# A Robust Frequency Offset Estimation Scheme for OFDM System with Cyclic Delay Diversity

Won-Jae Shin and Young-Hwan You

Abstract—Cyclic delay diversity (CDD) is a simple technique to intentionally increase frequency selectivity of channels for orthogonal frequency division multiplexing (OFDM).This paper proposes a residual carrier frequency offset (RFO) estimation scheme for OFDMbased broadcasting system using CDD. In order to improve the RFO estimation, this paper addresses a decision scheme of the amount of cyclic delay and pilot pattern used to estimate the RFO. By computer simulation, the proposed estimator is shown to benefit form properly chosen delay parameter and perform robustly.

Keywords-OFDM, cyclic delay diversity, FM system, synchronization

#### I. INTRODUCTION

**O**RTHOGONAL frequency division multiplexing (OFDM) is commonly used for broadband wireless transmission nowadays because of the ability of combating the multi-path channel problem and has been adopted by many high speed transmission standards such as 802.11a/g WLAN and ultra wideband (UWB) [1][2]. Besides, OFDM is extensively used in broadcasting domain, such as digital audio broadcasting (DAB), digital radio mondiale (DRM), and terrestrial digital video broadcasting (DVB-T) [3]-[6].

In wireless communications, transmit diversity techniques are used to combat channel fading and improve transmission reliability. One of those methods is cyclic delay diversity (CDD), which is effortlessly applicable to the existing broadcasting systems without modifications to its physical layer because its signal processing is performed on the baseband OFDM symbol in time domain [7]. In contrast to other diversity techniques, CDD transforms a multiple-input single-output (MISO) frequency-selective channel into an equivalent singleinput single-output (SISO) channel with increased frequencyselectivity so that the available spatial diversity is transformed into additional frequency diversity [7]-[10].

However, the advantages of the OFDM system are assured only when the transmitter and the receiver are precisely synchronized. The OFDM system is very sensitive to symbol timing errors and frequency offsets. After coarse synchronization in OFDM systems, there might still be a residual frequency offset (RFO) which can seriously degrade the performance of the systems. The uncompensated RFO can cause inter-carrier interference (ICI), and large signal constellation rotation. The RFO is introduced by small differences in oscillator frequencies between the transmitter and receiver, which causes the

Won-Jae Shin and Young-Hwan You are with the Department of Computer Engineering, Sejong University, Seoul, Korea, e-mail: (wjshin@sju.ac.kr and yhyou@sejong.ac.kr). time and subcarrier to vary the phase rotation as well as causing a fast Fourier transform (FFT) window shift [11]-[16].

DRM is an OFDM based digital radio standard which was designed to fit in with the existing AM broadcast band plan [6]. DRM consortium intended DRM to be used on FM frequency up to 174 MHz, called DRM plus (or DRM robustness mode E). This mode requires low transmitter powers and cost for mobile TV than via either DMB or DVB-H at UHF frequencies.

DRM+ will be used on FM frequency up to 174MHz. This bandwidth is much smaller than the coherence bandwidth of the channels in typical urban environment that is about a few MHz. Thus, the major challenge in FM band is to cope with severe flat fading conditions. One possible solution to combat flat fading is CDD. However, the increased frequency selectivity of channel characteristic can be a problem for post FFT estimation such as the channel estimation and residual carrier frequency offset (RFO) estimation [11]-[16].

This paper deals with how to determine the delay parameter and the pilot pattern for improved RFO estimation in OFDMbased DRM+ systems with two transmit antennas. Delay selection is performed by finding the delay which makes different power of channel transfer function (CTF). Afterward, estimation of the RFO is carried out once in four repetitive pilot symbol which maximizes the power of CTF. Thus, using these conditions will improve the performance of RFO synchronization scheme. Finally we analyze the performance of estimation algorithm of RFO. Based on the simulation, the robust RFO estimation scheme is tested and verified within the framework of the OFDM parameter and pilot pattern defined in the DRM+ system.

#### **II. SIGNAL MODELS**

When the CDD scheme is adopted in OFDM systems, cyclic delayed copies of the time domain data signal  $x_l(n)$  over further transmit antennas are transmitted, thus the *l*-th OFDM signal of the *t*-th transmit antenna is

$$x_{l,t}(n) = \frac{1}{\sqrt{N_T}} x_l(n - \delta_t)_N, \quad t = 0, 1, \cdots, N_T - 1, \quad (1)$$

where N is the number of FFT (IFFT) points,  $(\cdot)_N$  is the modulo-N operation,  $N_T$  is the number of transmit antennas, and  $\delta_t$  is the amount of cyclic delay at the *t*-th antenna. As discussed in many literatures [7]-[10], choosing

$$\delta_{t+1} = N/N_T + \delta_t, \quad t = 0, 1, \cdots, N_T - 1,$$
 (2)

maximizes delay between transmit antennas.

Manuscript received April 19, 2005; revised January 11, 2007.

By assuming perfect symbol timing recovery at the receiver, the received signal of the *l*-th symbol in the presence of small RFO is given by [12],[13]

$$R_l(k) \cong H_l(k) X_l(k) e^{j2\pi\Delta_c(lN_s + N_g)/N} + W_l(k), \quad (3)$$

where  $N_g$  is the number of guard interval samples,  $N_s = N + N_g$ ,  $\Delta_c$  is the RFO normalized by subcarrier spacing and  $W_l(k)$  is the contribution of the zero-mean AWGN with variance  $\sigma_W^2$ . Therefore, the equivalent CTF is  $H_l(k) = (\sqrt{N_T})^{-1} \sum_{t=0}^{N_T-1} H_l^t(k) e^{j2\pi k \delta_t/N}$ , where  $H_l^t(k)$  is the frequency channel response from the *t*-th transmit antenna with zero-mean and variance  $\sigma_H^2$ .

# III. CONVENTIONAL RFO ESTIMATION METHOD

A post-FFT pilot aided RFO estimator proposed in [12] is considered in this paper. Since DRM+ system does not have frequency reference cell (FRC), so that the RFO estimation is performed by gain reference cell (GRC). Then a cumulative correlation on GRCs is

$$\Lambda_l = \sum_{k \in S_p} R_l^*(k) R_{l+D_t}(k)$$

$$\cong \sum_{k \in S_p} E_s |H_l(k)|^2 e^{j2\pi\Delta_c D_t \rho} + \hat{W}_l(k),$$
(4)

with

$$\hat{W}_{l}(k) = H_{l}^{*}(k)X_{l}^{*}(k)W_{l+D_{t}}(k)e^{-j2\pi\Delta_{c}D_{t}(lN_{s}+N_{g})/N} + H_{l+D_{t}}(k)X_{l+D_{t}}(k)W_{l}^{*}(k)e^{j2\pi\Delta_{c}D_{t}((l+D_{t})N_{s}+N_{g})/N} + W_{l}^{*}(k)W_{l+D_{t}}(k),$$
(5)

where  $D_t$  is the periodicity of the GRC pattern in time domain,  $\rho = (N + N_g)/N$  and  $S_p$  is the set of GRCs having  $N_p$  elements. For simple description, we have assumed that  $H_l(k) \approx H_{l+D_t}(k)$  in (4).

By using (4), the estimation of RFO can be calculated by [12]

$$\hat{\Delta}_r = \frac{1}{2\pi\rho D_t} \arg\left\{\sum_{k\in S_p} \Lambda_l\right\},\tag{6}$$

where  $arg\{\cdot\}$  denotes the argument of a complex number.

#### IV. PROPOSED RFO ESTIMATION METHOD

In order to improve the performance of the RFO estimator in (6) when the CDD is adopted in DRM+, we determine how to select the amount of cyclic delays for two-antenna case. For simplicity, we consider the CDD scheme employing  $N_T = 2$ transmit antennas and zero delay at the first transmit antenna, i.e.,  $\delta_0 = 0$  is assumed without loss of generality.

Assuming that channels  $\{H_l^t(k), t=0,1\}$  are frequency flat, the CTF in (4) can be decomposed into

$$\sum_{k \in S_p} |H_l(k)|^2 = \frac{N_p}{2} (|H_l^0(k)|^2 + |H_l^1(k)|^2) + S(N_p) \sqrt{A^2 + B^2} \cos\left(2\pi (k_1 + k_{N_p})\delta_1 / N + \theta\right),$$
(7)

with

$$A = H_l^{0,I}(k)H_l^{1,I}(k) + H_l^{0,Q}(k)H_l^{1,Q}(k),$$
(8)

and

$$B = -H_l^{0,I}(k)H_l^{1,Q}(k) - H_l^{0,Q}(k)H_l^{1,I}(k),$$
(9)

where  $S(N_p) = \sin(\pi N_p D_f \delta_1 / N) / \sin(\pi D_f \delta_1 / N)$ ,  $\theta = \tan^{-1}(B/A)$ ,  $x^I$  and  $x^Q$  denote real and imaginary part of x, respectively. As we can see in (8), the channel term of the received signal heavily depends on the cyclic delay of second antenna  $\delta_1$  in the second term on the right-hand side for given system parameters  $N_p$ ,  $D_f$ , and N. If we choose  $\delta_0 = 0$  and  $\delta_1 = N/2$  as in (2), one can find that the CTF in (8) has same value at every GRC patterns.

In order to improve the performance of the RFO estimator, we decide  $\delta_1$  such that the second term on the right hand side of (8) has different values at each GRC pattern. To this end, the delay parameter of second antenna is chosen by

$$\hat{\delta}_1 = \left\langle \frac{N \cdot i}{D_f} \right\rangle, \quad i = 1, 2, \cdots, D_f - 1$$
 (10)

where  $\langle x \rangle$  rounds x to the nearest integer.

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After that RFO estimation is carried out by one of GRC pattern which has maximum power of CTF

$$\hat{l} = {}^{\arg\max}_{l} \left\{ \sum_{k \in S_p} |H_{(l)_4}(k)|^2 \right\}$$
(11)

$$\hat{\Delta}_r = \frac{1}{2\pi\rho D_t} \arg\left\{\sum_{k\in S_p} \Lambda_{\hat{l}}\right\}$$
(12)

Since  $\sqrt{A^2 + B^2} = |H_l^0(k)||H_l^1(k)|$  in (8) and  $S(N_p) \approx N_p$ in the case of integer valued (11), then we can approximate the maximum value of (8) by

$$\mathcal{H}_{\hat{l}}(k) = \sum_{k \in S_p} |H_{\hat{l}}(k)|^2 \approx \frac{N_p}{2} \Big\{ |H_{\hat{l}}^0(k)| + |H_{\hat{l}}^1(k)| \Big\}^2, \quad (13)$$

Then, based on (4) and (13), we obtain

$$\Lambda_{\hat{l}} \approx \frac{E_s N_p \mathcal{H}_{\hat{l}}(k)}{2} e^{j2\pi\Delta_c D_t \rho} + \sum_{k \in S_p} \hat{W}_{\hat{l}}(k).$$
(14)

## V. SIMULATION RESULTS AND DISCUSSIONS

To verify the usefulness of the algorithm presented in the previous section, we simulate the proposed frequency estimator under urban channel with maximum channel delay  $3\mu$ s, which is widely in the performance analysis of FM [17]. N = 214,  $N_g = 24$ ,  $N_p = 14$ ,  $D_t = 4$ , and  $D_f = 16$  are chosen according to the DRM+ draft. For antenna configuration,  $N_T =$ 2 is assumed and the antennas are placed such that their CTFs can be considered as uncorrelated. In the following examples,  $\Delta_c = 0.01$ , BW = 95 kHz, and center frequency is 90MHz.

Fig. 1 depicts the MSE of the estimated RFO versus  $\delta_1$  in urban channel when the mobile speed v=0km/h and SNR=40dB. From the figure, there are  $\delta_1$ 's that gives the lower MSE, i.e.  $\delta_1 = \{13, 27, 40, 67, 80, 94, 120, 134, 147, 174, 187, 201\}$ , which exactly coincides with the values obtained by (10). However, when every GRC patterns have same CTF power, i.e.  $\delta_1 = \{53, 107, 161\}$ , there are no performance differences. As we can see in Fig. 1, there is a trade off between the



Fig. 1. MSE of the RFO estimators versus cyclic delay  $\delta_1$  under urban channel.



Fig. 2. MSE of the RFO estimator versus mobile speeds

performance of RFO estimator and the amount of spatial diversity.

Fig. 2 shows the MSE of the RFO estimation scheme with respect to various mobile speeds v. The increase of v causes severe performance degradation for frequency estimators at high SNR, which primarily stems from the impact of the time selectivity of the channel. The proposed scheme still outperforms the conventional scheme regardless of the mobile speed.

# VI. CONCLUSION

In this paper, an estimation problem for OFDM-based DRM+ system due to employing CDD are addressed. Then, we suggested an improved frequency tracking scheme by selection of cyclic delay and pilot pattern of DRM+ system. The performance of the proposed RFO estimator scheme was compared with that of the conventional RFO estimator scheme. It has been found by extensive simulations that the proposed RFO estimation scheme endowed with the properly chosen cyclic delay and the pilot pattern has the advantages of being more robust against frequency selective fading and outperforms the conventional estimation scheme.

### ACKNOWLEDGMENT

This work was supported by the IT R&D program of MKE/KEIT [10039988], and is supported by Seoul R&BD Program (SS100009)

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# International Journal of Information, Control and Computer Sciences ISSN: 2517-9942 Vol:6, No:5, 2012

**Won-Jae Shin** was born in Seoul, Korea in 1982. He received the B.S, M.s. degrees in computer engineering, Sejong University, Seoul, Korea, in 2007 and 2009. He is working toward to PH.D. degree in the Department of computer engineering, Sejong University, Seoul, Korea. His research interests are in the areas of wireless communication system design and wireless digital broadcasting systems.

Young-Hwan You received the B.S., M.S., and Ph.D. degrees in electronic engineering from Yonsei University, Seoul, Korea, in 1993, 1995, and 1999, respectively. From 1999 to 2002 he had been a senior researcher at the wireless PAN technology project office, Korea Electronics Technology Institute (KETI), KyungGi-Do, Korea. Since 2002 he has been an associate professor of the Department of Internet Engineering, Sejong University, Seoul, Korea. His research interests are in the areas of wireless/wired communications systems design, spread spectrum transceivers, and system architecture for realizing advanced digital communications systems, especially, for wireless personalarea networks (WPAN) and wireless internet.