

# A Novel Pilot Scheme for Frequency Offset and Channel Estimation in 2x2 MIMO-OFDM

N. Promsuwanna, P. Uthansakul and M. Uthansakul

**Abstract**—The Carrier Frequency Offset (CFO) due to time-varying fading channel is the main cause of the loss of orthogonality among OFDM subcarriers which is linked to inter-carrier interference (ICI). Hence, it is necessary to precisely estimate and compensate the CFO. Especially for mobile broadband communications, CFO and channel gain also have to be estimated and tracked to maintain the system performance. Thus, synchronization pilots are embedded in every OFDM symbol to track the variations. In this paper, we present the pilot scheme for both channel and CFO estimation where channel estimation process can be carried out with only one OFDM symbol. Additional, the proposed pilot scheme also provides better performance in CFO estimation comparing with the conventional orthogonal pilot scheme due to the increasing of signal-to-interference ratio.

**Keywords**—MIMO, OFDM, carrier frequency offset, channel, estimation.

## I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) has enables a high data rate transmission over multipath fading channels because of the transformation of entire frequency selective channel into a parallel set of frequency flat sub-channels. It has been widely adopted for standards such as DAB, DVB and WLAN [1]. However, OFDM signal is very sensitive to carrier frequency offset (CFO) which cause a loss of BER performance. Thus CFO is needed to be estimated and compensated in order to maintain a good system performance.

There have been several works that are related to CFO estimation. In [2], the authors proposed the frequency domain maximum likelihood CFO estimation by using the repeat data symbol. The estimation length of this technique is 0.5 of subcarrier spacing. Based on training symbol, there are many works using the repeat data symbol such as [3] and [4] which are designed for burst transfer mode and suitable for slow fading channel. In broadcasting applications, the pilot schemes are usually employed by inserting the pre defined data in every OFDM symbol in order to track the variations of channel and CFO. Then CFO is able to be estimated by using various techniques such as the method described in [1] or [5]. However, the effect of data-interference to pilots causes the degradation of estimation performance. Due to this issue, the work in [6] proposes data-pilot multiplex schemes to reduce

the effect of data-interference to pilot and the authors in [7] proposes a cluster pilot to provide higher signal to ICI power ratio in order to improve CFO estimation. However, the work in [6] and [7] is based on single input single output OFDM system.

In this paper, we proposed pilot scheme for both CFO and channel estimation for mobile broadband MIMO-OFDM system. Where channel estimation process of the proposed scheme can be carried out with only one OFDM symbol. This provides the same benefit as the orthogonal pilot scheme proposed in [8]. But the proposed scheme can improve the CFO estimation performance by the increasing of signal-to-interference ratio (SIR). Thus, better CFO estimation leading to better in channel estimation.

The remainder of this paper is organized as follows. The system models of MIMO-OFDM with the effect of CFO are described in section II. The performance investigation of the conventional pilot scheme and the proposed pilot scheme are described in section III and IV respectively. In section V, the simulation results of proposed pilot scheme for 2x2 MIMO system are presented and finally the conclusion is given in Section VI.

## II. SYSTEM MODEL

In the MIMO-OFDM system with  $N_t$  transmitted antennas,  $M_r$  received antennas and  $N$  subcarriers, we denote  $\mathbf{X}_i^n = [X_i^n(1) \ X_i^n(2) \ \dots \ X_i^n(N)]^T$  is transmitted signal vector in the frequency domain of  $n$ th transmitted antenna on  $i$ th OFDM symbol,  $\mathbf{z}_i$  is an AWGN and  $N_{cp}$  is the cyclic prefix (CP) length to prevent intersymbol interference. The received signal vector in time domain of  $m$ th received antenna is effect by normalized CFO ( $\varepsilon$ ) which is given by

$$\mathbf{y}_i^m = \sum_{n=1}^{N_t} \mathbf{D}_\varepsilon e^{j\phi_i} \mathbf{V} \mathbf{H}^{m,n} \mathbf{X}_i^n + \mathbf{z}_i \quad (1)$$

where  $\mathbf{V}$  is the inverse DFT matrix  $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_N]$

$$\mathbf{v}_k = (1/\sqrt{N}) [1 \ e^{j2\pi k/N} \ e^{j2\pi(2k)/N} \ \dots \ e^{j2\pi((N-1)k)/N}]^T$$

$$\mathbf{D}_\varepsilon = \text{diag}[1, e^{j2\pi\varepsilon/N}, \dots, e^{j2\pi\varepsilon(N-1)/N}]$$

$$\phi_i = 2\pi\varepsilon i(N + N_{cp})/N$$

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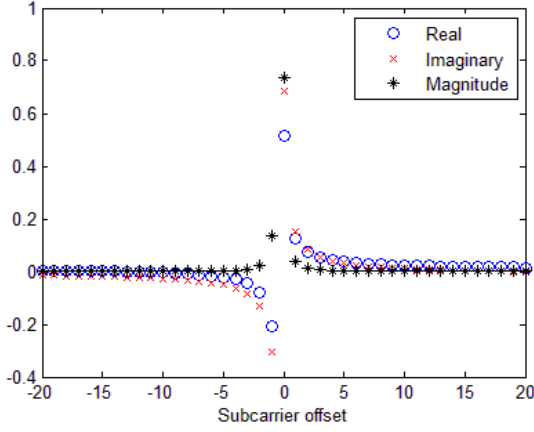


Fig. 1 Real part, imaginary part and magnitude of the ICI coefficients

and  $\mathbf{H}^{m,n} = \text{diag}[H^{m,n}(1), H^{m,n}(2), \dots, H^{m,n}(N)]$  where  $H^{m,n}(k)$  is channel frequency response on  $k$ th subcarrier of  $m$ th received antenna and  $n$ th transmitted antenna. Then the received signal in frequency domain on  $k$ th subcarrier after using DFT can be shown by

$$Y_i^m(k) = \sum_{n=1}^{N_t} \mathbf{v}_k^H \mathbf{D}_e e^{j\phi_i} \mathbf{V} \mathbf{H}^{m,n} \mathbf{X}_i^n + Z_i(k) \quad (2)$$

$$= e^{j\phi_i} \sum_{n=1}^{N_t} \left\{ \alpha_0 H^{m,n}(k) X_i^n(k) + \sum_{l=0, l \neq k}^{N-1} \alpha_{l-k} H^{m,n}(l) X_i^n(l) \right\} + Z_i(k)$$

where  $H$  represents hermitian, ICI coefficient ( $\alpha$ ) is given by

$$\alpha_{l-k} = \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi n(l-k+\varepsilon)}{N}} \quad (3)$$

Fig.1 shows the complex weighting coefficients for the case of  $\varepsilon = 0.3$  and  $N = 64$ . It reveals that both the real and the imaginary parts of the ICI coefficients are slightly changing for each subcarrier except for several coefficients around the zero subcarrier. This effect can be stronger when  $\varepsilon$  is higher.

### III. CONVENTIONAL PILOT SCHEME

In this paper, we use the orthogonal pilot scheme in [8] be the conventional pilot scheme. This pilot scheme converts the complexity of multiple antenna channel estimation into simple single antenna system where the channel estimation process can be completed with only one OFDM symbol. Thus, it is a useful technique especially for mobile broadband communication. This pilot scheme can be shown in Fig. 2.

#### A. CFO Estimation

Based on the orthogonal pilot scheme in Fig. 2, CFO can be estimated by measuring the phase shift of pilot symbols in two consecutive OFDM symbols. By giving  $a$  is the pilot symbol and  $\Gamma$  is the set of pilot tone indexes. By ignoring the effect of

noise from (6), we can write received signal of pilot tone on  $i$ th and  $i+1$ th OFDM symbol by

$$Y_i^m(k)_{k \in \Gamma} = e^{j\phi_i} (\alpha_0 H^{m,n}(k) a + I_i^{m,1}(k) + I_i^{m,2}(k)) \quad (4)$$

$$Y_{i+1}^m(k)_{k \in \Gamma} = e^{j\phi_{i+1}} (\alpha_0 H^{m,n}(k) a + I_{i+1}^{m,1}(k) + I_{i+1}^{m,2}(k)) \quad (5)$$

where ICI term can be given by

$$I_i^{m,n}(k) = \sum_{l=0, l \neq k}^{N-1} \alpha_{l-k} H^{m,n}(l) X_i^n(l) \quad (6)$$

Based on [2], the estimated phase can be given by

$$\phi_{i+1} - \phi_i \approx \hat{\phi}_\Delta = \left[ \frac{\text{Im}(Y_i^m(k)_{k \in \Gamma}^* Y_{i+1}^m(k)_{k \in \Gamma})}{\text{Re}(Y_i^m(k)_{k \in \Gamma}^* Y_{i+1}^m(k)_{k \in \Gamma})} \right] \quad (7)$$

where  $*$  stand for conjugate,  $\text{Re}(\cdot)$  and  $\text{Im}(\cdot)$  denote real and imaginary part respectively. Then the estimated normalized CFO can be given by

$$\hat{\varepsilon} = \frac{\hat{\phi}_\Delta}{2\pi(N + N_{CP})/N} \quad (8)$$

By assuming that, transmitted signals on each transmitted antenna are uncorrelated,  $|X_i^n|^2 = |X|^2$ ,  $E\{X_i^n\} = 0$  and  $\text{Cov}(X_i^n(k), X_i^n(m))_{m \neq k} = 0$ . Then SIR of pilot tone based on (4) can be given by

$$\text{SIR} = \frac{|\alpha_0 H^{m,n} a|^2}{|I_i^{m,1}(k)|^2 + |I_i^{m,2}(k)|^2} \quad (9)$$

$$|I_i^{m,n}(k)|^2 = \sum_{l=0, l \neq k}^{N-1} |\alpha_{l-k}|^2 |H^{m,n}(l) X_i^n(l)|^2 \quad (10)$$

The work in [9] indicated that the performance of CFO estimation on (7) can be improved with the increasing of SIR. Thus, it is interest of designing pilot scheme that can improve SIR to provide more accurate estimated CFO.

#### B. Channel Estimation

After estimating and compensating the CFO, channel estimation of the orthogonal pilot scheme can be achieved easily as in SISO-OFDM system by

$$H^{m,n} = Y_i^m(k; k \in \Gamma^n) / a \quad (11)$$

where  $\Gamma^n$  is the set of pilot indexes from  $n$ th antenna.

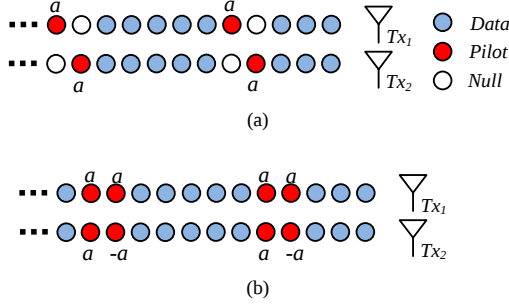


Fig. 2 (a) orthogonal pilot scheme (b) proposed pilot scheme

#### IV. PROPOSED PILOT SCHEME

The proposed pilot scheme can be shown in Fig. 2. We cluster two adjacent pilot tones as group for each antenna. We set pilot tones in each group of the first antenna are identical while pilot tones in each group of the second antenna are antipodal.

##### A. CFO Estimation

Assuming that  $\Gamma_l$  is the set containing the left pilot tone indexes and channel response is assumed to be flat in the preliminary study where  $H^{m,n}(k) = H^{m,n}$ . The received signal in frequency domain of the left pilot tone and the right pilot tone in each pilot group based on (2) without noise can be given by

$$Y_i^m(k)_{k \in \Gamma_l} = e^{j\phi} a \left[ \alpha_0 H^{m,1} + \alpha_0 H^{m,2} + \alpha_1 H^{m,1} - \alpha_1 H^{m,2} + I_i^{m,1}(k) + I_i^{m,2}(k) \right] \quad (12)$$

$$Y_i^m(k+1)_{k \in \Gamma_l} = e^{j\phi} a \left[ \alpha_0 H^{m,1} - \alpha_0 H^{m,2} + \alpha_1 H^{m,1} + \alpha_1 H^{m,2} + I_i^{m,1}(k+1) + I_i^{m,2}(k+1) \right] \quad (13)$$

Equation (12) and (13) represent the received signal of the left and the right pilot tone at  $m$ th received antenna respectively. From (12) and (13),  $\alpha_1 H^{m,1}$ ,  $\alpha_1 H^{m,2}$ ,  $\alpha_{-1} H^{m,1}$  and  $\alpha_{-1} H^{m,2}$  represent the ICI from the adjacent pilot tone. Then the clustered pilot tones can be given by

$$\begin{aligned} (12) - (13) &= Y_i^m(k)_{k \in \Gamma_l} - Y_i^m(k+1)_{k \in \Gamma_l} \\ &= e^{j\phi} a \left[ 2\alpha_0 H^{m,2} + \alpha_1 H^{m,1} - \alpha_{-1} H^{m,1} - \alpha_1 H^{m,2} - \alpha_{-1} H^{m,2} \right. \\ &\quad \left. + I_i^{m,1}(k) - I_i^{m,1}(k+1) + I_i^{m,2}(k) - I_i^{m,2}(k+1) \right] \\ &= e^{j\phi} a \left[ H^{m,2} (2\alpha_0 - \alpha_1 - \alpha_{-1}) + H^{m,1} (\alpha_1 - \alpha_{-1}) + I_{\Delta}^{m,1} + I_{\Delta}^{m,2} \right] \end{aligned} \quad (14)$$

where

$$I_{\Delta}^{m,n} = I_i^{m,n}(k) - I_i^{m,n}(k+1) = \sum_{\substack{l=0 \\ l \neq k, k+1}}^N (\alpha_{l-k} - \alpha_{l-k-1}) H^{m,n} X_i^n(l) \quad (15)$$

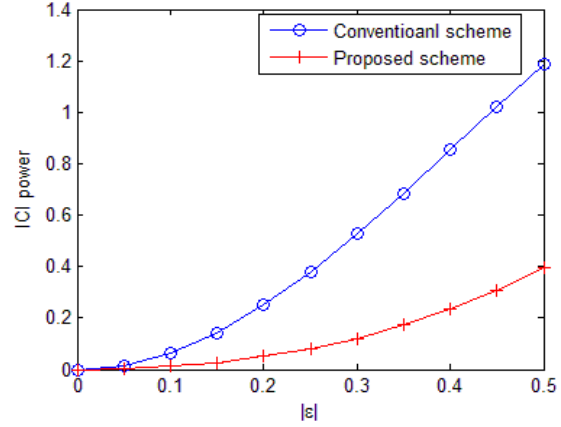


Fig. 3 ICI power comparison

As same as (9), we can write SIR of the propose pilot scheme by

$$SIR = \frac{|a|^2 |H^{m,2} (2\alpha_0 - \alpha_1 - \alpha_{-1}) + H^{m,1} (\alpha_1 - \alpha_{-1})|^2}{|I_{\Delta}^{m,1}(k)|^2 + |I_{\Delta}^{m,2}(k)|^2} \quad (16)$$

$$|I_{\Delta}^{m,n}(k)|^2 = |H^{m,n}|^2 |X|^2 \sum_{\substack{l=0 \\ l \neq k, k+1}}^N |(\alpha_{l-k} - \alpha_{l-k-1})|^2 \quad (17)$$

Based on (17) and (10), we can show the performance of ICI reduction by the proposed scheme in Fig. 3. Where  $|H^{m,n}|^2$  and  $|X|^2$  are assumed to be 1. As seen in Fig. 3, the proposed scheme provides less ICI power than the conventional scheme for any  $\varepsilon$  where the reduction is about 6 dB on the average.

In addition, the proposed scheme can improve signal power of pilot as shows in (16). However signal power from (16) is based on  $H^{m,2}$  and  $H^{m,1}$  thus the performance of several case of  $H^{m,2}$  and  $H^{m,1}$  should be investigated. Fig. 4 shows the average performance of signal power where  $H^{m,2}$  and  $H^{m,1}$  are random from 5000 Gaussian channels. We categorize case of  $H^{m,2}$  and  $H^{m,1}$  into 5 events which are  $|H^{m,2}|_{dB}^2 - |H^{m,1}|_{dB}^2 = 7, 3, 0, -3, -7$  and  $-13$  dB and we normalize channel by  $|H^{m,2}|^2 + |H^{m,1}|^2 = 2$ . As seen in Fig. 4, the proposed scheme can enhance signal power of pilot tone for above  $|H^{m,2}|_{dB}^2 - |H^{m,1}|_{dB}^2 \geq -3$  dB while  $|H^{m,2}|_{dB}^2 - |H^{m,1}|_{dB}^2 \leq -7$  dB provide better performance in signal power when  $|\varepsilon| \geq 0.25$ .

However, the deviation of  $|H^{m,2}|_{dB}^2 - |H^{m,1}|_{dB}^2 \leq -7$  dB is very limit situations in a broadband channel where it should occur when there is rich of multipath like an indoor channel. Thus the proposed scheme still provides better average performance than the conventional scheme if a broadband channel is considered.

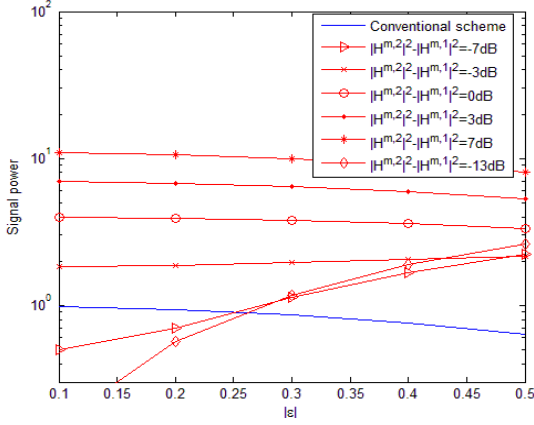


Fig. 4 Signal power comparison

The estimated CFO of the proposed scheme based on (7) can be given by

$$\hat{\phi}_\Delta = \frac{\text{Im}\left(K_i^m(k)_{k \in \Gamma_i}^* K_{i+1}^m(k)_{k \in \Gamma_i}\right)}{\text{Re}\left(K_i^m(k)_{k \in \Gamma_i}^* K_{i+1}^m(k)_{k \in \Gamma_i}\right)} \quad (18)$$

where  $K_i^m(k)_{k \in \Gamma_i} = Y_i^m(k)_{k \in \Gamma_i} - Y_i^m(k+1)_{k \in \Gamma_i}$ .

In addition, we also combine estimated phase from (7) with (18) in order to improve the performance of phase estimation.

#### B. Channel Estimation

The proposed pilot scheme can estimate MIMO channel by using only one OFDM symbol as same as the orthogonal pilot scheme. This based on the assumption that even if channel response of a broadband channel is selective but channel gain between adjacent subcarrier can be assumed to be the same  $H^{m,n}(k) \approx H^{m,n}(k+1)$ . Thus we can use adjacent subcarrier to complete channel estimation process of MIMO-OFDM system. The estimated channel of the proposed pilot scheme based on least square Estimation (LSE) can be given by

$$\hat{\mathbf{H}}(k) = \mathbf{Y}(k) \mathbf{X}^\dagger \quad (18)$$

where  $^\dagger$  represent pseudo-inverse,

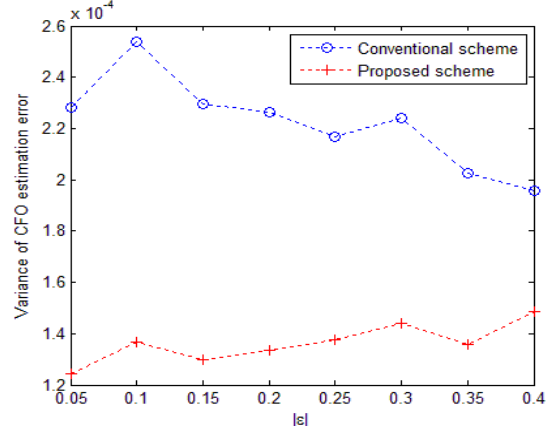
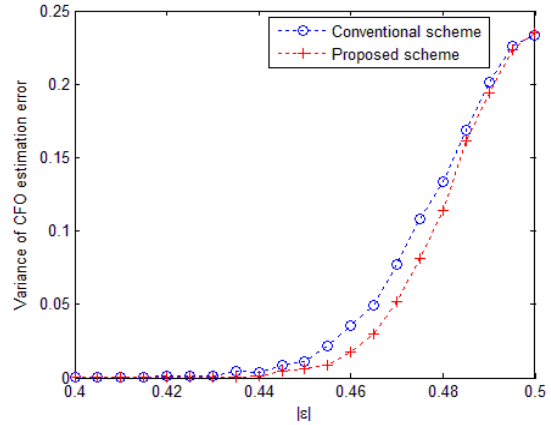
$$\mathbf{Y}(k) = \begin{bmatrix} Y^1(k) & Y^1(k+1) \\ Y^2(k) & Y^1(k+1) \end{bmatrix} \quad (19)$$

$$\mathbf{X} = \begin{bmatrix} a & a \\ a & -a \end{bmatrix} \quad (20)$$

The channel responses of remain subcarriers can be achieved by interpolation between estimated subcarrier.

## V. SIMULATION RESULTS

In this section, the simulation experiments will be presented to investigate the performance of the proposed technique for

Fig. 5 Variance of estimation errors versus  $\varepsilon$  ( $0 < \varepsilon \leq 0.4$ )Fig. 6 Variance of estimation errors versus  $\varepsilon$  ( $0.4 \leq \varepsilon \leq 0.5$ )

frequency selective channels. Pilots are placed by equi-space along the subcarrier axis and the performance was averaged for 5,000 random Gaussian channels. The number of pilot subcarriers for each technique is equal in order to make a peer comparison. The other simulation parameters are as follows:

- Number of subcarriers ( $N$ ) = 256
- Modulation = QPSK
- Number of pilots = 32
- Subcarrier spacing = 10.93kHz
- Cyclic prefix length =  $N/8$

In order to investigate the estimation performance in frequency selective channel, this paper adopts the ITU vehicular A to be a channel model where this model has been adopted in the WiMAX forum. The relative multipath delay ( $\tau$ ) and the normalized path gain ( $\rho$ ) can be given by

$$\tau = [0 \ 0.31 \ 0.71 \ 1.09 \ 1.73 \ 2.51] \mu\text{s}$$

$$\rho = [0 \ -1 \ -9 \ -10 \ -15 \ -20] \text{dB}$$

Fig. 5 shows the estimation performance in terms of variance of estimation errors versus  $\varepsilon$  ( $0 < \varepsilon \leq 0.4$ ) when OFDM symbol's SNR is 5dB. As seen in Fig. 5, the proposed scheme provides better CFO estimation performance than the conventional scheme which is about 80% in an error variance reduction. For other values of SNR, the proposed scheme still provides similar benefit over the conventional scheme as shown in Fig. 5. Fig. 6 shows the estimation performance where  $0.4 \leq \varepsilon \leq 0.5$ . As seen in Fig. 6, the proposed scheme still provides better CFO estimation performance but the error variance become larger than Fig. 5. This is because the interference from data subcarriers becomes larger when there is high value of  $\varepsilon$ .

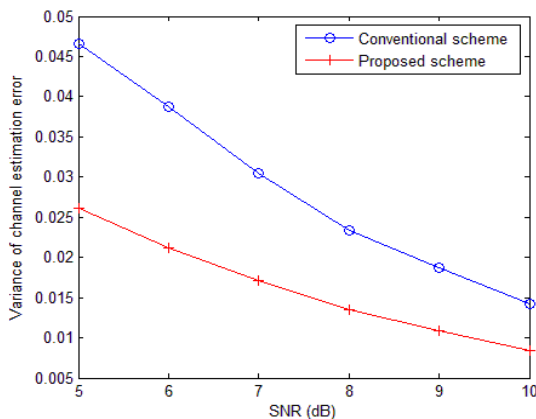


Fig. 7 Variance of channel estimation errors versus SNR

Fig. 7 shows variance of channel estimation error of  $H^{L,1}$  versus OFDM symbol's SNR for both the proposed scheme and the conventional scheme when  $\varepsilon = 0.3$ . The whole channel frequency responses of both schemes are achieved by channel estimation and linear interpolation. Channel estimation for both techniques are performed after compensates the CFO for received signal. As seen in Fig. 7, the proposed scheme provides better performance by reducing channel estimation errors. This is because the proposed scheme provides better performance in CFO estimation thus it causes less CFO effect from the compensated received signal. For other values of  $\varepsilon$ , the proposed technique still provides similar benefits over the conventional scheme where variance of estimation error become larger when  $\varepsilon$  is higher.

## VI. CONCLUSION

This paper proposes pilot scheme for both channel and CFO estimation for mobile broadband MIMO-OFDM system. Orthogonal pilot scheme is a useful technique especially for MIMO-OFDM channel estimation where the estimation process can be carried out with only one OFDM symbol. However using null subcarriers in orthogonal pilot scheme cause information on these subcarriers is wasteful. The proposed pilot scheme converts these null subcarriers into pilot tone where the information on pilot tones are carefully

design in order to improve CFO estimation performance. In addition, the proposed scheme still provides the same benefit of channel estimation as the conventional orthogonal pilot scheme.

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