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A Note on MHD Flow and Heat Transfer over a Curved Stretching Sheet by Considering Variable Thermal Conductivity

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***Abstract*—**The mixed convective flow of MHD incompressible, steady boundary layer in heat transfer over a curved stretching sheet due to temperature dependent thermal conductivity is studied. We use curvilinear coordinate system in order to describe the governing flow equations. Finite difference solutions with central differencing have been used to solve the transform governing equations. Numerical results for the flow velocity and temperature profiles are presented as a function of the non-dimensional curvature radius. Skin friction coefficient and local Nusselt number at the surface of the curved sheet are discussed as well.

***Keywords*—**Curved stretching sheet, finite difference method, MHD, variable thermal conductivity.

I. Introduction

FLOW and heat transfer over a stretching sheet is widely discussed due to its applications in engineering and industry sectors such as polymer extrusion, paper product, manufacture of plastic sheet, wire drawing, drawing plastic films, glass fiber and so on. This product quality depends on the stretching, rate of heat transfer and skin friction at the sheet. The boundary layer flow and heat transfer over a linear or nonlinear stretching sheet, surface or flat plate were investigated by many researchers. However, very few literatures indicated the fluid flow past over a curved stretching surface. Abbas et al. [1] studied the fluid flow and heat transfer in the presence of magnetic field over a curved stretching sheet. Siddheshwar et al. [2] discussed the flow and heat Transfer over a curved stretching sheet and they considered temperature distribution in the flow is prescribed surface temperature and heat flux. Further Abbas et al. [3] also investigated flow and heat transfer in boundary layer over a curved stretching sheet with heat generation and thermal radiation in the presence of electrical conductivity. Flow of viscous fluid past a curved stretching sheet with applied magnetic field is studied by Hayat et al. [4]. Naveed et al. [5] reported the effect of MHD on micropolar fluid over a curved stretching surface. Rosca and Pop [6] presented the unsteady fluid flow over a curved stretching surface. Hayat et al. [7] examined the flow of ferrofluid due to nonlinear stretching curved sheet and they considered that fluid is electrically conducting in the presence of magnetic field. Sanni et al. [8] examined the flow analysis with nonlinear stretching velocity over a curved stretching surface.

In this paper, the MHD flow and heat transfer over a curved stretching sheet in the presence of significant buoyance force and variable thermal conductivity is presented. The governing equations induced are due to curved surface in the form of curvilinear coordinates system. Computations are performed for a wide range of governing parameters such as magnetic field parameter, power law exponent temperature parameter, and other involved parameters, and the effect of these parameters on the velocity and temperature field is presented. Besides, the finite difference solutions are generated for skin-friction coefficient and rate of heat transfer.

II. Model Analysis

We consider the flow of a viscous incompressible fluid passing over a curved stretching surface. The geometry of the model is described by curvilinear coordinates (r,s,z). R is the radius of the curvature. The sheet is stretched due to the two opposite and equal forces which act along the s direction where r is perpendicular to it (see Fig. 1). The stretching velocity of the sheet is $u = as$, a is the stretching constant. A uniform magnetic field B_0 is applied to the r direction.

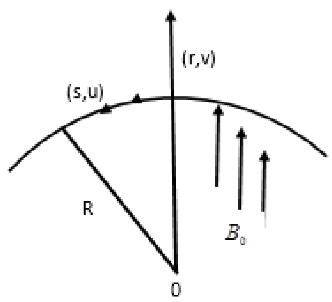

Fig. 1 Model geometry

The equation of motion which governs the flow is

$$\frac{\partial}{\partial r}\{(r+R)v\} + R\frac{\partial u}{\partial s} = 0$$

$$\frac{u^2}{r+R} = \frac{1}{\rho}\frac{\partial p}{\partial r}$$

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$$v\frac{\partial u}{\partial r}+\frac{R}{r+R}u\frac{\partial u}{\partial s}+\frac{uv}{r+R}=\frac{1}{\rho}\frac{R}{r+R}\frac{\partial p}{\partial s}+$$
$$\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+R}\frac{\partial u}{\partial r}-\frac{u}{(r+R)^2}\right)-\frac{\sigma B_0^2 u}{\rho}+g\beta\left(T-T_\infty\right)$$
$$\rho c_p\left(v\frac{\partial T}{\partial r}+\frac{R}{r+R}u\frac{\partial T}{\partial s}+\sigma B_0^2 u^2\right)=\frac{1}{r+R}\frac{\partial}{\partial r}\left\{(r+R)k\frac{\partial T}{\partial r}\right\}$$

These partial differential equations are to be solved with appropriate boundary conditions

$$u=as,\ v=-v_w,\ T=T_w\quad at\quad r=0$$

$$u\to 0,\ T\to T_\infty,\ p+\frac{1}{2}\rho_\infty q^2=const\ as\ r\to\infty$$

Introduce $u=asf',\ \ v=-\frac{R}{r+R}\sqrt{a\nu}f,\ p=\rho a^2 s^2 P(\eta),$

$$\eta=\sqrt{\frac{a}{\nu}}R,\ \ \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

Assume that the fluid thermal conductivity varies as a linear function of temperature, i.e.

$$k=k_\infty\left(1+b\left(T-T_\infty\right)\right)=k_\infty\left(1+m\theta\right)$$

where m is the thermal conductivity parameter, and k_∞ is the fluid thermal conductivity far away from the sheet.

Momentum and energy equations reduce to

$$P'=\frac{f'^2}{\eta+K}$$

$$f'''+\frac{K}{\eta+K}ff''+\frac{K}{(\eta+K)^2}ff'+\frac{1}{\eta+K}f''-\frac{K}{\eta+K}f'^2$$
$$-\frac{1}{(\eta+K)^2}f'-\frac{2K}{\eta+K}P-Mf'+\lambda_1\theta=0$$
$$(1+m\theta)\theta''+(1+m\theta)\frac{\theta'}{\eta+K}+m\theta'^2+\Pr\frac{K}{\eta+K}f\theta'-M\lambda f'^2=0$$

The boundary conditions are

$$f=A,\ f'=1,\ \theta=1\ \ at\quad \eta=0$$

$$f'\to 0,\ \theta\to 0,\ P\to 0\ \ as\ \eta\to\infty$$

where

$$K=\sqrt{\frac{a}{\nu}},\ M=\frac{\sigma B_0^2}{\rho a},\ \Pr=\frac{\mu c_p}{k},\lambda_1=\frac{g\beta(T_w-T_\infty)}{a^2 s}$$

which are defined as a curvature parameter, magnetic number, Prandtl number and mixed convection parameter, respectively.

The skin friction coefficient and the local Nusselt number of the problem along the s-directions are defined by

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho u_w^2},\ \ where\ \tau_w=\mu\left(\frac{\partial u}{\partial r}-\frac{u}{(r+R)}\right)\Bigg|_{r=0}$$
$$\Rightarrow \mathrm{Re}_s^{1/2}C_f=f''(0)-\frac{f'(0)}{K}=f''(0)-\frac{1}{K}$$

and local Nusselt number is

$$Nu=\frac{sq_w}{k(T_w-T_\infty)},where\ \ q_w=-k\frac{\partial T}{\partial r}\bigg|_{r=0}$$
$$\Rightarrow \mathrm{Re}_s^{-1/2}\,Nu=-\theta'(0)$$

III. Results and Discussion

The boundary value problem along with the boundary conditions is solved numerically using finite difference solutions with central differencing by Kafoussias and Williams [9]. We have computed our results of the skin friction coefficient with those tabulated by Sanni et al. [8] and they are found to be in good agreement.

TABLE I

Expansion of Skin Friction Coefficient $-\mathrm{Re}_s^{1/2}C_f$

Curvature parameter K	Sanni et al. [8]	Present
5	1.1576	1.1591
10	1.0734	1.0745
20	1.0355	1.0363
30	1.0235	1.0241
40	1.0176	1.0181
50	1.0140	1.0145
100	1.0070	1.0074
200	1.0036	1.0039
1000	1.0008	1.0011

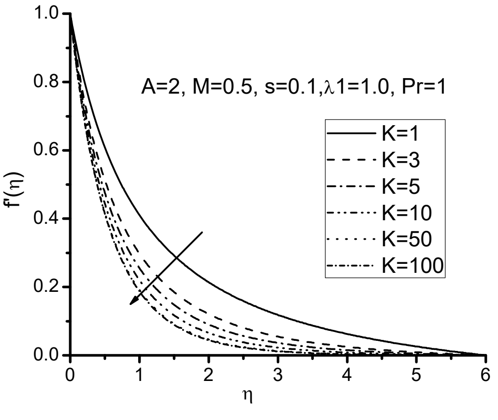

Fig. 2 Influence of k on f'

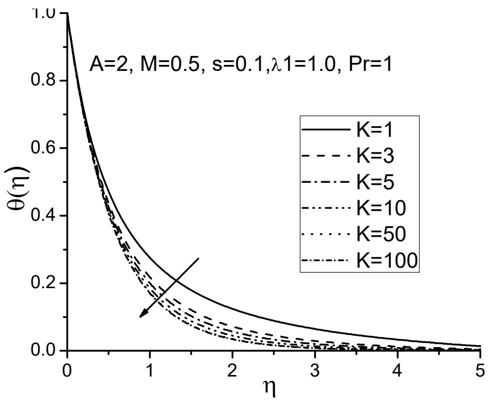

Fig. 3 Influence of k on θ

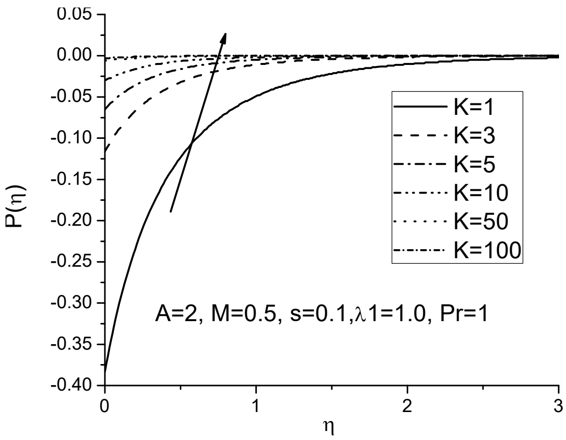

Fig. 4 Influence of k on P

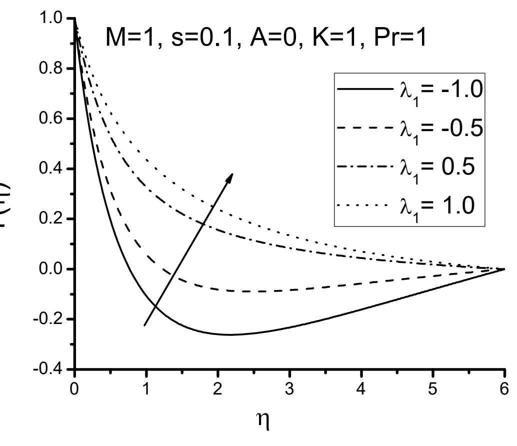

Fig. 5 Influence of λ_1 on f'

Figs. 2-4 illustrate the impact of curvature parameter on velocity, temperature and pressure profiles and we see from Fig. 2 that, for larger value of curvature parameter, the velocity profile and the momentum boundary layer decrease. It also appears that the higher values of K are significantly smaller than the small value of K in all flow cases. From Fig. 3, we observe that the temperature decreases with the increases of radius of curvature parameter. Note that for increasing value of radius of curvature parameter, the sheet becomes flat and heat transfer rate from the sheet to fluid is slower than the curved sheet. From Fig. 4, we see that the pressure distribution increases with the increase of radius of curvature parameter and for flat surface (for the larger value of radius of curvature parameter) and the pressure distributions tend to zero.

Figs. 5-7 respectively show the effect of mixed convection parameter on velocity, temperature and pressure distribution. From these figures, it can be seen that the velocity profile increases with increasing convection parameter and reduces temperature distribution. It is noted that, $\lambda_1 > 0$ means temperature transfer from the sheet to the fluid. In the other words, heating of the fluid and cooling of the sheet and for $\lambda_1 < 0$ means the fluid temperature transfer to the sheet from the fluid, i.e. sheet is heated. The mixed convection parameter λ_1 increases with the increase in the temperature difference between sheet and fluid. Due to this velocity profile and boundary layer thickness increases. For increasing value of convection parameter, the rate of heat transfer from sheet to fluid increases and hence decreases the thermal boundary layer thickness. Also, we observe that pressure distribution decreases as mixed convection parameter increases.

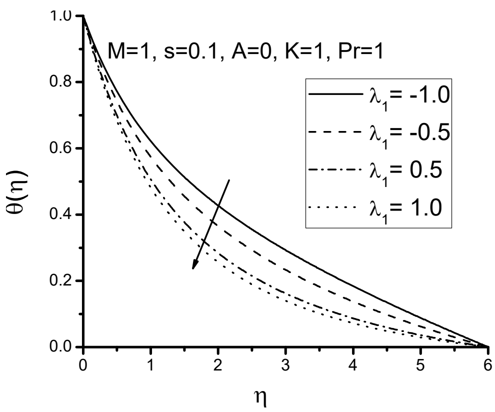

Fig. 6 Influence of λ_1 on θ

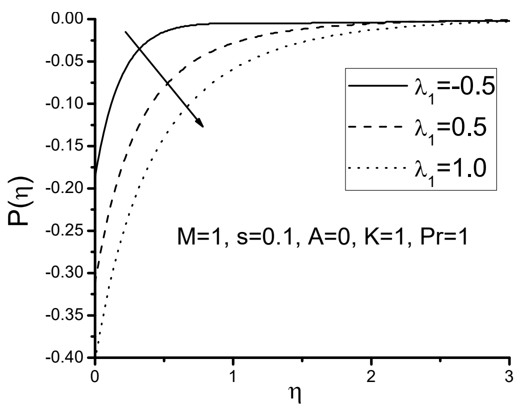

Fig. 7 Influence of λ_1 on P

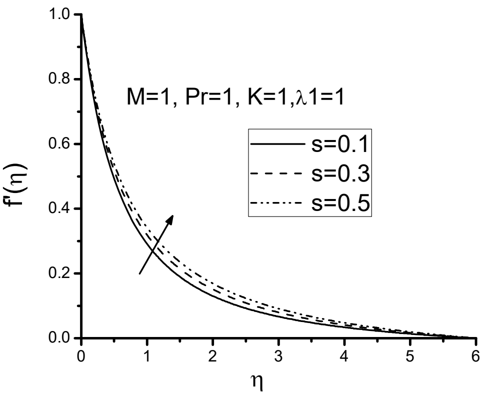

Fig. 8 Influence of Son f'

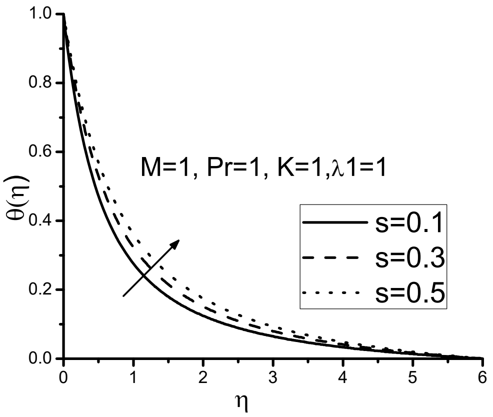

Fig. 9 Influence of Son θ

The effect of thermal conductivity parameter on velocity and temperature distributions are shown in Figs. 8 and 9. We observe that velocity and temperature profile is increases as increases thermal conductivity parameter. This fact is due to magnitude of temperature which increases with the increase in thermal conductivity parameter, and both boundary layer thicknesses increase as well.

The effect of stabilization parameter A [suction parameter A>0 and injection parameter A<0] on velocity and temperature profiles are shown in Figs. 10 and 11. It is observed that the increasing value of stabilization parameter decreases the velocity. From Fig. 11, it is clear that temperature decreases with stabilization parameter. For injection (A<0), we observed as curved surface satisfying boundary conditions.

Figs. 12 and 13 depict the effects of curvature parameter, magnetic number, thermal conductivity parameter and mixed convection parameter on skin friction and rate of heat transfer. From these figure, we see that for increasing magnetic number and curvature parameter, skin friction coefficient increases, whereas it decreases with mixed convection parameter and thermal conductivity parameter. On the other hand, rate of heat transfer decreases with magnetic number and thermal conductivity parameter.

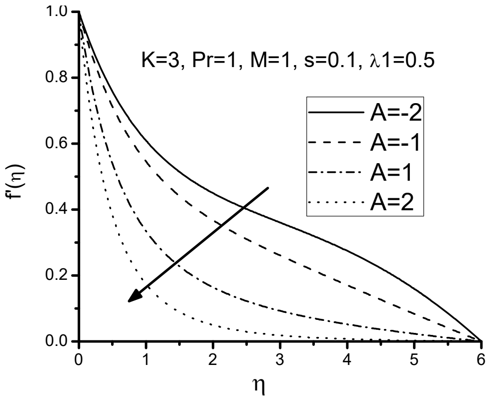

Fig. 10 Influence of A on f'

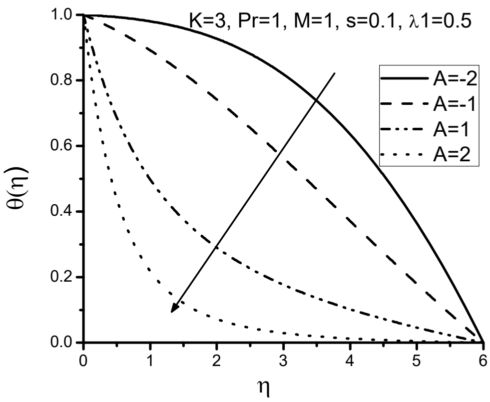

Fig. 11 Influence of A on θ

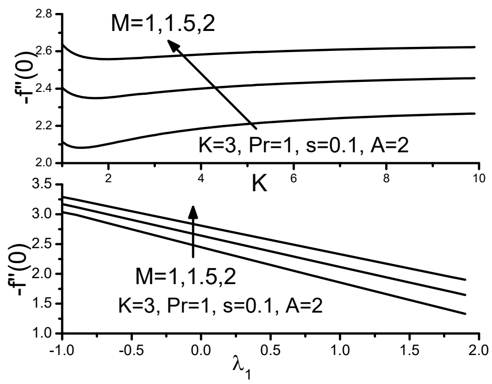

Fig. 12 Influence of M, λ_1, K on $f''(0)$

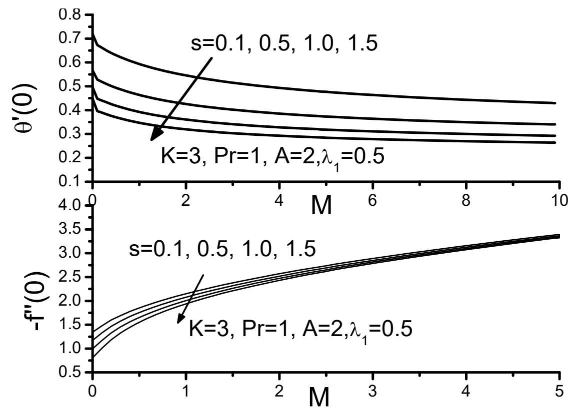

Fig. 13 Influence of M, s on $f''(0)$

IV. Conclusion

The following observations can be made from the present findings:

(i) Velocity and temperature profiles decrease with an increase in the radius of curvature, and pressure distribution tends to zero for larger value of radius of curvature parameter.

(ii) Impact of magnetic parameter on velocity and temperature profile is opposite

(iii) Velocity increases with an increase in mixed convection parameter, but temperature and pressure have opposite impact.

(iv) Increasing thermal conductivity leads to enhance the velocity and temperature boundary.

(v) The effect of the magnetic number and curvature parameter is to reduce the surface heat transfer but to induce the skin friction coefficient. Also, mixed convection has a reverse effect on velocity gradient (skin friction coefficient).

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