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A New Velocity Expression for Open Channel and its Application to Lyari River

Rana Khalid Naeem and Asif Mansoor

Abstract**—**In this communication an expression for mean velocity of waste flow via an open channel is proposed which is an improvement over Manning formula. The discharges, storages and depths are computed at all locations of the Lyari river by utilizing proposed expression. The results attained through proposed expression are in good agreement with the observed data and better than those acquired using Manning formula.

Keywords**—**Comparison, Depth, Flow, Open Channel, Proposed Model, Storage

I. INTRODUCTION

CHEZY [1] developed an equation which computes mean velocity in an open channel flow and this equation is,

$$\mathrm{V} = C\sqrt{RS} \tag{1}$$

where R is the hydraulic radius in meter and S is the channel slope. In (1) C is called Chezy's coefficient and is dimensional having dimensions $(\text{length})^{1/2}$ per time. The Chezy's coefficient is determined by experiments. Manning performed series of experiments and found that dependence on hydraulic radius is actually not as given in (1), and modified (1). The modified equation, called Manning equation, is

$$\mathrm{V} = \frac{1}{n} R^{2/3}\sqrt{S} \tag{2}$$

in which n is Manning's resistance coefficients and its value depends on the surface material of the Channel's, wetted perimeter and is determined from experiments. This formula is more accurate then Chezy and is widely used now a days. The relationship between Chezy's C and Manning's n is easily shown to be

$$C = \frac{1}{n} R^{1/6} \tag{3}$$

After the appearance of the Manning work, many formulae are proposed for C by researchers. Here we quote some of them and for others the reader can refer to references there in [2]-[6].

Bazin [7], considered Chezy C to be a function of R but not of S and presented the following formula for computation of C

$$C = \frac{87}{1+\dfrac{M}{\sqrt{R}}} \tag{4}$$

in which M is the coefficient of roughness of the channel. Pavlovsky (1925), modified C in Manning formula. The modified C is

$$C = \frac{1}{n} R^{y} \tag{5}$$

where $y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}(\sqrt{n} - 0.10)$. It depends upon the roughness coefficient (n) and hydraulic radius (R). This formula is widely used in U.S.S.R.

Kennedy [8] gave a classic empirical equation correlating mean velocity V in a regime canal with vertical depth D, measured on the approximately horizontal silted bed. The mean velocity involves critical velocity $\mathrm{V_o}$ is obtained by

$$\mathrm{V_o} = 0.55\, D^{0.64} \tag{6}$$

The hydraulic diagrams given by Kennedy are extensively used in India for designing irrigation canals. These diagrams enable to design any number of channel section for given slope with different bed width and depth.

In the present work we propose the following formula for computing the mean velocity through an open channel. The formula is

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} C_{model} \tag{7}$$

This formula is an improvement over Manning formula and C_{model} is a dimensionless number defined in (13). It is worth noting that in the proposed V, the C_{model} is non-dimensional while in the earlier works in which Chezy C is modified, are all dimensional. The formula is employed in computing V through the Lyari river. The computed V is compared with the observed values and the values obtained through Manning formula. It is seen that the computed values through the proposed V (7) are in good agreement with the observed values, and better than the values through Manning formula.

II. MODEL CONSTRUCTION

The hydraulic condition of open channel flow is a function of the channel geometry, discharge, roughness, and slope

The discharge Q through cross sectional area A is given by [9], [10]

$$Q = AV \tag{8}$$

where V is the mean velocity of flow.

Rana Khalid Naeem, PhD, Professor of Mathematics, is with University of Karachi, Karachi-75270, Pakistan. (e-mail: 2rknaeeem@gmail.com).

Asif Mansoor is research scholar Department of Mathematics University of Karachi, Karachi-75270, Pakistan (phone: 92-301-2990882; e-mail: asifmansure@hotmail.com).

The Lyari river can be nearly approximated as an open channel with trapezoidal cross section. The mean velocity for flow through it, employing Manning formula, is given by

$$V = \frac{h^{\frac{2}{3}} W^{\frac{2}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}} \tag{9}$$

where W, h and z are river width, channel flow depth and side slope respectively.

Since the cross sectional area A is given by

$$A = Wh \tag{10}$$

Equation (8), utilizing (9) and (10), yields

$$Q_{manning} = \frac{W^{\frac{5}{3}} h^{\frac{5}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}} \tag{11}$$

Equation (9) and (11) are used for computing the mean velocity and discharge through Lyari river. The subscript in (11) indicates that discharge Q utilizes Manning formula.

In order to obtain dimensionless expression for C in the proposed formula (7), we set

$$C(= C_{model}) = C(R, W) \tag{12}$$

The dimensional analysis tells that C can be expressed in terms of π -term 'R/W' and therefore, we write.

$$C_{model} = C_{model}(R/W) \tag{13}$$

Many rational approximations for C_{model} are attempted and it is found that ratio of two cubic for C_{model} produces very good results. The rational approximation for C_{model} is

$$C_{model} = \frac{1 + ax + bx^2 + cx^3}{1 + dx + ex^2 + fx^3} \tag{14}$$

where $x = R/W$ and

$a = 1.09E\text{-}3$

$b = 8.29E\text{-}3$

$c = 1.08E\text{-}3$

$d = 8.99E\text{-}4$

$e = 1.17E\text{-}3$

$f = 9.15E\text{-}4$

The discharge using C_{model} is

$$Q_{model} = \left(\frac{1 + .00109x + .00829x^2 + .00108x^3}{1 + .000899x + .00117x^2 + .000915x^3}\right)\left[\frac{W^{\frac{5}{3}} h^{\frac{5}{3}} \sqrt{s}}{n\left[W + zh\sqrt{1+z^2}\right]^{\frac{2}{3}}}\right] \tag{15}$$

The balance of the surface water store, S_s, within two river bridges is given by

$$\frac{dS_s}{dt} = I - Q \tag{16}$$

in which I is the inflow.

We now represents equations for surface water store, S_s, and depth, h which already exists in literature and used in the present study.

The surface water, S_s, is assumed to be a linear function of outflow discharge [11] and is given by

$$S_s = \tau Q \tag{17}$$

in which 'τ' is the travel time between two bridges under consideration. If 'L' is the distance between two bridges, then

$$\tau = \frac{L}{V} \tag{18}$$

Equation (16), utilizing (8), (10) and (17), yields

$$\frac{dh}{dt} = \frac{1}{LW}\left[I - WhV\right] \tag{19}$$

Equation (19) describes the flow in terms of rate of change of the flow depth for a given river section.

On using an explicit forward step finite difference approximation for (19), we obtain

$$h_{t+1} = h_t + \frac{\Delta t}{LW}\left[I_t - WhV_t\right] \tag{20}$$

Equation (16) is solved numerically. The discretization of (19) on xt-plane (Fig. 1) leads to

$$\frac{I_t + I_{t+1}}{2} - \frac{Q_t + Q_{t+1}}{2} = \frac{S_{t+1} - S_t}{2} \tag{21}$$

in which

I_t = inflow at time level 1
I_{t+1} = inflow at time level 2
Q_t = outflow at time level 1
Q_{t+1} = outflow at time level 2
S_t = storage at time level 1
S_{t+1} = storage at time level 2
Δt = time interval

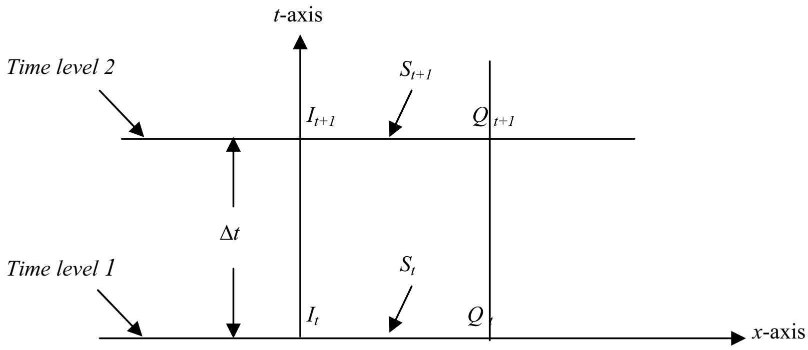

Fig. 1 Discretization of storage equation in *xt*-plane

Then Storage (S_{t+1}) at any time 't+1', after rearranging (20), is given by

$$S_{t+1} = S_t + \frac{1}{2}\left[I_t + I_{t+1}\right]\Delta t - \frac{1}{2}\left[Q_t + Q_{t+1}\right]\Delta t \tag{22}$$

III. Comparison of Computed and Observed Values

Comparisons are made between discharges (mean monthly, mean annual); mean monthly storage, mean monthly depth obtained from Manning *V* and proposed *V* and observations at all eleven bridges of the Lyrai river. It is seen that at all locations of the Lyari river the proposed *V* produces better results than the Manning *V*. There are thirty two figures of comparisons, and in here we, for the sake of completeness, give only four of them (Fig. 2-5). In Fig. (2-5), by calculated flow, we mean using Manning *V*.

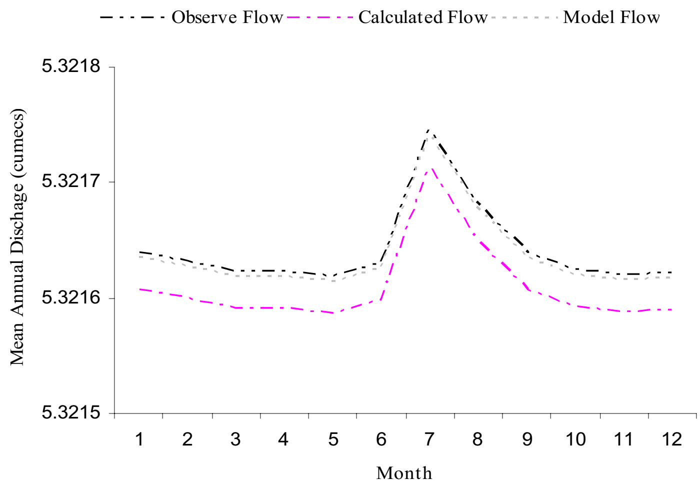

Fig. 2 Comparison of mean annual discharge.

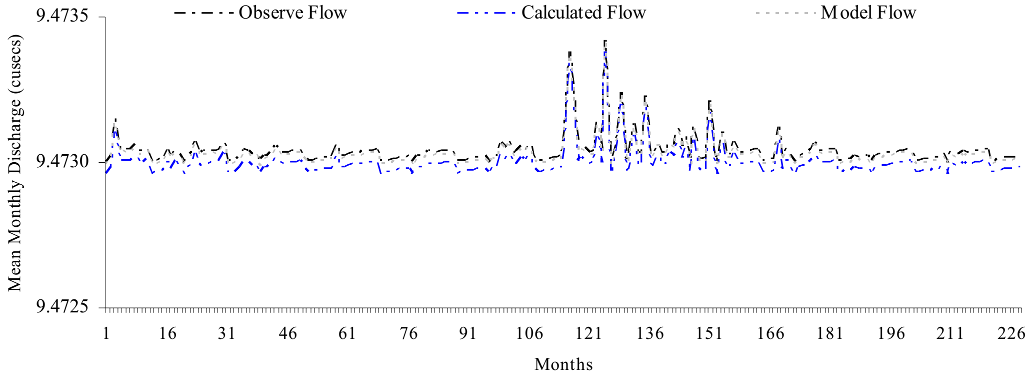

Fig. 3 Comparison of mean monthly discharge.

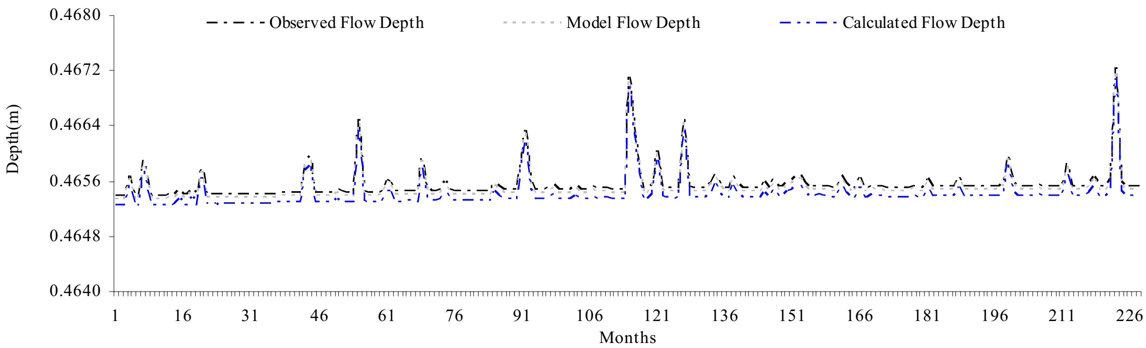

Fig. 4 Comparison of depth of Lyari waterways.

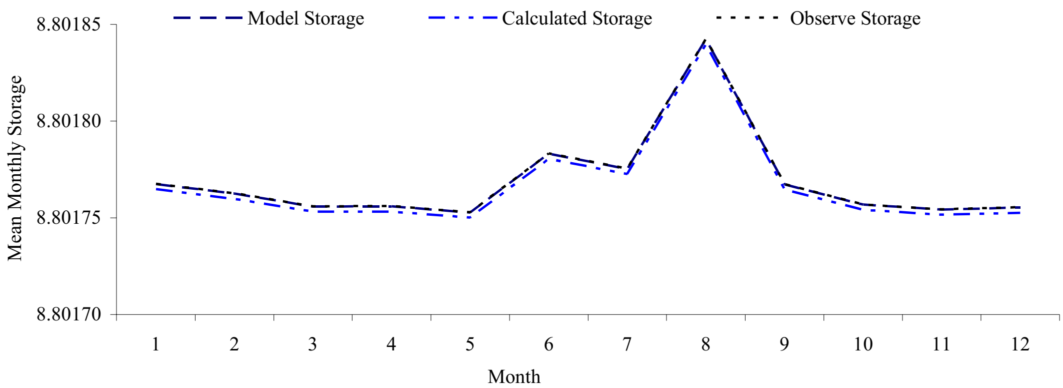

Fig. 5 Comparison of mean monthly storage between two consecutive bridges.

IV. CONCLUSION

In the present work, an improved version for the model proposed by Manning for computing mean velocity of open channel is proposed. These involves non-dimensional *C*, while in the earlier works in which Chezy *C*, is modified, are all dimensional. The discharges, storages and depths are computed at all bridges of Lyari river using proposed *V*, and Manning *V*. It is observed that results obtained through proposed *V* are good agreement for the recent data set than those obtained by means of Manning *V*.

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