A Novel Approach for Coin Identification using Eigenvalues of Covariance Matrix, Hough Transform and Raster Scan Algorithms

J. Prakash, and K. Rajesh

Abstract—In this paper we present a new method for coin identification. The proposed method adopts a hybrid scheme using Eigenvalues of covariance matrix, Circular Hough Transform (CHT) and Bresenham's circle algorithm. The statistical and geometrical properties of the small and large Eigenvalues of the covariance matrix of a set of edge pixels over a connected region of support are explored for the purpose of circular object detection. Sparse matrix technique is used to perform CHT. Since sparse matrices squeeze zero elements and contain only a small number of non-zero elements, they provide an advantage of matrix storage space and computational time. Neighborhood suppression scheme is used to find the valid Hough peaks. The accurate position of the circumference pixels is identified using Raster scan algorithm which uses geometrical symmetry property. After finding circular objects, the proposed method uses the texture on the surface of the coins called texton, which are unique properties of coins, refers to the fundamental micro structure in generic natural images. This method has been tested on several real world images including coin and non-coin images. The performance is also evaluated based on the noise withstanding capability.

Keywords—Circular Hough Transform, Coin detection, Covariance matrix, Eigenvalues, Raster scan Algorithm, Texton.

I. INTRODUCTION

ONE of the most significant problems in object recognition is to find the important features of an image. Successful identification of the features can be helpful in various fields including scientific research, industrial and biomedical applications. The detection of circular objects from an image is one of the basic tasks of model based computer vision. The problem of detecting circular features arises in many areas of image processing, analysis and object detection. The Hough Transform (HT) [1] and its variants [2] constitute a popular method for extracting geometrical features. Over decades many researchers modified the basic HT to detect geometrical primitives more efficiently.

The typical Hough based approaches to the problem of

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circular object detection employs an edge detector and uses edge information to deduce the centre locations. A variety of approaches have been suggested for detecting the circle and estimating the associated parameters. In particular, the Standard Hough Transform (SHT) method [3] is used to characterize the analytical curves such as lines, circles and ellipses. It is less sensitive to noise, shape discontinuities and partial occlusions. However, it suffers the disadvantage of a massive requirement of computations and memory storage and the complexities grow exponentially with the number of feature points. To solve this problem many improvements have been suggested. Tsuji and Matsumoto [4] decomposed the parameter space and used the symmetric property of circles. Based on gradient information, Ballard D.H et al. [5] [6] presented a method which simplifies the parameterization by deriving the circle centre using the coordinates and tangent of an edge pixel. Illingworth et al. [7] finds better centre finding process and used Adaptive Hough Transform (AHT) to estimate other parameters. Davis [8] used a two pass algorithm to locate a circle centre in two 1-D arrays. Based on both magnitude and direction of image gradients Michele et al. [9] detected the circular objects using orientation matching. It is capable to overcome the problems arising due to gradient based voting schemes. Several other methods [10] [11] utilize the randomized selection of edge points and geometrical properties of circles, instead of using the information of edge pixels and evidence gathering histograms in parameter space.

On the other hand, some non-HT methods for circle detection were proposed by other researchers. In particular Chun Ta Ho et al. [12] proposed a simple algorithm to detect circles using geometric symmetry properties. Peng-Yeng Yin [13] proposed a new method for circle detector, which adopts a hybrid scheme, consists of a Genetic Algorithm (GA) phase and a local search phase. This method considerably reduces the computation and storage cost. Based on the statistical parameters such as Eigenvalues of covariance matrix of boundary points over a small region of support, Du-Ming Tsai et al. [14] identifies the corner and circular arcs. D.S. Guru et al. [15] proposed statistical and geometrical properties of the small Eigenvalues of the covariance matrix for the purpose of straight line identification. In this method small Eigenvalue analysis is used to decide on the prominence of a pixel as a linear pixel. The same concept along with HT and Raster scan

technique is utilized to extract linear, circular and elliptical objects in images [16] [17].

From the above discussion, it is evident that, the parametric space analysis of Hough Transform, statistical analysis of edge images and geometrical properties of objects are the three parameters used to detect the circular primitives in images. In our proposed method, we utilized the features of all the three parameters. In this paper, our problem is to detect coins from a combined image of coins and circular objects. To distinguish the coins from general circular objects, we used the texture on the surface of the coins.

II. PROPOSED METHODOLOGY

The proposed method for detecting coins in real world images has the following steps.

Step 1: For the given grayscale image, find the edge image using suitable edge detection operators.

Step 2: Find the covariance matrix for the edge image and obtain the small and large Eigenvalues.

Step 3: Obtain the CHT for Eigenvalue image using sparse matrix technique.

Step 4: Find the meaningful set of distinct Hough peaks using neighborhood suppression scheme [18].

Step 5: Once the candidate peaks and their locations are identified, find center of the circular objects and obtain the circumference pixel locations using Bresenham's Raster scan algorithm [19].

Step 6: After extracting circular objects, identify the coins based on the texton.

III. CIRCULAR OBJECT DETECTION

Extracting geometrical primitives such as circles from digital image has received more attention for several decades. The Circular Hough Transform (CHT) identifies the circular shape with a given radius. For unknown radii the algorithm should be run for all possible radii to form a 3-D parameter space. In our work we used the fact that the Eigenvalues of the covariance matrix characterize the shape information of the geometric objects.

To Find the Eigenvalues of the covariance matrix for the set of edge pixels covered by the window 'w'

$$\lambda = \frac{1}{2}\left[a_{11} + a_{22} \pm \sqrt{(a_{11} - a_{22})^2 + 4a_{12}^2}\right] \quad (1)$$

where $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is the covariance matrix and the coefficients a_{11} or a_{22} is the variance and a_{12} or a_{21} is the covariance of 'w'.

$$\text{var(x)} = a_{11} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \text{ , where } \bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\text{var(y)} = a_{22} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 \text{ ,where } \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i \text{ and}$$

$$\text{cov(x, y)} = a_{12} = a_{21} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

From Eqn. (1) the small and large Eigenvalues of a covariance matrix can be expressed as

$$\lambda_s = \frac{1}{2}\left[a_{11} + a_{22} - \sqrt{(a_{11} - a_{22})^2 + 4a_{12}^2}\right] \quad (2)$$

$$\lambda_l = \frac{1}{2}\left[a_{11} + a_{22} + \sqrt{(a_{11} - a_{22})^2 + 4a_{12}^2}\right] \quad (3)$$

In our work, we used the fact that, the small and large Eigenvalues for the circular segment in the continuous domain will be equal regardless of the size and orientation of the circular segment [14]. So

$$\lambda = \begin{cases} \lambda_s \cong \lambda_l = t & f(x,y) \text{ is a point of a circular segment} \\ \infty & \text{Noisy pixel} \end{cases}$$

where 't' is the predefined threshold value.

To test the characteristics of small and large Eigenvalues for a circle, an experiment has been conducted for different radii circles. The small and large Eigenvalues for the covariance matrix of the circles with different window sizes is listed in the Table I. From this we observe that the increment of circle radius yields the decrement of the λ_s and λ_l values. If the range of the desired radius is known priori, the centre of the circle can be easily found by finding local maxima of the Hough peaks. For unknown radii, the window size can be varied until the correct radius is obtained. The correctness of the coordinate points and the coordinates of the discontinuity region are calculated using the Bresenham's or midpoint raster scan circle algorithms. In Bresenham's algorithm, to eliminate the unequal spacing between points along the circular boundary, parametric polar form, $x = x_c + r\cos\theta$ and $y = y_c + r\sin\theta$ is used. In midpoint algorithm [19] the circle function is defined as $f(x,y) = x^2 + y^2 - r^2$. For unit step in 'x' direction use a decision parameter to determine, which of the two possible 'y' positions is closer to the circle path. The initial value of the decision parameter is $p_0 = 5/4 - r$. At each x_k position, starting at $k = 0$, if $p_k < 0$, the next point along the circle is (x_{k+1}, y_k) and $p_{k+1} = p_k + 2x_{k+1} + 1$. Otherwise the next point along the circle is $(x_k + 1, y_k - 1)$ and $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$, where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$. In the same way symmetry points are calculated for other octants also.

TABLE I

SMALL AND LARGE EIGENVALUES OF A CIRCLE $x^2 + y^2 = r^2$

Radius 'r'	Region of support W=5x5		Region of support W=7x7		Region of support W=9x9	
	λ_s	λ_L	λ_s	λ_L	λ_s	λ_L
30	0.249	0.249	1.397	1.398	2.542	2.544
50	0.089	0.090	0.465	0.465	0.840	0.841
70	0.042	0.042	0.231	0.231	0.419	0.422
90	0.023	0.023	0.140	0.140	0.256	0.257

IV. EXPERIMENTAL RESULTS

In this section, we present the results of experiment in which the proposed method is applied to several coin and non-coin images. In the first phase, we extract the circles from the edge data. This involves, locating all edge fragments in an image and the small Eigenvalues λ_s and large Eigenvalues λ_L of covariance matrix constituted by the edge image. The CHT is performed for the Eigenvalue image to find the centre and radius of the circles which uses highest Hough peaks in the accumulator space. A circle is drawn at the identified centre and radius using Bresenham's algorithm.

The circles extracted from the first phase correspond to both coin and non-coin structures. In the second phase to distinguish coins from non-coins, we use texton [20] of the coins. Since texton as a centre of the cluster in filter response space [21], we used k-means clustering [22] [23] in our experiments.

To describe the first phase of the proposed work, an experiment is conducted on a real world image which has only Indian coins. Fig. 1(a) shows 25 paise, 50 paise, 1 Rupee, 2 Rupees and 5 Rupees coins. The corresponding edge image using canny operators is as shown in Fig. 1(b). The CHT for the Eigenvalues of the covariance matrix of edge image is as shown in Fig. 1(c). The circles extracted for the desired Hough peaks representing the circumference of the coins is as shown in Fig. 1(d). To reveal the correctness and accuracy, the circles identified are overlapped with original image is as shown in Fig. 1(e).

To test the robustness of circular object identification capability in our proposed approach, an experiment is conducted on an image which has circular and non-circular coins. Fig. 2(a) shows 25, 50 paise and 01, 05 rupees circular coins along with non-circular coins. Among non-circular coins 05 paise has square structure, 10 paise has half sine wave edged and 20 paise has hexagon structure. The edge mapped image is as shown in Fig. 2(b). Since the small Eigenvalues λ_s for line segments is zero [14], we can eliminate the 5 paise and 20 paise coins from the test set easily. The large and small Eigenvalues of 10 paise are not equal, since it is not representing the either circular feature or linear feature. So it can also be eliminated from the test set easily. The CHT with valid Hough peaks for edge image is as shown in Fig. 2(c). The circles extracted correspond to only circular coins from the proposed approach are as shown in Fig. 2(d). The identified circular coins from non-circular coins are as shown in Fig. 2(e). From this, it is evident that the proposed approach is very effective in identifying circular features.

(a)   (b)   (c)
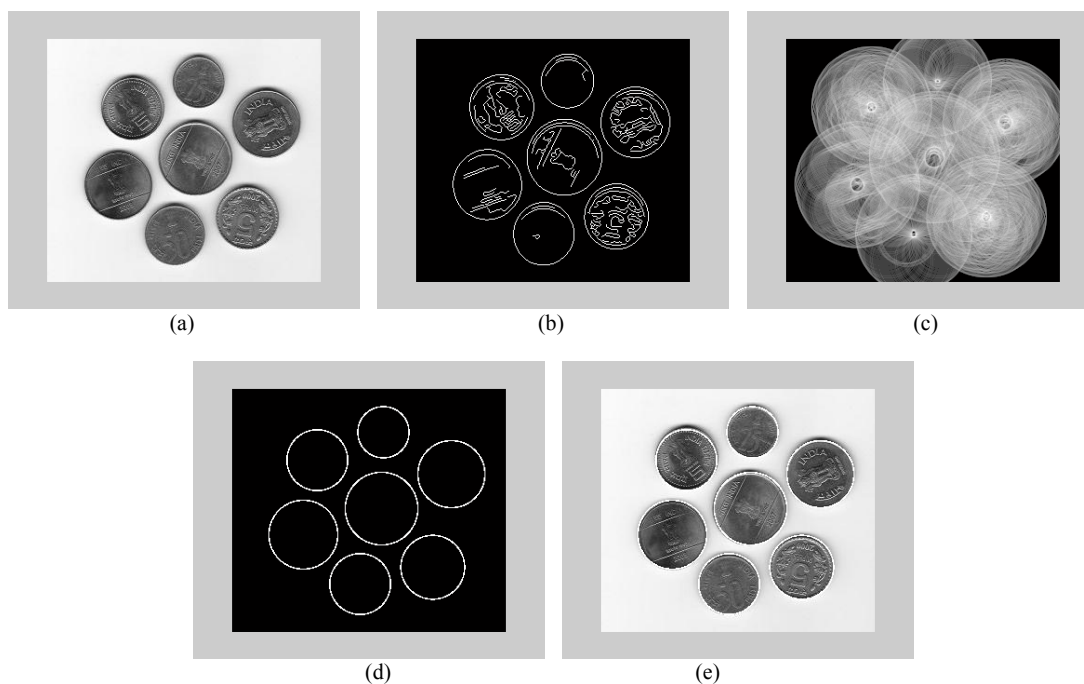

(d)   (e)

Fig. 1 Identification of circular features (a) Original coins image (b) Edge extracted image (c) CHT image (d) Circles identified (e) Circles overlapped with original image
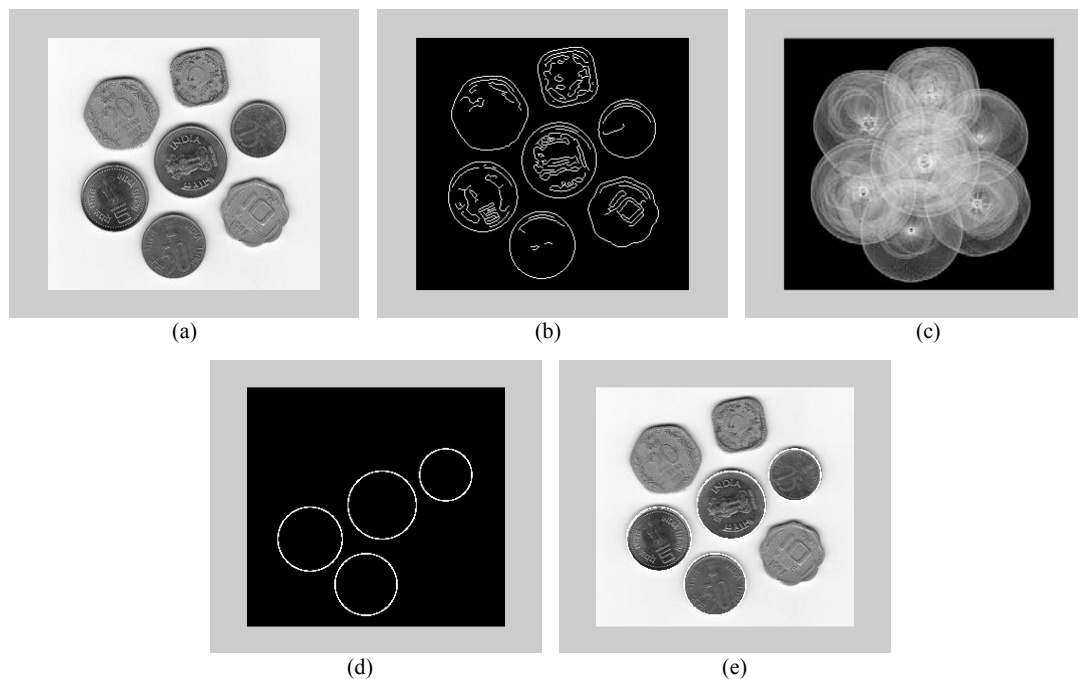
(a)　　　　　　　　　　(b)　　　　　　　　　　(c)

(d)　　　　　　　　　　(e)

Fig. 2 Identification of circular features from circular and non-circular coins (a) Original Image (b) Edge image (c) CHT with valid Hough peaks (d) Circles extracted (e) Circles overlapped with original image

To describe the second phase of the proposed method and to identify the valid coins from an image, we conducted an experiment based on texton structure of the coins. Since textons represent the cluster centre in the filter response space, we used the classical k-means algorithm for clustering. For experimentation the two faces of the coins, King and Queen are considered. A set of 5 coins with two faces each and 32 combinations for each coin are taken into account. Fig. 3 represents the clustered images with k=5 for the king face side of 25, 50 paise and 01, 02 and 05 rupees images. In the modelling stage, each pixel of the given image is mapped to its nearest texton. Since each texton represents the microstructure to discriminate or recognize the smallest element, each pixel is labeled exactly as one texton. Fig. 4 shows the Probability Density Function (PDF) of normalized histogram model of the texton images. From this figure we noticed that, all texton histogram graphs of statistical models are almost same. This fact is used in our approach to distinguish coin and non-coin in images. The PDF of the non-coin structures is as shown in Fig. 5, which represents the false positive circular feature from Hough Transform. From this, we observed that the PDFs of circular coin and circular non-coin images are absolutely different from each other.

We can classify the coin and non-coins by just comparing two PDF models. For identification purpose, if the minimum distance among the PDFs of the known coins and the PDF of the test image is less than a given threshold 'T', then it may be considered as a coin or non-coin otherwise. The following equation can be used for identification purpose.

$$C = MIN\left(\sum_{i=1}^{N} \frac{\left(H(i) - h(i)\right)^2}{\left(H(i) + h(i)\right)} \right)$$

where $H(i)$ and $h(i)$ represents the PDFs of testing and original coin images respectively, N denotes the number of test classes. If $C<T$, then 'H' is considered as PDF of the coin image. Fig. 6(a) shows the circular features identified from the Standard Hough Transform method. It extracts the all the circles, but does not distinguish the coins and non-coins. The circular features extracted are overlapped with original image is as shown in Fig. 6(b). The circles which represent the non-coins are marked with arrows. The result of our proposed approach is as shown in Fig. 6(c). It identifies only the circular features corresponds to coins. It discards the circles of non-coin images. To test the accuracy the circles identified are overlapped with original image is as shown in Fig. 6(d). From this, it is evident that, the proposed technique based on texton can identify the coins very accurately and also distinguishes the coins from non-coin images.

Another evaluation is done to test the robustness of our method against noise. In this evaluation for the original image shown in Fig. 7(a), a Gaussian white noise with variance of 0.01 and 20 % density of salt and pepper noise is added as shown in Fig 7(b). The edge image shown in Fig. 7(c) is a very complex image, since it has considerable amount of noise. The best fit circles correspond to coins extracted from the proposed approach are as shown in Fig. 7(d). To test the accuracy the extracted circles are overlapped with the noisy image and original image are as shown in Fig. 7(e), 7(f) respectively. From this evaluation, we conclude that the proposed method gives accurate circular features for coins in noise conditions also.
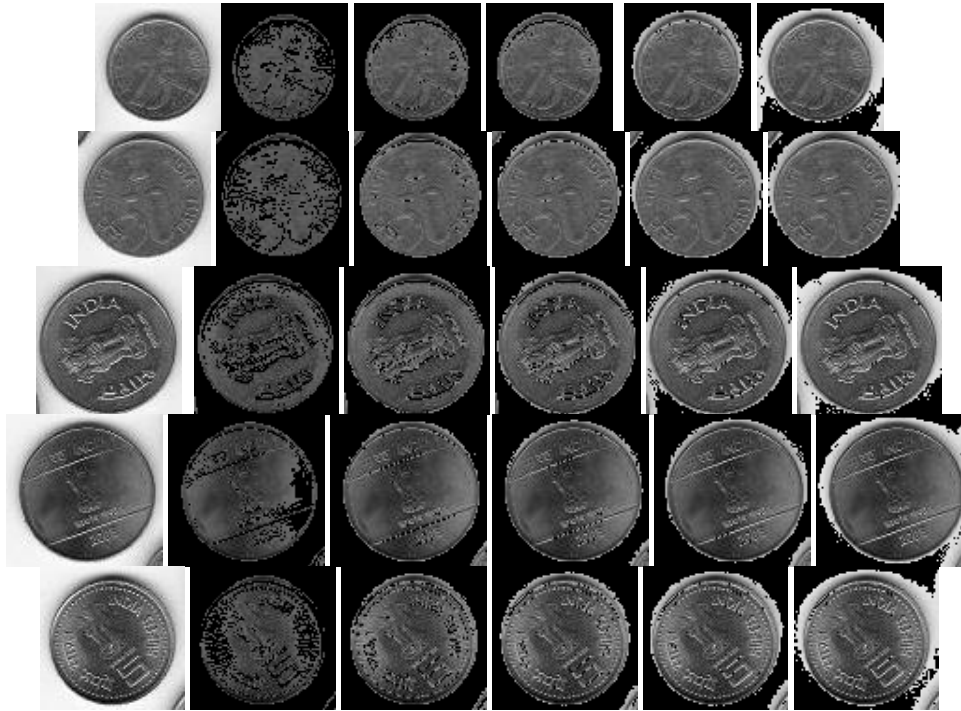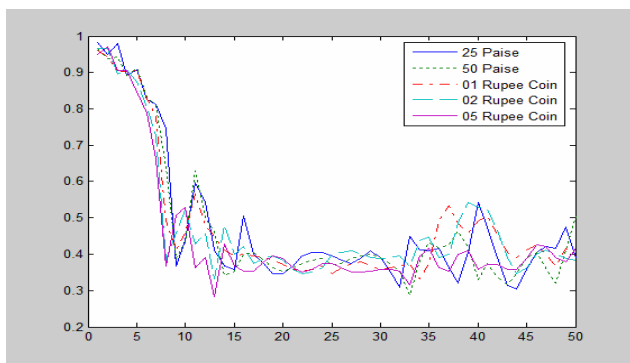
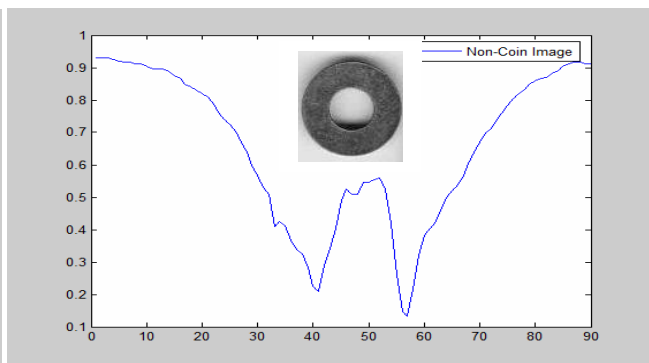Fig. 3 Circular coins clustered with k=5 iterations

Fig. 4 PDFs of Coin images

Fig. 5 PDF of Non-Coin image

(a)   (b)   (c)   (d)
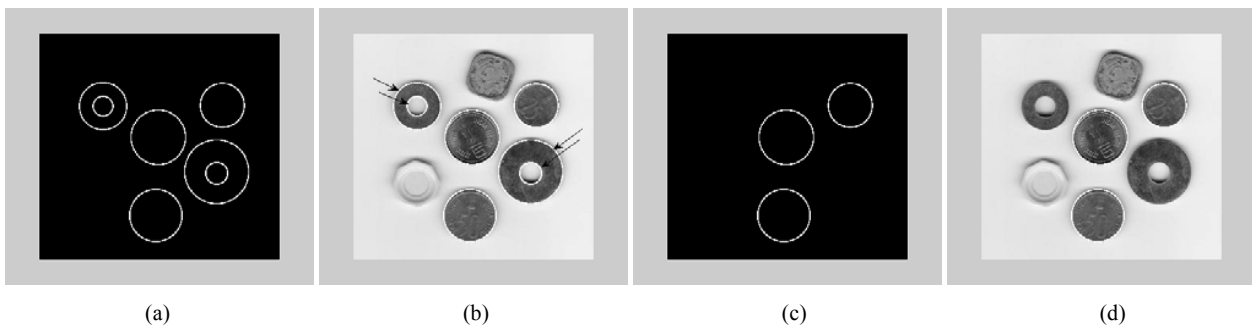
Fig. 6 Coins Identified using Texton structures (a) Circles extracted from HT method (b) Circles overlapped wit original image (c) Circles correspond to Coins (d) Identified Coins
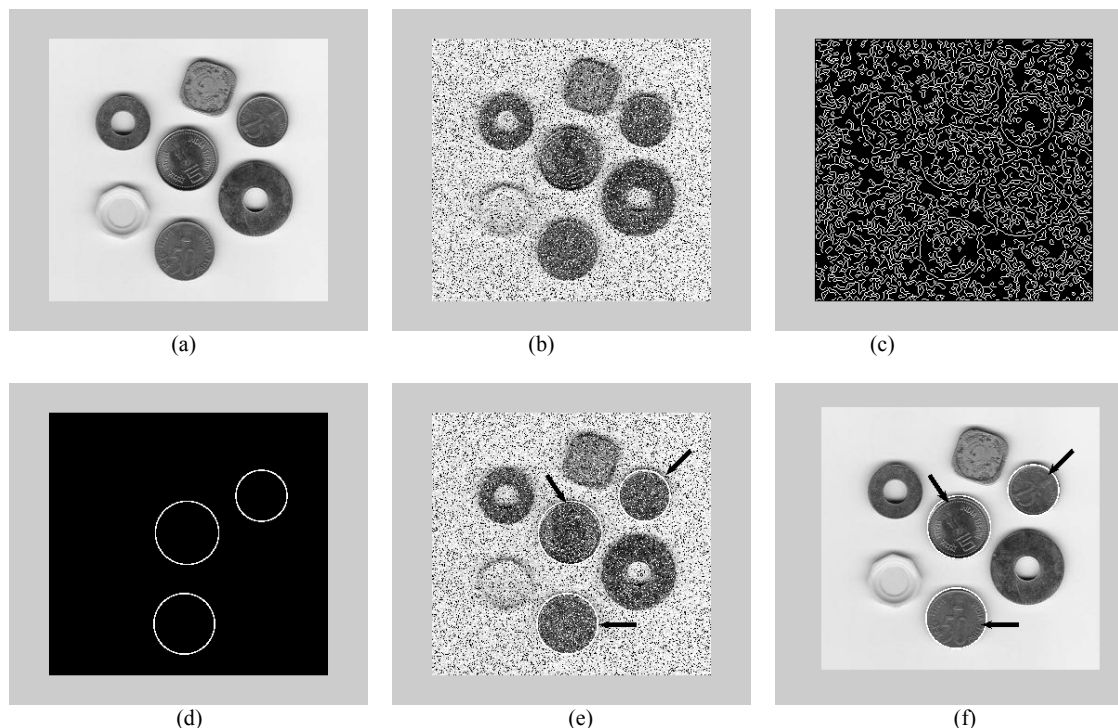
Fig. 7 Coins detected in noisy image (a) Original image (b) Noisy image (c) Edge image (d) Circles detected for coins (e) Circles overlapped with noisy image (f) Circles overlapped with original image

To test the noise withstanding capability of our method, an additional experiment is conducted using salt and pepper noise with different densities. In this evaluation the density of salt and pepper noise was increased from 0 to 80 percent and the identification rate was evaluated. Results of this experiment show that the noise with less than 34% density has no effect on the accuracy of the coin identification. Further increasing the noise density increases the complexity in CHT causing the centre and Eigenvalues of the circle changes. Hence the identification rate reduces. Fig. 8 draws the identification rate versus noise density. From this it is evident that our proposed method can withstand noise and is efficient to identify the coins up to 56 percent of noise density.
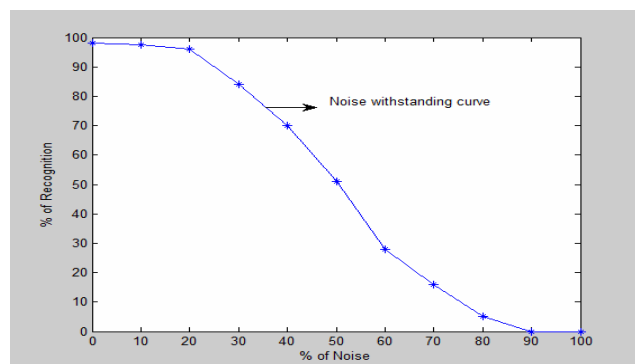

Fig. 8 Noise resistance curve

V. CONCLUDING REMARKS

In this paper we discussed an efficient and accurate method for coin detection. The distinct features of our proposed method from HT based methods are that, it is a hybrid scheme consisting of Eigenvalues approach, CHT and Raster scan algorithms. As compared to conventional HT methods, the main strengths of our method are its low computational time, less memory requirement and accuracy of detection. Since sparse matrix technique is used for CHT, the amount of data required for circle detection and coin identification is reduced. The proposed method also uses the texture on the surface of the coins for its identification. The PDFs of coins are compared with PDFs of non-coin images to distinguish the coins from non-coins. Performance evaluation is done by conducting the experiments on non-coin images and noisy images. The limitations of our approach includes, scaling of texton classifier and identification of spatial texture property of coin and non-coin images.

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