures are 213.8MPa and 425.9MPa respectively, and the corresponding relative errors are -0.93% and -1.30% respectively. We can expect that when  $\Delta$  decreases downward to 0, the relative error could also decrease. This shows that our numerical results from the "self-consistent model" have good accuracy.

The results indicate that for elastic contact, the average pressure is approximately proportional to  $\Delta$ . In the problem we analyzed, the deformation is very small because  $\Delta$  is in the order  $O(10^{-3})$  of  $l$ , so the strain in the upper half-space is proportional to  $\Delta$  if we ignore the higher order term.

But when the materials are ideal-plastic, the average pressure is close to the yielding stresses of the materials.

#### V. CONCLUSION REMARKS AND DISCUSSION

Our numerical results show that the "self-consistent model" can capture the interaction between multiple asperities in the rough surface contact. The numerical results for the two plane strain cases with  $\alpha = 0$  coincide well with the existing analytical solution.

The results show that for elastic contact, the average pressure for full contact is proportional to ratio  $\Delta/l$ . But for the plastic contact, the average pressure is determined mainly by the yielding stress for full contact and is close to the yielding stress.

For both the elastic contact and the plastic contact, the fatter configuration (smaller  $\alpha$ ) has larger average pressure for full contact.

Generally the real contact area is very small [2], for instance, in the order of 1% or less. Fortunately, the wafer bonding technique in MEMS packaging industry generates large contact area up to full contact. Our results can be applied to this technique straightforwardly.

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