A Practical Scheme for Transmission Loss Allocation to Generators and Loads in Restructured Power Systems

M.R. Ebrahimi and M. Ehsan

Abstract—This paper presents a practical scheme that can be used for allocating the transmission loss to generators and loads. In this scheme first the share of a generator or load on the current through a branch is determined using Z-bus modified matrix. Then the current components are decomposed and the branch loss allocation is obtained. A motivation of proposed scheme is to improve the results of Z-bus method and to reach more fair allocation. The proposed scheme has been implemented and tested on several networks. To achieve practical and applicable results, the proposed scheme is simulated and compared on the transmission network (400kv) of Khorasan region in Iran and the 14-bus standard IEEE network. The results show that the proposed scheme is comprehensive and fair to allocating the energy losses of a power market to its participants.

Keywords—Transmission Loss, Loss Allocation, Z-bus modified matrix, current Components Decomposition and Restructured Power Systems

I. INTRODUCTION

THE transmission loss contributes to a small fraction of the total transmission costs. However, this small proportion is still notable, since it can propose great effects on the decisions the power market participants make. As a result, a proper loss allocation method, whose purpose is to allocate each generator and/or load a fraction of losses, will have substantial influences on the market functions.

The previous monopolistic structure used a simple pricing scheme based on a uniform distribution of the approximated loss of 2% to 5% of generated power. This simple loss allocation, however, is not sufficient for the restructured electricity market as it does not encourage competition between market participants [1].

Power loss is the quadratic function of power flows. As a result, it has a nonlinear nature, which makes it impossible to unbundle the power loss in a way that assures the fraction of loss given to every generator or load is exactly the amount of loss that generator or load causes. This also results in lack of a standard criterion for evaluating the efficiency of different loss allocation techniques.

It is important to note that loss allocation does not change power flows. It only determines how to distribute loss cost among generators and loads of system. In other words, loss allocation follows the power flow process and the total system loss is determined by the latter one. However, loss allocation changes the planning and expansion policies. For instance, a loss allocation method, like pro rata, which is merely based on the power level of a generation or consumption unit, does not encourage new generation units to be installed near load centers and hence increases total system loss. This will result in an inequitable loss allocation, since the other market participants will have to pay for a loss cost which is only due to an improper loss allocation method. In contrast, an equitable loss allocation method, which rewards the loads located in areas of high generation and the generators in areas of high load profile, will conduct the planning policies of system agents in a more proper manner.

There are several issues which should be considered for a loss allocation method, such as clarity and fairness. A fair loss allocation technique distributes the total loss among the generators and loads according to their transmission network usage. The loss allocation should be not only considered fair by most of the system participants, but also simple to be performed.

Several methods have been proposed for loss allocation, such as Pro-Rata, Proportional Sharing (PS) [6-8], Incremental Transmission Loss (ITL) [9], Loss Weight Factor (LWF) allocation method [10], and Z-bus method [4]. The incremental transmission loss allocation, which is based on how a slight change in power injections at a single bus affect system losses, depends on the choice of the slack generator and the direct application of the coefficients typically results in over-recovery of losses. The proportional sharing method is based on the proportional sharing principle, which assumes that the inflows are proportionally shared among the outflows at each network node. LWF loss allocation method is based on tracing the power of a particular load or generator. Z-bus method, which is based on the network z-bus matrix, does not require any simplifying assumptions.

In this paper, a practical scheme for allocating the transmission loss to the generators and loads is rendered which is based on using the Z-bus method and the Z-bus modified matrix.

M. R. Ebrahimi is with Sama Technical and Vocational College Islamic Azad University,Tehran Branch, Tehran-Iran
(email: Mr_ebrahimi@alum.sharif.edu)
M. Ehsan is with the Department of Electrical Engineering, Sharif University of Technology, Azadi Ave, Tehran, Iran (email: ehsan@sharif.edu)

II. THE PROPOSED SCHEME

A. Implementation of the Impedance Matrix for Calculating the share of Bus Bars in the Branch Currents

The proposed method, which is based on the modified Z-bus loss allocation method [2], uses the circuit characteristics to determine the loss. Fig. 1 depicts the π-equivalent model of the line ij.
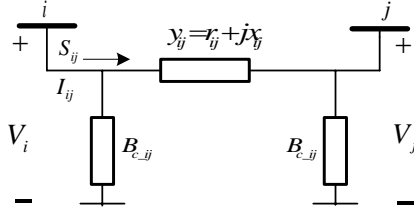

Fig. 1. The π-equivalent model of the line ij

First it is assumed that the direction of power flow is from the bus bar i to the bus bar j. In other words it is assumed that $P_{ij} \geq 0$, in which P_{ij} is the active power flow from the bus bar i to the bus bar j. Thus the current of the line ij, I_{ij}, can be written as:

$$I_{ij} = (V_i - V_j) y_{ij} + V_i B_{C\_ij},\qquad(1)$$

in which V_i and V_j are respectively the voltages at bus bars i and j, y_{ij} is the ij th element of the Y-bus matrix, and B_{C_ij} is the shunt admittance of the π-equivalent model of the line ij. Also using:

$$V_i = \sum_{k=1}^{n} z_{ik} I_k,\qquad(2)$$

in which z_{ik} is the ik th element of the z-bus matrix, I_k is the injected current at bus bar k, and n is the number of the system nodes, (1) can be rewritten as

$$I_{ij} = (\sum_{k=1}^{n} z_{ik} I_k - \sum_{k=1}^{n} z_{jk} I_k) y_{ij} + \sum_{k=1}^{n} z_{ik} I_k B_{C\_ij}$$

$$\Rightarrow I_{ij} = \sum_{k=1}^{n} [(z_{ik} - z_{jk}) y_{ij} + z_{ik} B_{C\_ij}] I_k.\qquad(3)$$

The expression $(z_{ik} - z_{jk}) y_{ij} + z_{ik} B_{C\_ij}$ written in (3) is a constant value, which can be expressed as the characteristic between bus bar x and the line ij, and shown as C_{ij}^k. In other words

$$c_{ij}^k = (z_{ik} - z_{jk}) y_{ij} + z_{ik} B_{c\_ij}.\qquad(4)$$

Using (3) and (4), the share of the bus bar x in the line flow of the branch ij can be calculated as

$$I_{ij}^k = c_{ij}^k I_k.\qquad(5)$$

The line through branch ij can be calculated as

$$I_{ij} = \sum_{k=1}^{n} I_{ij}^k.\qquad(6)$$

B. Loss Allocation to the System Participants Using Current Components Decomposition

Calculating the share of each bus bar in the current through each line using (5), the share of each bus bar in the loss of each line can be determined using the concept of current components decomposition.

If the current of the line ij, I_{ij}, is considered the reference vector, the line currents which is the result of the injected currents, can be decomposed to two vectors: one which is parallel to I_{ij} and the other which is perpendicular to it. As it is shown in Fig. 2, the current of each line is the summation of the share of line currents due to each current injection.
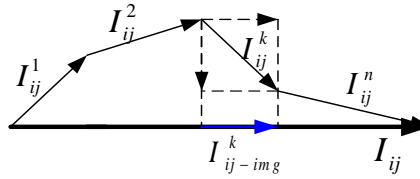

Fig. 2. The current of line ij due to current injections

If I_{ij-img}^k is the image of the vector I_{ij}^k in the direction of I_{ij}, then

$$I_{ij\_img}^k = \left| I_{ij}^k \right| \cos(\phi_{ij}^k - \phi_{ij}) e^{j\phi_{ij}},\qquad(7)$$

in which ϕ_{ij} and ϕ_{ij}^k are the phase angle of I_{ij}^k and I_{ij} with respect to the phase angle of the reference bus bar voltage.

The power flow of the sending bus bar of the line ij due to the injected current at bus bar k, $S_{ij-f-img}^k$, can be calculated as

$$S_{ij-f-img}^k = V_{ij-f} \cdot I_{ij-img}^{k*} = \left| V_{ij-f} \right| \left| I_{ij}^k \right| \cos(\phi_{ij}^k - \phi_{ij}) e^{j(\theta_{ij-f} - \phi_{ij})},\qquad(8)$$

In (8), θ_{ij-f} is the voltage angle of the sending bus bar of the line ij.

Likewise, the power flow of the receiving bus bar of the line ij due to the injected current at bus bar k, $S_{ij-t-img}^k$, is

$$S_{ij-t-img}^k = V_{ij-t} \cdot I_{ij-img}^{k*} = \left| V_{ij-t} \right| \left| I_{ij}^k \right| \cos(\phi_{ij}^k - \phi_{ij}) e^{j(\theta_{ij-t} - \phi_{ij})},\qquad(9)$$

in which θ_{ij-t} is the voltage angle of the receiving bus bar of the line ij.

The share of the injected current at the bus bar k in the loss of the line ij can be obtained as

$$L_{ij}^k = S_{ij-f-img}^k - S_{ij-t-img}^k,\qquad(10)$$

which, using (8) and (9), can be written as

$$L_{ij}^k = (V_{ij-f} - V_{ij-t}) I_{ij-img}^{k*}$$
$$= \left| I_{ij}^k \right| \cos(\phi_{ij}^k - \phi_{ij}) e^{-j\phi_{ij}} I_{ij} z_{ij} \quad L_{ij}^k = S_{ij-f-img}^k - S_{ij-t-img}^k \qquad(11)$$
$$= \left| I_{ij}^k \right| \left| I_{ij} \right| \cos(\phi_{ij}^k - \phi_{ij}) z_{ij}$$

In this paper the active power loss of the line ij is allocated as

$$P_{loss\,ij}^k = \left| I_{ij}^k \right| \left| I_{ij} \right| \cos(\phi_{ij}^k - \phi_{ij}) r_{ij}.\qquad(12)$$

If l is the set of all the system branches, the total loss

allocated to the participant at the bus bar k is

$$P_{loss}{}^k = \sum_{ij \in l} P_{loss\,ij}{}^k \;. \qquad (13)$$

III. CASE STUDY

A. The Transmission Network of Khorasan Region in Iran

In order to demonstrate the use of the proposed scheme in Iran power market, the suggested algorithm is run on the 400kv transmission network of Khorasan region in Iran.

The single-line diagram of this power system is shown in Fig. 3.
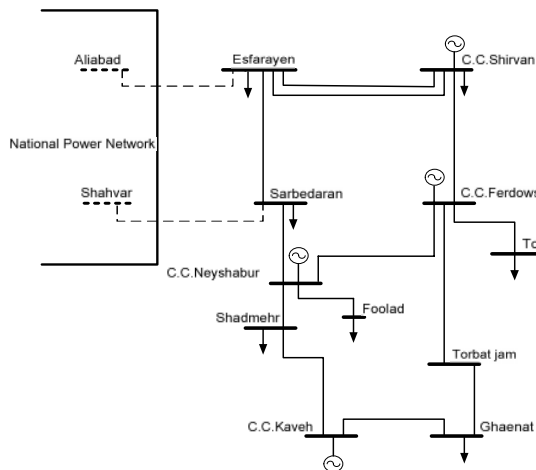

Fig. 3 The single-line diagram of 400kV Khorasan transmission network

The system data is available in Tables I and II. The values of the tables are calculated at 10:30 AM on 3.12.2009.

TABLE I
KHORASAN TRANSMISSION NETWORK DATA

Line No.	From bus bar	To bus bar	Line resistance	Line Reactance	Line shunt admittance
1	Shiravan	Esfarayen	0.00141	0.01498	0.198
2	Shiravan	Esfarayen	0.00143	0.01523	0.2012
3	Shiravan	Ferdousi	0.00318	0.0337	0.4455
4	Esfarayen	Sarbedaran	0.0032	0.0327	0.44
5	Sarbedaran	Neyshabur	0.00235	0.02496	0.33
6	Ferdowsi	Toos	0.001	0.01	0.1275
7	Ferdowsi	Neyshaboor	0.00148	0.01511	0.2035
8	Ferdowsi	Torbate-jam	0.0041	0.042	0.565115
9	Neyshabur	Shadmehr	0.00278	0.0284	0.38225
10	Torbate-jam	Ghaenat	0.00478	0.04885	0.65725
11	Shadmehr	C.C.Kaveh	0.00362	0.037	0.49775
12	Ghaenat	C.C.Kaveh	0.0002	0.0017	0.022

Khorasan region is connected to the national power network through the two lines Aliabad-Esfarayen and Sarbedaran-Shahdar. As a result, the two bus bars Aliabad and Shahdar are assumed as the reference bus bars. Since it is intended to investigate only the mentioned region, the regions having voltages less than 400 kV are modeled as loads.

TABLE II
KHORASAN TRANSMISSION NETWORK POWER FLOW DATA

Bus	Type	Generation		Load		Voltage
		P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	(PU)
Aliabad	Slack	-	-	224.5	-45	1.05
Shahvar	Slack	-	-	183.8	-40	1.05
C.C.Shiravan	PV	700	-	280	-6	1.04
Esfarayen	PQ	0	0	125	101	1.03
Sarbedaran	PQ	0	0	140	90	1.03
C.C.Ferdousi	PV	55	-	0	0	1.03
C.C.Neyshabur	PV	810	-	85	-31	1.02
Torbat-jam	PQ	0	0	144	59	1.01
Shadmehr	PQ	0	0	224	40	1.01
Toos	PQ	0	0	31	69	1.03
Ghaenat	PQ	0	0	200	40	1.01
C.C.Kaveh	PV	260	-	0	0	1.01

The results of the proposed scheme loss allocation and z-bus loss allocation are shown in Table III.

TABLE III
RESULTS OF LOSS ALLOCATION FOR KHORASAN REGION

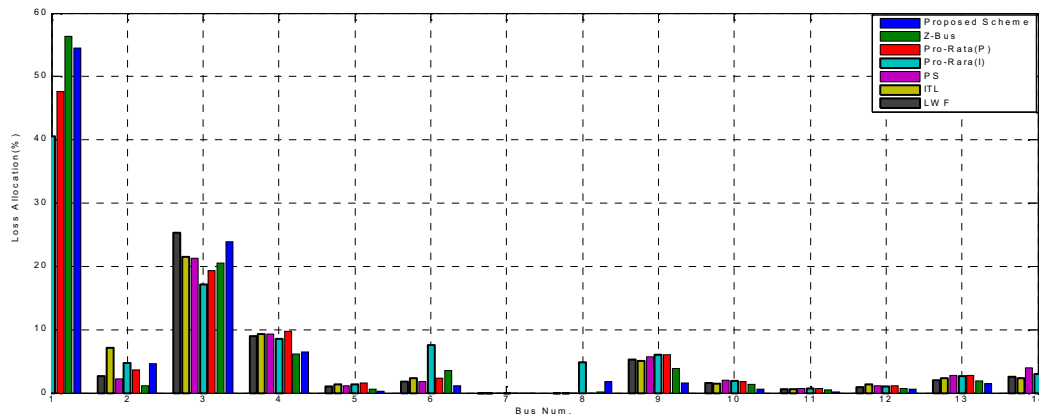| Generator / load	Proposed scheme	Z-bus method
C.C.Shiravan	1.65	1.95
Esfarayen	1.00	0.47
Sarbedaran	1.37	0.97
C.C.Ferdowsi	0.07	0.11
Toos	0.58	0.68
C.C.Neyshaboor	2.12	2.95
Torbate-jam	1.33	1.18
Shadmehr	1.29	0.88
Ghaenat	1.51	0.74
C.C.Kaveh	-1.70	-0.71
Total	9.22	9.22

Fig. 4 The results of different loss allocation methods

As it is shown in Table III, the total loss in the region is 9.22 MW. The allocated loss to the generation unit located at C.C.Neyshaboor is the most, which is 23% for the proposed scheme and 32% for the z-bus method. Among the loads, the most allocated loss for the proposed scheme belongs to the Ghaenat load with 16% of the total loss. For the Z-bus method, it belongs to Shadmehr bus bar which is allocated 10% of the total loss. Since the generator located at C.C.Kaveh decreases loss through producing the predominant current, its allocated loss is negative in both methods. The negative loss allocation could be physically understandable with the concept of counter flows and dominant flows [3]. Whether the negative loss allocations are acceptable or not depends on the market and participants.

B. The 14-bus Standard IEEE Network

In this section the results of the proposed scheme and five other methods of loss allocation for the pool market are obtained for the standard 14-bus IEEE network. The data of the network are available in [4]. The five methods analyzed are z-bus loss allocation [4], two pro-rata methods (one based on the active power and the other on the current magnitude injections) [5], proportional sharing method [6-8], incremental transmission loss allocation method [9], and loss weight factor (LWF) allocation method [10]. The unit for the evaluation of the methods in this case study is chosen to be "dollar per hour", which clearly describes the monetary impact of loss allocation and the significant differences among the various methods. The system marginal price is assumed to be 50 $/MWh. The results of loss allocation to the system bus bars are shown in Table IV. Fig. 4 shows the results of different loss allocation methods.Table IV compares the loss components allocated to each bus for different methods. Columns 5 through 11 represent the cost of the allocated bus losses for the seven different allocation methods. These are respectively: proposed method, Z-bus, *pro rata* based on active power injections (P), *and pro rata* based on current magnitude injections (I), proportional sharing (PS), incremental transmission loss method (ITL), and loss weight factor (LWF).Also Fig. 4 shows the result of proposed method against other methods.Because the power injection of bus 7 is zero, its share on the allocated losses equal to zero. It is clear

that among generators, generator 1, gets the highest allocated, according to all methods. Similarly, among loads, all seven methods allocate the highest cost to the load at bus 3. The proposed method allocates 369$/h to generator 1 and 162$/h to load 3.It is evident that other methods are available in Table IV, the most present of loss allocated to generator 1 that seems unfair, but the proposed method is decreased this value that lead to "fair" results.

IV. CONCLUSIONS

In this paper a new scheme for transmission loss allocation problem has been developed, which is based on decomposing the function transmission loss and the current projection concept. The scheme has been compared to a number of common loss allocation methods. Numerical results and comparisons have demonstrated that the proposed scheme has a good performance, and it is consistent from the point of view of *fairness* and *transparency*.

TABLE IV
LOSS ALLOCATION RESULTS

Bus Num	Active power gen. Pg	Active load dem. Pd	Bus current inject. |I|	Distribution of active power losses Ploss=13.5 MW ; λ=50$/MWh						
				Proposed Scheme	Z-bus	Pro-rata		PS	ITL	LWF
						P	I			
	MW	MW	A	$/h	$/h	$/h	$/h	$/h	$/h	$/h
1	232.7	0	1598	369.71	382	323	275	324	307	320
2	40	21.7	188	31.16	8	25	32	15	48	18
3	0	94.2	676	162.20	139	131	116	144	146	172
4	0	47.8	339	44.12	42	66	58	63	63	61
5	0	7.6	55	2.20	4	11	9	8	9	7
6	0	11.2	298	8.04	24	16	51	12	16	12
7	0	0	0	0	0	0	0	0	0	0
8	0	0.1	190	11.94	1	0	33	0	0	0
9	0	29.5	239	10.71	26	41	41	39	34	36
10	0	9	76	4.04	9	12	13	14	10	11
11	0	3.5	27	1.33	3	5	5	5	4	4
12	0	6.1	43	4.00	5	8	7	8	9	6
13	0	13.5	102	9.85	13	19	18	19	16	14
14	0	14.9	112	18.69	22	21	20	27	16	17
Sum	272.7	259.1	-	678	678	678	678	678	678	678

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Mohammad Reza Ebrahimi was born in Tehran, Iran in 1983. He received his B.Sc. on 2006 from ferdowsi University of Mashhad(UM), Mashhad, Iran and M.Sc. on 2009 from Sharif University of Technology (SUT), Both in Electrical Engineering. Currently he is a faculty memberin EE Department of Sama Technical and Vocational College Islamic Azad University, Tehran Branch. His research interests are Power & Energy System Analysis, Power Systems Restructuring and Deregulation, Power Markets, Reliability of Power Systems and Distributed Generation.

Mehdi Ehsan Received B.Sc. & M.Sc. in Electrical Engineering from Technical College of Tehran University 1963, PhD & DIC from Imperial College University of London in 1976. Since then He has been with EE Dept of Sharif University of Technology in different responsibilities. He has an extended cooperation with research centers in Iran and also abroad. Currently he is a professor in EE Department of Sharif University. His research interests are Power system Simulation, Dynamic & Transient Stability, Application of Expert Systems in Identification, Operation, and Planning & Control of power system.