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A Novel Instantaneous Frequency Computation Approach for Empirical Mode Decomposition

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***Abstract*—**This paper introduces a new instantaneous frequency computation approach α -Counting Instantaneous Frequency for a general class of signals called simple waves. The classsimple wave contains a wide range of continuous signals for which the concept instantaneous frequency has a perfect physical sense. The concept of α -Counting Instantaneous Frequency also applies to all the discrete data. For all the simple wave signals and the discrete data, α-Counting instantaneous frequency can be computed directly without signal decomposition process. The intrinsic mode functions obtained through empirical mode decomposition belongs to simple wave. So α-Counting instantaneous frequency can be used together with empirical mode decomposition.

***Keywords*—**Instantaneous frequency, empirical mode decomposition, intrinsic mode function.

I. Introduction

THE physical meaning of the instantaneous frequency (IF) at a time moment t is the averaging vibrating times of a certain vibration during the 2π-length interval. It should be a non-negative quantity varying along with time. IF stems from the fact that in practice frequencies of a signal vary with time [1]. Due to theoretical limitation, some exists IF concepts cannot meet various practical demands. Researchers have been making efforts in finding more practical, acceptable and workable ways to define and computer IF [1-3].

There are various methods to define frequency. With Fourier series and Fourier integral representations one first defined it in terms of harmonic waves. To make the representation more local one started with window Fourier transforms. Many researchers studied analytic signals and accordingly defined analytic time-varying IF [2]. Lately wavelet transforms were studies, where frequencies correspond with the dilation parameters. In 1998 the so called empirical mode decomposition (EMD) algorithm was proposed by Huang et al [4] that provide a highly local and adaptive representation, via a finite number of basic functions called IMFs.

The EMD decomposition is based on the assumptions: (1) the signal has at least two extrema--one maximum and one minimum; (2) the characteristic time scale is defined by the time lapse between the extrema; and (3) if the data were totally devoid of extrema but contained only inflection points, then it can be differentiated once or more times to reveal the extrema. Final results can be obtained by integration(s) of the components [4]. An intrinsic mode function (IMF) is a function that satisfies two conditions: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero [4].

Note that EMD is dependent on envelops made by splines, as well as a threshold. The question is how to reasonably define IFs for the obtained IMFs, which are required to be non-negative. The EMD algorithm rests in the expectation that the analytic IFs obtained via the associated analytic signal of an IMF is non-negative [2, 5]. That is, if f is an IMF, then by expressing:

$$f(t) + iHf(t) = \rho(t)e^{i\theta t},$$

One expects

$$\theta' \geq 0 \tag{1}$$

Unfortunately, it is not the case. In [6] it is shown that there exist IMFs, obtained through EMD, which do not enjoy the property (1). Specifically, it means that IMFs obtained through EMD do not behave well with Hilbert transformation, and, in fact, it has no relation with Hilbert transform [4, 5].

There are two ways to get out of this embarrassing. One is to give a new definition of IF without using the Hilbert transformation or analytic signal. This paper is to propose one such method. The other way is to introduce algorithms to adaptively decompose signals into those of non-negative analytic IFs. The newly developed adaptive Fourier decomposition (AFD) is a method that can decompose a signal into mono-components, which are functions with non-negative derivatives (IFs) [7, 8]. In this paper we define for a large class of signals called simple waves (SWs) the α -Counting Instantaneous Frequency (α-Counting IF). The approach is straight forward with simple formula and algorithm to compute the IF values. It is suitable for engineering applications.

The paper is organized as follows. The new mathematical definition of α-CountingIF for EMD is introduced in section II. The computation algorithm of the proposed IF is presented in section III. An experiment result is shown in section IV. The conclusions are drawn in section V.

II. New Mathematical Definition of IF for EMD

A. Empirical Mode Decomposition

The EMD produces a number of functions$f_i, (i = 1, \cdots, m)$ and r, where m is determined by the algorithm and not known in advance, f_i are real-valued functions. Then

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$$f = \sum_{i=1}^{m} f_i + r.$$

The EMD relies crucially upon the extraction of the envelope of f, which consists of two distinct functions, the upper and the lower envelopes f_u and f_l, where f_u is obtained through the cubic spline interpolation of all local maxima of f, and similarly f_l is obtained through the cubic spline interpolation of all local minima of f. The envelop, in turn, leads to the definition of the *local mean* of f point wise as $l = \frac{f_u+f_l}{2}$ [4].

An IMF has no mathematical definition. There is only one method to verify whether a function is an IMF of some EMD that is to run the EMD algorithm to it: If the EMD cannot decompose the given function, then the given function is an IMF with respect to that particular EMD, and otherwise not. Nevertheless, an IMF has a description. Each IMF satisfies the following two properties.

(i) Between every adjacent pair of local extrema there is a zero crossing.

(ii) The average of upper envelop and lower envelop is zero.

Note that this description strongly dependent on the splines and the threshold in use. In this note an IMF is considered to be a continuous function. By the zero-mean property we have that has all its local maxima larger than 0 and all its local minima less than 0. A typical IMF is shown in Fig. 1.

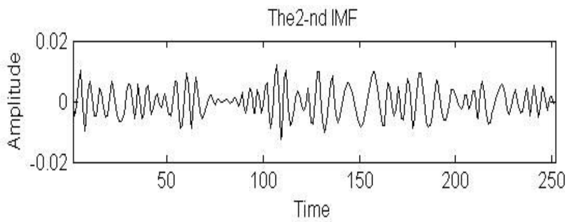

Fig. 1 A sample of IMF decomposed by EMD

B. New Mathematical Definition of IF

A qualified instantaneous frequency should satisfy the following three conditions. The first is the time varying property; the second is non-negativity; and the third is that it should reflect the vibrating frequency. The last requirement amounts to require that the definition of the instantaneous frequency should give rise to n if is applied to cosnt.

Definition 1: We call a function on a finite or an infinite intervala *simple wave* (SW) if it is a continuous function with finitely many strict local extrema.

We say that f has a *strict local maximum* at t_0 if

$$f(t_0 + \Delta t) < f(t_0)$$

for all sufficiently small $\Delta t > 0$. *Strict local minimum* is defined similarly. By saying that f is *monotone* in an interval we mean

$$f(t+\Delta t) \geq f(t) \quad \text{for all} \quad t \quad \text{and} \quad \Delta t > 0$$

whenever both t and $t+\Delta t$ are in the interval; or

$$f(t+\Delta t) \leq f(t) \quad \text{for all} \quad t \quad \text{and} \quad \Delta t > 0$$

whenever both t and $t+\Delta t$ are in the interval. The above two cases are said to be, respectively, *monotonously increasing* or, *monotonously decreasing*, or, in brief, *increasing* or *decreasing*.

Definition 2: For any $\alpha > 0$ the quantity

$$\frac{1}{2}[\max\{2k : t-\alpha \leq t_{2k} \leq t+\alpha\} - \min\{2k : t-\alpha \leq t_{2k} \leq t+\alpha\}]$$

represents the (integer) number of the vibrations that f has in the interval $[t-\alpha, t+\alpha]$. Then the average of the above quantity over the interval of length 2α multiplied by 2π represents the number of vibrations over the interval of length 2π. The last quantity is given by

$$\frac{\pi}{2\alpha}[max\{2k : t-\alpha \leq t_{2k} \leq t+\alpha\} - \min\{2k : t-\alpha \leq t_{2k} \leq t+\alpha\}]$$

that is defined the α-Counting IF of f at the moment t.

Note that it is technical to choose $\alpha > 0$ for practical problems. If α is chosen too large, then what we are about average of the frequency, that dose not the time-varying property; and if α is chosen too small, say, smaller than the time needed to have a full vibration, then we get $\theta'_\alpha(t) = 0$.

Since there is no requirement for the maxima being positive and minima being negative, In particular, the method can be used to weak IMFs defined in [6]. The method can be used to IMFs and the basic components in wavelet [9] and AFD decompositions.

Assumption the defined α-Counting IF satisfies the first two of the required three conditions for instantaneous frequency, viz. the time-varying and non-negativity conditions. Now we show that they also satisfy the third condition. For $s(t) = \cos nt$ take $\alpha \geq \pi/n$. Simple computation shows that α-Counting IF $\theta'_\alpha(t) = n$ for all t. If α is taken smaller then π/n, then at all t there holds $\theta'_\alpha(t) = 0$.

Below we give a justification on the defined IF in relation to the frequencies in the existing time-frequency distributions. To our knowledge, besides the analytic instantaneous frequency for mono-components, in all the other cases, what we have are not IF, but just frequency. In any given time-frequency model, if there exists uncertainty principle, then there does not exist IF.

In general, a complicated signal is decomposed into a series of basic functions. For each basic function one can define reasonable “frequency”. For Fourier decomposition the basic functions are trigonometric functions of which each has a constant frequency. This kind of basic functions do not possess time-varying frequencies and do not offer good representations. For wavelet decomposition the basic functions are dilations and translations of the wavelet function. In such setting the dilation factor can be treated as frequency. Again, it is not time-varying. Window Fourier transform is widely used in practice. It is designed to have localization property.

It is expected that the related time-frequency distributions would exhibit much local properties of the signal, including the local frequencies. The effectiveness, however, is again restricted: it is bounded by uncertainty principle. For the time-frequency representations in the Cohen class the situation is similar. There could be only two types of instantaneous frequencies: the mono-component type (analytic instantaneous frequency) and the α-counting frequency. Each of them can only be used to a particular type of basic functions, of which the α-counting method is applicable to the largest class of functions. Based on each of these methods one can create a Dirac type time-frequency distribution [10]. To explore the effective usage of the respective time-frequency distributions is the task of the further studies.

III. The Algorithm of α–Counting IF

Based on the above mathematical definition, the flowchart of the α–Counting IF is illustrated in Fig. 2.

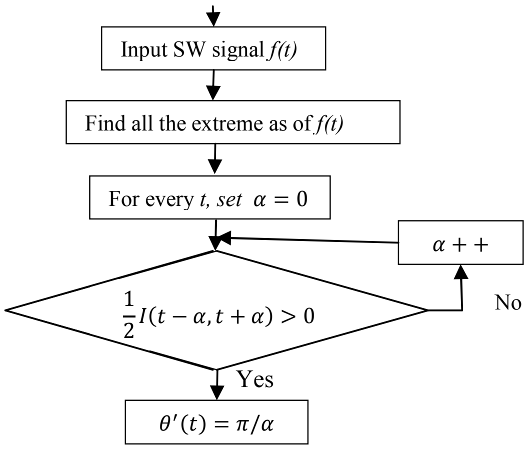

Fig. 2 The flowchart of the α –Counting IF computation

IV. Experiment Results

There are two experiments conducted in this section.

A. Experiment 1

In experiment 1, the IF of $\cos(10t)$ is computed in two ways: direct α –Counting IF computation and average IF computation through EMD.

As $\cos(10t)$ is a SW, the α –Counting IF computation can be applied directly. The original signal of $\cos(10t)$ and the IF result are shown in Fig. 3 and Fig. 4, respectively.

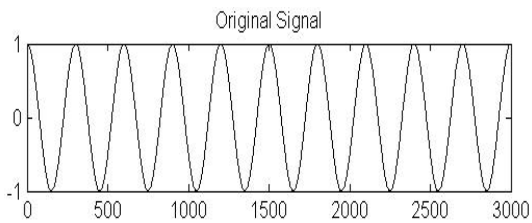

Fig. 3 $\cos(10t)$

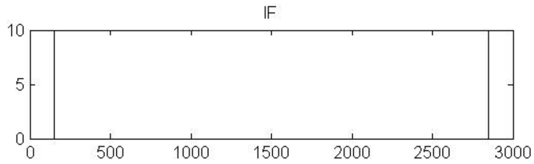

Fig. 4 The IF of $\cos(10t)$

Fig. 4 demonstrates that the α –Counting IF satisfies the third required condition of IF.

$\cos(10t)$ is not an IMF. It can be decomposed by EMD. The decomposed IMFs are illustrated in Fig. 5. Each IMF is a SW. α –Counting IF can be used to compute the IFs of IMFs. The corresponding IFs are illustrated in Fig. 6. The average IF through EMD is shown in Fig. 7. Fig. 7 demonstrates that the average IF through EMD does not consistent with the required third condition of IF.

B. Experiment 2

The data of experiment 2 comes from the closing prices of Hong Kong's Hang Seng Index. There were total 300 data used in the experiment. The original signal is illustrated in Fig.8. As the discrete data belongs to SW, the IF value can be computed directly by α –Counting IF approach. The IF result is illustrated in Fig. 9.

The real data signal is not an IMF. It can be decomposed by EMD. The IMFs of the real data through EMD is illustrated in Fig. 11. The IFs of IMFs computed by α –Counting IF is illustrated in Fig. 12. The average IF through EMD is illustrated in Fig. 13. The average IF through EMD does not consistent with the required third condition of IF for real data either.

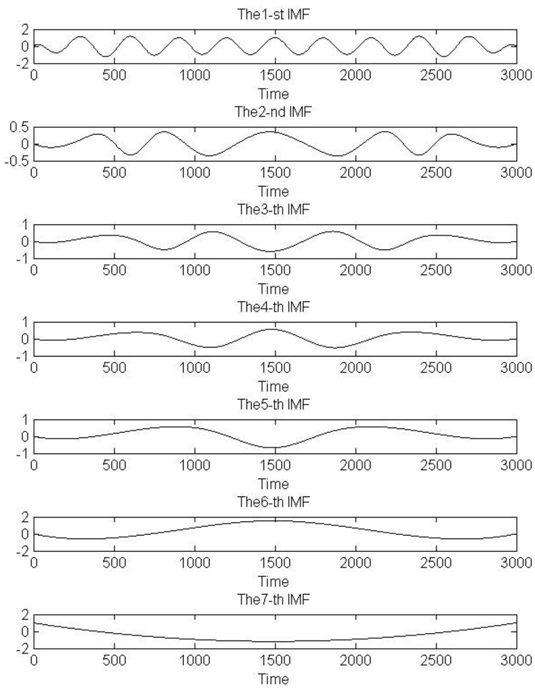

Fig. 5 The IMFs of $\cos(10t)$ through EMD

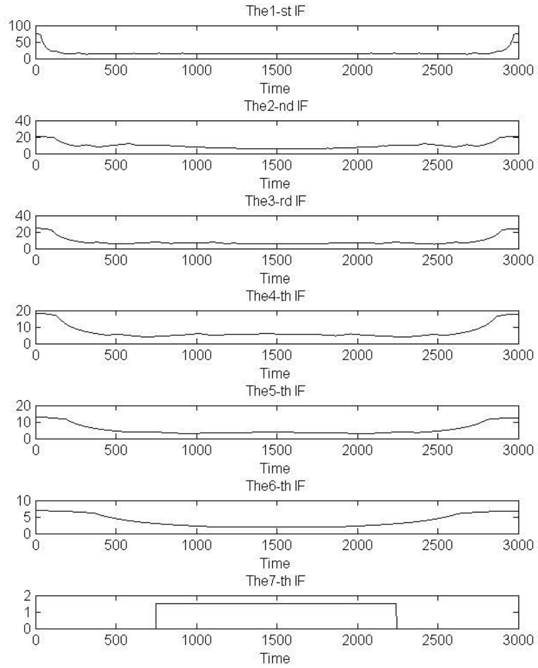

Fig. 6 The IFs of IMFs

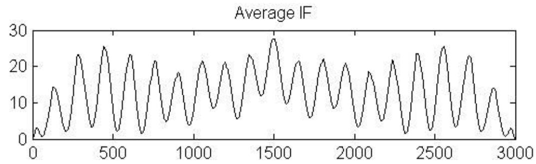

Fig. 7 The average IF through EMD

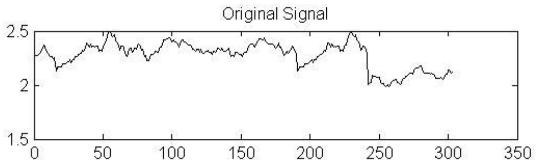

Fig. 8 The original signal of the real data

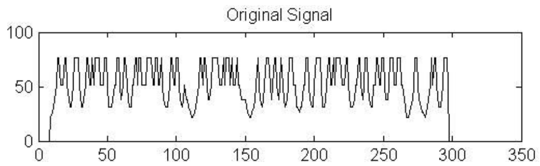

Fig. 9 The IF directly computed by α –Counting IF

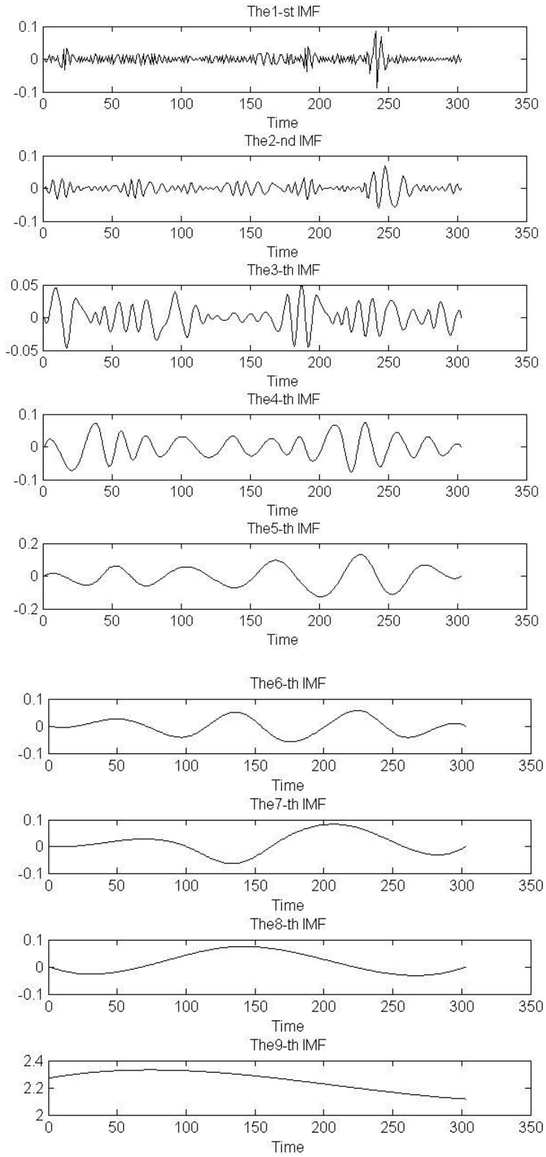

Fig. 10 The IMFs of the real data through EMD

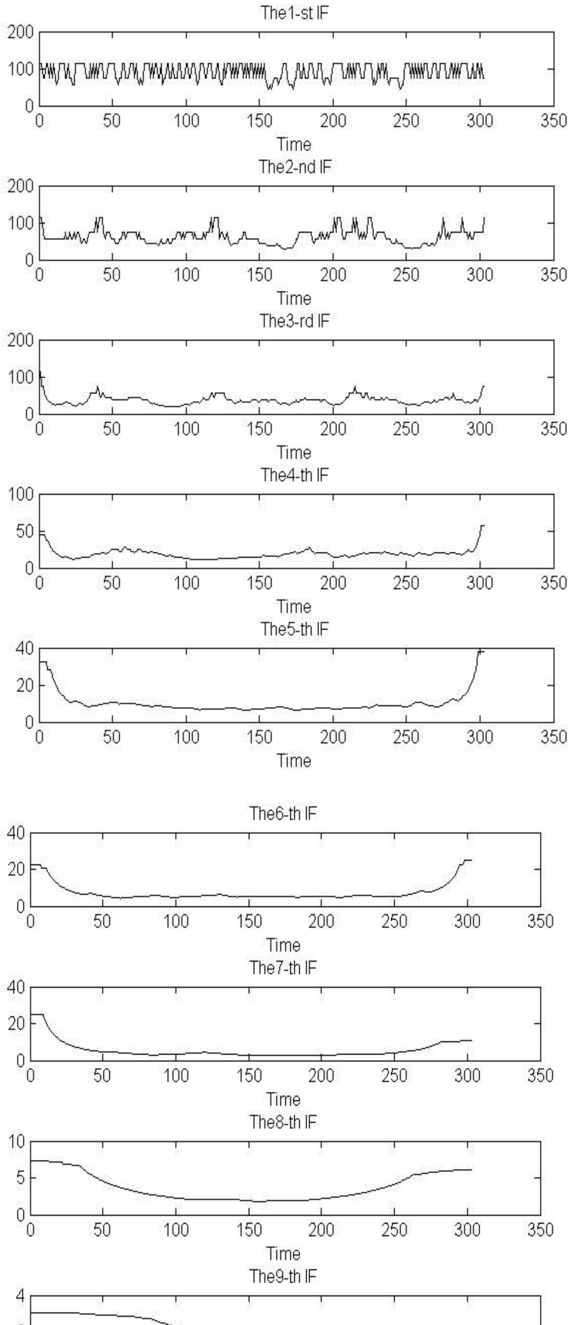

Fig. 11 The IFs of the IMFs

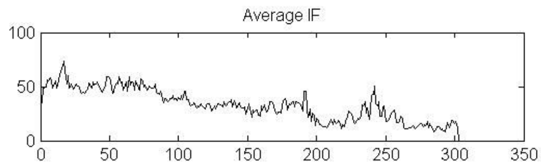

Fig. 12 The average IF through EMD

V. Conclusion

A novel Instantaneous frequency computation approach α –Counting IF is proposed in this paper. It can be used to a large class signals of SW. IMFs through EMD belong to SW. α –Counting IF can be used together with EMD to compute the IFs. α –Counting IF approach can also be used directly to any signal belonging to SW or discrete data. It should be noted that the directly computed IF does not consistent with the average IF through EMD.

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