# A Program for Solving problems in Inorganic Chemistry based on Knowledge Base

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Abstract—The Model for Knowledge Base of Computational Objects (KBCO model) has been successfully applied to represent the knowledge of human like Plane Geometry, Physical, Calculus. However, the original model cannot easyly apply in inorganic chemistry field because of the knowledge specific problems. So, the aim of this article is to introduce how we extend the Computional Object (Com-Object) in KBCO model, kinds of fact, problems model, and inference algorithms to develop a program for solving problems in inorganic chemistry. Our purpose is to develop the application that can help students in their study inorganic chemistry at schools. This application was built successful by using Maple, C# and WPF technology. It can solve automatically problems and give human readable solution agree with those writting by students and teachers.

*Keywords*—artificial intelligence, automated problem solving, knowledge base system, knowledge representation, reasoning strategy, education software/educational applications.

## I. INTRODUCTION

SUPPORTING program in education is really important for students and teachers. After recently surveying, many students agree that they need a computer program can help them in learning inorganic chemistry. The program have ability for reading problems and solving them automaticaly, then give students the results in the way they usually use in schools

To develop the program can solve problems automatically, the techniques of artifical intelligence, knowledge presentation and reasoning methods are very important and nessessary. In this case, the main programing tool is Maple (version 15), which is a symbolic programing language, and can help us in solving equations, logic expressions automatically. Morever, to make a program can read problems, show solutions like human and have friendly user interface, we have used C# language and WPF technology .

In this article, we present how to build a program has introduced above. First, we base on KBCO model [4] and make new extensions to build knowledge base used for inorganic chemistry field. Then, we write about the problem model and use design the reasioning methods to develop the inference engine. At last, we introduce something about the application and the example.

Our research have extended the KBCO model to apply in inorganic chemistry field, proposed new reasoning methods

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used in inference engine. The important extensions are type of attribute in Com-Object, Set computational relations and some new kinds of fact. We have developed the education application which can become a part of system that contain many supported applications in education. Morever, we hope our research will help us easy to apply KBCO model in others fields

### II. MAIN FUNCTIONS OF PROGRAM

Main function of program is to solve some problems automatically in inorganic chemistry field. User only declare hypothesis and goal of problems by using a simple languague but strong enough for specifying them. The hypothesis need consist of objects (all objects appear in problems such as: elements, solutes, mixtures, gas or solid products in reactants etc...), and relations between them or between their attributes. It can alse contain formulas, relation formulas and some facts about the objects and their attributes. The goal need to be objects or attributes of objects which user want to identify or compute.

After specifying a problem, user click to a button to request the program to solve it automatically. The program will give a human readable result and solution when it finish solving. Of course, user can ask the program to save the problems and solutions for retrieval later.

# III. KNOWLEDGE REPRESENTATION

There are many methods and techniques ([7], [8], [1]) for knowledge representation, especially the KBCO model which was introduced in [4] and was successfully used in [2], [3], [5], [6]; is the most suitable to represent the Chemical knowledge domain and design inference engine for the solving problems, main feature described in section II.

# A. Extensive Computational Object

The structure of a Com-object (modeled in [4]) is defined as a tuple 4 (Attrs, F, Facts, Rules) where Attrs is a set of attributes, F is a set of equations called computation relation, Facts is a set of properties or event of object, and Rules is a set of deductive rule on facts.

1) Collection kinds attribute: Each attribute in Attrs of a Com-object is one of two kind below:

Computational information

like numeric (real, integer, complex, ...), boolean ... Another lower level object

for example Com-object AB represents a segment of

line has attribute start and end correspondingly to 2 objects A and B which represent 2 end points.

The second kind of attributes represents the constructional relation of two Com-objects. Because the Attrs set is declared explicitly for a class of Com-objects due to the number of this kind relations also statically defined. But in the real world and particularly in Chemical field, there are exist objects that have the number of kinds of relations indefinitly. For example: a solution object mays have many substances as its solutes as

Because of the above problem, beside the 2 kinds (1) Computational information and (2) Lower level object, a attribute of Attrs may also is (3) a Collection. There is 3 kinds of a "collection" attribute can be:

represent a ordered set of elements. List<T> tell that the attribute is a list of elements type T.

Set represent a unordered set of elements. Set <T> tell that the attribute is a set of elements type T.

represent a key-value pair of dictionary. Dict<TK, TV> tell that the attribute is a dictionary where the keys is type TK and the values is type TV.

For example, a MIXTURE object has the attribute Component is type of Set<SUBTANCE> which represents the set of substances mixed in the mixture. A EXPERIEMNT object has the attribute State is List<Set<SUBSTANCE>> to recorded the states (each state is a set of substances) from the begining to the end of the experiment. Or the  $\rho$  attribute of a SOLUTION object is a Dict<SUBSTANCE, real> is the mole concentration table maps the substance s to it's mole concentration  $\rho_s$ .

2) Set computational relation: F is a set of computational relations. Each computational relation is simply a computational equation among the computational attributes inside a Com-object.

Corresponding the extension of attribute's kind, we introduce a new kind of computational relations call Set computational relation. There are two kind of Set computational

Definition 1 (Set computational relation kind 1):

$$\forall x \in f(la_1, \dots, la_n) \quad fr(x, na_1, \dots, na_m)$$

where

is a "collection" type attribute (for  $1 \le i \le n$ ).  $la_i$ 

is a function that return a (finite) set. f takes part of f a filter, a combinator, ...

is a computational value type attribute (for  $a \leq j \leq$ 

fris a fuction that return a formula relations.

An example of set computational relation kind 1:

$$\forall \{x,y\} \in \binom{S}{2}, \quad \frac{r_x}{r_y} = \frac{n_x}{n_y}$$

Notation: with S is a set and k is a positive interger then  $\binom{S}{k} = \{X : X \subseteq S \text{ and } |X| = k\}$ 

Definition 2 (Set computational relation kind 2):

$$fr(la_1,\ldots,la_n,na_1,\ldots,na_m)$$

where

is a "collection" type attribute (for  $1 \le i \le n$ ).  $la_i$ 

is a computational value type attribute (for  $a \leq j \leq$  $na_i$ 

is a function that return a formula relation. fr usually contain the set operator like  $\sum_{i}$ ,  $\prod_{j}$ ,  $\bigcup_{i}$ ...

An example of set computational relation kind 2:

$$m_{\text{mixture}} = \sum_{s \in Component} m_s$$

- 3) Com-Object Structure: The structure of a Com-object can be modeled by (Attrs, F, Facts, Rules) where
  - Attrs is a set of attributes. Attrs can be partitioned to three seperate sub-collections

$$Attrs = Attrs_N \bigcup Attrs_O \bigcup Attrs_S$$

where

 $Attrs_N$  is a set of computational kind attributes.

 $Attrs_O$  is a set of lower level object kind attributes.

 $Attrs_S$  is a set of collection kind attributes.

•  $F = F_E \bigcup F_S$  is a set of computational relations which is constructed from two subset

is a set of equation kind of computational rela-

 $F_S$ is a set of set computational relations. It is combined from 2 separate subset  $F_{S_1}$  and  $F_{S_2}$ corresponding two kind of set computational relation.

- Facts is set of properties or event of objects
- Rules is set of deductive rules on facts.

For example, a model for class of a SOLUTION Com-object are as follows:

$$Attrs = Attrs_N \bigcup Attrs_O \bigcup Attrs_S$$

 $Attrs_N = \{m_{Solution}, V, m_{Solutes}, pH, \rho_0\}$  (solution's mass, volume of solution, solutes's mass, mass

 $Attrs_S = \{Solutes, c, \rho, w\}$  where Solutes is type set (SUBSTANCE), is a set of dissolved substance;  $c(\rho, w)$  is molar concentration (mass concentration, mass fraction), is type of dict(SUBSTANCE  $\rightarrow$ NUMERIC). For each dissolved SUBATANCE object s in this SOLUTION object then  $c_s$  is molar concentration of s in solution.

 $\{Solvent\}$  where Solvent is a  $Attrs_O$ = SUBSTANCE object.

$$F = F_E \bigcup F_S$$

$$F_E = \left\{ \rho_0 = \frac{m_{Solution}}{V} \right\}$$
  
$$F_S = F_{S_1} \bigcup F_{S_2}$$

$$F_{S_{1}} = \left\{ \begin{array}{c} \forall s \in Solutes \, c_{s} = \frac{n_{s}}{V}, \\ \forall s \in Solutes \, \rho_{s} = \frac{m_{s}}{V}, \\ \forall s \in Solutes \, w_{s} = \frac{m_{s}}{m_{Solution}}, \\ \forall s \in Solutes \, c_{s} = \frac{\rho_{s}}{M_{s}}, \\ \forall s \in Solutes \, w_{s} = \frac{\rho_{s}}{\rho_{o}} \end{array} \right\}$$

$$F_{S_2} = \left\{ \begin{array}{c} \rho_0 = \sum_{s \in Solutes} \rho_s, \\ m_{Solutes} = \sum_{s \in Solutes} m_s \end{array} \right\}$$

 $Facts = \{m_{Solution} > 0, \rho_0 > 0, V > 0, m_{Solutes} > 0\}$  $Rule = \{...\}$ 

4) Behaviors: As a major characteristics, a Com-object must be equipped abilities to solve problems by applying the internal dedutive information F, Rules with the current state of the attributes. Because of the structure's extension of Comobject, the behaviors of object must also be able to handle the new Set computational relations. The algorithms 1 and 2 is the simple and naive appoarch algorithm to deduce information from a Set computational relation f.

# **Algorithm 1** Deduce from $f \in F_{S_1}$

- 1: if  $la_i$  is determined  $\forall i$  then
- 2:  $C \leftarrow f(la_1, \dots, la_n)$
- 3: **return**  $\{fr(x, na_1, \dots, na_m) | \forall x \in C\}$
- 4: end if

# Algorithm 2 Deduce from $f \in F_{S_2}$

- 1: if  $la_i$  is determined  $\forall i$  then
- 2: **return**  $fr(la_1,\ldots,la_n,na_1,\ldots,na_m)$
- 3: end if

For example, when a SOLUTION object has  $Solutes = \left\{ \mathrm{Na}^+, \mathrm{Ba}^{2+}, \mathrm{OH}^- \right\}$  then when apply the  $f = \left( \forall s \in Solutes \, c_s = \frac{n_s}{V} \right)$  will give us 3 equation kind formular relations  $\left\{ c_{\mathrm{Na}^+} = \frac{n_{\mathrm{Na}^+}}{V}, c_{\mathrm{Ba}^{2+}} = \frac{n_{\mathrm{Ba}^{2+}}}{V}, c_{\mathrm{OH}^-} = \frac{n_{\mathrm{OH}^-}}{V} \right\}$ .

## B. Chemistry's Knowledge Model

1) Components of model: The Chemistry's Knowledge model is based on the KBCO model with the Extensive Computational object model in III-A.

Definition 3 (Chemistry's KB model):

- C is a set of concepts of Com-object. Each concepts is a class of Com-objects.
- H is a set of hierarchy relation on the concepts.
- R is a set of relations on the concepts.

Rules is a set of rules.

The concepts in C include the SUBSTANCE class and its derived, the ELEMENT, SOLUTION, MIXTURE, REACTION and EXPERIMENT class. The H represent the ordered relation "specialization" of the concepts (especially the SUBTANCE and its subclass) in C.

The R other kind relation on C. For example the "similar" relation of 2 solutions objects tell the objects has the same kind of solutes, same of mole concentration for each solute. And finally the R represent for deductive rules usually in form **if** hypothetical-facts **then** conclusive-fact

- 2) Kinds of Facts: Because the lacking of components to the general KBCO model in [4], the requirement of program does not meet all of 11 kinds in [4] except the first 6 kinds of facts do be useful. Beside that, there are more 4 new kinds of facts that necessary to define for the chemistry's domain. The kinds of facts are as follows: (the first 6 is derive from [4]).
  - Fact of kind 1: information about object kind, like H<sub>2</sub>SO<sub>4</sub> is a acid.
  - Fact of kind 2: a determined of an object or an attribute of an object.
  - Fact of kind 2: a non-determined of an object or an attribute of an object.
  - Fact of kind 3: a determined of an object or an attribute of an object by a value or a constant, for example:  $n_{\rm Ba(OH)_2} = 0.5$ .
  - Fact of kind 4: equality on object or attributes of object, for example: n<sub>FeO</sub> = n<sub>FeOO2</sub>.
  - Fact of kind 5: a dependence of an object on other object by a general equation. For example: n<sub>OH</sub> = n<sub>NaOH</sub> + 2n<sub>Ba(OH)</sub>.
  - Fact of kind 6: a relation on objects or attributes of the objects. For example: solution X is "similar" with solution Y.

Following is the 4 new kinds of facts.

- Fact of kind 7: a comparison of an attribute with a constant, like  $n_{\rm H^+} > 0$ .
- Fact of kind 8: a determined of the domain of an object or an attribute. For example: element  $M \in \{\text{Na, Ba, Ca, K}\}.$
- Fact of kind 9: a fact of a collection kind attribute is a subset of a constant set or another collection kind attribute. For example: reaction  $r_1$ 's attribute Reactants is **subset** of experiment e's current state  $State_0$ .
- Fact of kind 10: a determined of the component of a collection kind attribute. For example: the solution X's Solutes attribute containts 2 kinds of salt. This fact has 3 parameter a, n, c where a is the collection kind attribute (X's Solutes); c is a concept in C; n can be a positive integer (in the example, n = 2), 0 (mean there are no c object in a) or none (mean there are c objects in a but the number of c objects is unknown).

### IV. MODELING PROBLEMS & REASONING

### A. Problems Model

The *Network of Computational Objects* in [4] is use as a model of the problems; and be able to specificate almost problems in Chemistry high school education program. The structure of problem model is denoted by:

$$(O,F)\to G$$

- O is a set of Com-objects.
- F is a set of facts or computational relations among the objects in O.
- G is a goal set, it consist the target attributes, objects that need to find.

For example:

Completely absorbed 4.48 liters of  $CO_2$  (at TOR) in 500 ml solution of NaOH 0.1 M and  $Ba(OH)_2$  0.2 M, generated m gram precipitate. The value of m is: A. 19,70. B. 17,73. C. 9,85. D. 11,82.

The above problem can be considered on the network of Comobjects (O, F) as follows:

$$O = \left\{ \begin{array}{ll} subCO2 & \texttt{SUBSTANCE CO}_2 \\ subNaOH & \texttt{SUBSTANCE NaOH} \\ subBaOH2 & \texttt{SUBSTANCE Ba(OH)}_2 \\ solX & \texttt{SOLUTION} \\ e & \texttt{EXPERIMENT} \\ objP & \texttt{unknown} \end{array} \right.$$

$$F = \begin{cases} subCO2_V = 4.48 \\ solX_{Solutes} = \{subNaOH, subBaOH2\} \\ solX_V = 0.5 \\ solX_{MC}[subNaOH] = 0.1 \\ x_{MC}[subBaOH2] = 0.2 \\ e_{Input} = \{subCO2, solX\} \\ objP = e_{SolidOutput} \end{cases}$$

and goal  $G = \{objP_m\}.$ 

### B. Inference modules

There are 4 important modules in the inference engine: Apply Rules, Deduce Inside Objects, Deduce computational relations and a bundle of deduction methods.

The Apply Rules is try to apply the rules in R. The module try to match the current facts with the hypothetical-facts of the rule. If all the hypothetical-facts are satisfied, then we get the conclusive-facts and record the case. If there exist at least one hypothetical-fact that inconsistent with the current facts, then in this case, the rule is never meet, we also record the case. Recording the case help to avoid multiple-time applying the same case to the rule. If the module can not determined the case is satisfied or not, it ignore the case in this time.

The Deduce inside objects module is try to use the solving problems behavior of Com-object to get more deduction. Also the Deduce computational relations is try to apply the computational relations among the objects.

### V. APPLICATION

The program for solving problems in inorganic chemistry was designed include:

• Four main components of program is knowledge base, inference engine, user inteface and utility functions. Knowledge base contain all of definations of inorganic chemistry. Each defination is a Com-Object such as: element, substance, mixture, solution etc... Inference engine or knowledge processing module is written in Maple, it has to read knowledge base, read problems and find the solution automatically. User interface is a window for inputting problems above special language and another window for showing solutions of problems and figures.

- Utilify functions support in processing special data in chemistry as the chemistry formula, the solution table or the periodic.
- The program require users declare hypothesis above special language, it's very simple and strong enough for specifying problems. The hypothesis can consist of objects, relations between objects or attributes. The goal can be to compute the attributes. After specifying problem, user clicks a button to request program solving problems automatically. After finishing, the program will give users the solutions and results. They are easy to read and understand by many students and teachers becouse of using a simple language and show solutions steps by steps.

Example: continue the previous example problem, the Solution found by the system as follow:

SUBSTANCE objects information:

Object	Formular
subCO2	$CO_2$
subNaOH	NaOH
subBaCO3	$BaCO_3$
subBaOH2	$Ba(OH)_2$
subBa2b	Ba <sup>2+</sup>
subBa2a	Ba <sup>2+</sup>
subCO32	$CO_3^{2-}$
subHCO3a	$HCO_3^-$
subHCO3b	$HCO_3^-$
subNa	Na <sup>+</sup>
subOHa	$OH^-$
subOHb	$OH^-$

SOLUTION objects information:

Object	Solutes	
solX	subNaOH, subBaOH2	
	subBa2a, subNa, subOHa	
solOut	subBa2b, subHCO3b, subNaS	

EXPERIMENT object e

$$Input = \{ subCO2, solX \}$$

State 1 = { subCO2, subBa2a, subNa, subOHa }

$$\begin{array}{c|cccc} CO_2 & 0.2 (mol) & from~(2) \\ Ba^{2+} & 0.1 (mol) & from~hypothesis \\ Na^+ & 0.05 (mol) & from~hypothesis \\ OH^- & 0.25 (mol) & from~(7) \\ \end{array}$$

Phase  $1 = \{ r1 \}$ 

$$CO_2$$
 +  $OH^ \rightarrow$   $HCO_3^-$   
 $x_1$   $x_1$   $x_1 = 0.5$ 

State  $2 = \{ \text{ subBa2a, subHCO3a, subNa, subOHb } \}$ 

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Ba<sup>2+</sup>
                          0.1(mol)
                                            from hypothesis
            HCO<sub>3</sub><sup>1-</sup>
                          0.2(mol)
                                            from hypothesis
            Na<sup>+</sup>
                          0.05(mol)
                                            from hypothesis
            OH-
                          0.05(\text{mol})
                                           from (6),(9)
      Phase 2 = \{ r2 \}
                        OH-
                                                             CO_3^2
      HCO_3^- +
                                         H_2O
                                                          x_2 = 0.05
                                           x_2
       State 3 = { subBa2a, subCO32, subHCO3b, subNa }
            Ba<sup>2+</sup>
                          0.1(mol)
                                            from hypothesis
            CO_3^{2-}
                          0.05(mol)
                                            from (11),(13)
            HCO<sub>2</sub><sup>1</sup>
                          0.15(mol)
                                            from hypothesis
            Na<sup>+</sup>
                         0.05(\text{mol})
                                           from hypothesis
       Phase 3 = \{ r3 \}
                              CO_3^{2-}
                                                    BaCO<sub>3</sub>
                                                  x_3 = 0.05
                                 x_3
       State 4 = { subBaCO3, subBa2b, subHCO3b, subNa
}
            BaCO<sub>3</sub>
                          0.05(mol)
                                            from (15), (16)
            Ba^{2+}
                          0.05(mol)
                                            from hypothesis
            HCO<sub>3</sub><sup>1-</sup>
                          0.15(mol)
                                            from hypothesis
            Na<sup>+</sup>
                         0.05(\text{mol})
                                           from hypothesis
       Output Solid = subBaCO3
       Output Solution = solOut
   The solving steps of the problems:
   (1) reaction r1 is complete \rightarrow n_{subsCO2} = x_1
   (2) V_{subCO2} = 4.48 and V_{subCO2} = 22.4 * n_{subsCO2}
\rightarrow n_{subsCO2} = 0.2
\begin{array}{lll} 7 & n_{subSCO2} = 0.5 \\ (3) & V_{solX} = 0.5 \text{ and } \rho_{solX}^{subBaOH2} = 0.2 \text{ and } \\ \rho_{solX}^{subBaOH2} = n_{subBaOH2}/V_{solX} \rightarrow n_{subBaOH2} = 0.1 \\ (4) & V_{solX} = 0.5 \text{ and } \rho_{solX}^{subNaOH} = .1 \text{ and } \rho_{solX}^{subNaOH} = 0.1 \end{array}
n_{subNaOH}/V_{solX} \rightarrow n_{subNaOH} = 0.05
   (5) n_{subsCO2} = 0.2 and n_{subsCO2} = x_1 \rightarrow x_1 = 0.2
    (6) after reaction r1 \rightarrow n_{subOHb} = n_{subOHa} - x_1
   (7) n_{subNaOH} = 0.05 and n_{subBaOH2} = 0.1 and
n_{subOHa} = n_{subNaOH} + 2 * n_{subBaOH2} \rightarrow n_{subOHa} =
.25
   (8) reaction r2 is complete \rightarrow n_{subOHb} = x_2
   (9) n_{subOHa} = .25 and n_{subOHb} = n_{subOHa} - x_1 and
x_1 = .2 \rightarrow n_{subOHb} = 0.05
   (10) n_{subOHb} = 0.05 and n_{subOHb} = x_2 \rightarrow x_2 = 0.05
   (11) after reaction r2 \rightarrow n_{subCO32} = x_2
   (12) reaction r3 is complete \rightarrow n_{subCO32} = x_3
   (13) n_{subCO32} = x_2 and x_2 = 0.05 \rightarrow n_{subCO32} =
0.05
   (14) n_{subCO32} = 0.05 and n_{subCO32} = x_3 \rightarrow x_3 =
0.05
   (15) after reaction r3 \rightarrow n_{subBaCO3} = x_3
   (16) n_{subBaCO3} = x_3 and x_3 = 0.05 \rightarrow n_{subBaCO3} =
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0.05

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(17) M_{subBaCO3}=197 and n_{subBaCO3}=0.05 and n_{subBaCO3}=m_{subBaCO3}/M_{subBaCO3} \rightarrow m_{subBaCO3}=9.85
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### VI. CONCLUSION

The KBCO model with extensions about type of attribute in Com-Object, kinds of fact and kind of relation give us more confortable when we represent knowledge on the computer. Beside, problem model and reasoning althorithms were upgraded to use in new extensions easily. The extensions of model and althorithms are used to apply in solving problems automatically in inorganic chemistry field successfully. We have built a program which can read problems, solve problems automatically and give the result in the way like people. In testing period, the program has solved many common problems in inorganic chemistry rapidly.

The program was built in Maple, C# language and WPF technogory, has a friendly user interface and easy to use by students and teachers. Our purpose in furture is continue to upgrade the model and althorithms to solve more problems. This methods can be used for another programs for studying knowledge and solving problems in the others fields.

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