A New Stabilizing GPC for Nonminimum Phase LTI Systems Using Time Varying Weighting

Mahdi Yaghobi, and Mohammad Haeri, *Member, IEEE*

Abstract—In this paper, we show that the stability can not be achieved with current stabilizing MPC methods for some unstable processes. Hence we present a new method for stabilizing these processes. The main idea is to use a new time varying weighted cost function for traditional GPC. This stabilizes the closed loop system without adding soft or hard constraint in optimization problem. By studying different examples it is shown that using the proposed method, the closed-loop stability of unstable nonminimum phase process is achieved.

Keywords—GPC, Stability, Varying Weighting Coefficients.

I. INTRODUCTION

STABILITY of MPC has drawn attention for many years, since local optimization in a finite preview horizon does not guarantee stability in general [1]. The most widely referenced approach to guarantee stability in MPC procedures is to add an equality constraint on the Final state in the prediction horizon (hard Constraint) [2, 3] or to put a weight on the final state in the cost function (soft constraint) [4-6]. Another approach is to use an infinite prediction horizon with a finite control horizon [7] that makes it possible to apply standard linear quadratic regulator (LQR) theory to guarantee stability.

We will show that for some processes, stability doesn't occur with the existing methods and therefore propose a new method to stabilize these processes. The main idea is to use a new time varying weighting matrices in the cost function of the GPC.

The paper is organized as follows. In Section II, GPC controller is described. In Section III, the proposed varying weighting method is discussed. Numerical simulations are carried out in Section IV for illustration and verification of the presented methodology. Finally some concluding remarks are given in Section V.

II. GPC CONTROLLER FORMULATION

Generalized predictive control (GPC) has been widely used in process control engineering, because of its good tracking performance and ability to manipulate constraints. Since it is

M. Yaghobi is with the Electrical Engineering Department, Islamic Azad University, Mashhad, Iran, (e-mail: yaghobi@mshdiau.ac.ir).

Dr. M. Haeri is with the Electrical Engineering Department, Sharif University of Technology, and the head of the Advanced Control System Lab. Iran (phone: +98-21-616-5964, fax: +98-21-602-3261, e-mail: haeri@sina.sharif.edu).

usually designed based on the transfer function representation of the process, formulations are given on this basis.

Consider the ARIMAX (Auto-Regressive Integrated Moving-Average eXogenous) model given by:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})}{\Delta}e(t), \quad \Delta = 1 - q^{-1} \tag{1}$$

where q is the shift operator, $u(t)$ and $y(t)$ are the input and the output of the process respectively. $e(t)$ represents the unpredictable parts of the process behavior. For Convenience, $C(q^{-1})$ is assumed to be one, which means that the disturbance sequences are white noise.

In order to derive a GPC formulation, the future outputs of the system are predicted based on the knowledge of the future inputs and the past inputs and outputs. The following relations including Bezout identity are employed in this regard.

$$y(t + j) = G_j \Delta u(t + j - 1) + f(t + j), \quad j = 1, 2, \dots, P \tag{2}$$
$$f(t + j) = H_j \Delta u(t - 1) + F_j y(t)$$
$$1 = (A.\Delta).E_j + q^{-j}.F_j$$
$$B.E_j = G_j + q^{-j}.H_j$$

The vector-matrix form of relation in (2) is given as:

$$\boldsymbol{y} = \boldsymbol{G}\Delta\boldsymbol{u} + \boldsymbol{f} \tag{3}$$

$$\Delta\boldsymbol{u} = [\Delta u(t) \quad \Delta u(t+1) \cdots \Delta u(t+P-1)]^T$$
$$\boldsymbol{y} = [y(t+1) \quad y(t+2) \cdots y(t+P)]^T$$
$$\boldsymbol{f} = [f(t+1) \quad f(t+2) \cdots f(t+P)]^T$$

$$\boldsymbol{G} = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ g_2 & g_1 & 0 & \cdots & 0 \\ \vdots & & & & \\ g_P & g_{P-1} & \cdots & & g_1 \end{bmatrix}$$

The cost function in an ordinary GPC is given by (4).

$$J = \sum_{j=1}^{P}(q_j(y_d(t+j) - y(t+j))^2 + r_j \Delta u(t+j-1)^2) \tag{4}$$

$y_d(t+i)$ is the desired trajectory for the closed loop output and is determined as the output of a first order filter described by the following transfer function.

$$y_d(t) = \frac{1-\alpha}{1-\alpha q^{-1}}r(t) \tag{5}$$

$r(t)$ is the set point signal and α assumes values between 0 and 1.

Minimizing of the cost function in (4) by adjusting input variations Δu results in the following performances;

1. Minimizing the difference between the process output and its desired trajectory.
2. Minimizing the variation of the future inputs.

The optimal input variations are derived as;

$$\Delta u = (G^T Q G + R)^{-1} G^T Q (y_d - f) \qquad (6)$$

Using the first term of $\Delta u(t)$ (i.e. $\Delta u(t)$) $u(t)$ is obtained from the following equation and applied to the process.

$$u(t) = \Delta u(t) + u(t-1)$$

Based on the obtained result and the latest measured data the whole procedure is repeated in the next sampling intervals.

III. TIME VARYING WEIGHTING COST FUNCTION

Since output of an unstable process grows with time, the error of predicted output grows with time as well. To stabilize an unstable process it is necessary to control the growth of prediction error. To do the job, we propose a new cost function in which the early sentences in the prediction horizon have greater weight than the later sentences. The goal is achieved by using variable parameters in the weight matrices of Q and R in the cost function of (5). Q, and R matrices are selected as follows:

$$Q = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \beta^1 & 0 & \dots & 0 \\ 0 & 0 & \beta^2 & & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & \beta^{p-1} \end{bmatrix}, \ R = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \alpha^1 & 0 & \dots & 0 \\ 0 & 0 & \alpha^2 & & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & \alpha^{p-1} \end{bmatrix}$$

When α and β take values between 0-1, the prediction error is penalized tightly in the early stages. This matter leads to the point that the trust of the first sentences in optimization is greater than the final sentences. This action has two outcomes: first, the error weight of the first predictive steps in the optimization is considered more than the others, and second, first terms of the predictive outputs play more roles in the optimization. By applying this method on an unstable process it is expected to have bounded prediction errors.

IV. COMPUTER SIMULATIONS

To compare the proposed method with the ordinary GPC as well as its stabilizing variants, different simulations were conducted and some of the results are illustrated here.

A. Process with Non-Repetitive Right Hand Side Poles

In analyzing this kind of process such as the process of equation (5), current methods such as Constrained Receding Horizon Predictive Control (CRHPC) that stabilize the closed loop system using the hard constraint, and GPC with End point state Weighting (WGPC) that stabilize the system using the soft constraint are used where in each case by suitable selection of P (predictive horizon) and M (control horizon) (e.g. $P = M = 6-8$) and the final constraint number (e.g. $m = 3$), stability of the closed loop system is achieved.

$$G(s) = \frac{0.25s + 0.025}{s^2 + 0.45s - 0.025} \qquad (7)$$

B. Process with Non-Repetitive Right Hand Side Poles and Zeros

The process of equation (8) has one pole and one zero in the right hand side of the s-plane.

$$G(s) = \frac{-0.25s + 0.025}{s^2 + 0.45s - 0.025} \qquad (8)$$

Using ordinary GPC with any combination of the control parameters was not resulted in a stable closed loop system. However, application of the CRHPC design method along with the control parameters of $P = M = 6-8$ and the final states constraint $m = 3$ stabilizes the closed loop system.

C. Process with Repetitive Right Hand Side Poles

The process of equation (9) has repetitive poles in the right hand side.

$$G(s) = \frac{s^2 + 0.2s + 0.26}{s^3 - 0.9s^2 + 0.15s + 0.025} \qquad (9)$$

The existing methods of stabilization such as CRHPC, WGPC, and Mixed Weighting GPC (MWGPC) [8] were examined in which all the mentioned conditions (of each method) are considered. We couldn't find any set of control parameters by which the closed loop system becomes stable.

The proposed method of this paper was implemented by the following set of control parameters. Stability of the closed loop was achieved and the results are shown in Fig. 1.

$$P = M = 5, \ \beta = 0.05, \ \alpha = 0.05$$

D. Process with Repetitive Poles and Non-Repetitive Zeros in the Right Hand Side

The process of equation (10) has real repetitive poles and non-repetitive zeros in the right hand side.

$$G(s) = \frac{s^2 - 0.2s + 0.26}{s^3 - 0.9s^2 + 0.15s + 0.025} \qquad (10)$$

The existing stabilizing methods were implemented, but none of them was able to stabilize the closed loop system. Conditions required in each method were satisfied and wide range of the control parameters was examined. Our proposed method could able to stabilize the closed loop system by the

following set of parameters. Results of the simulation are given in Fig. 2.

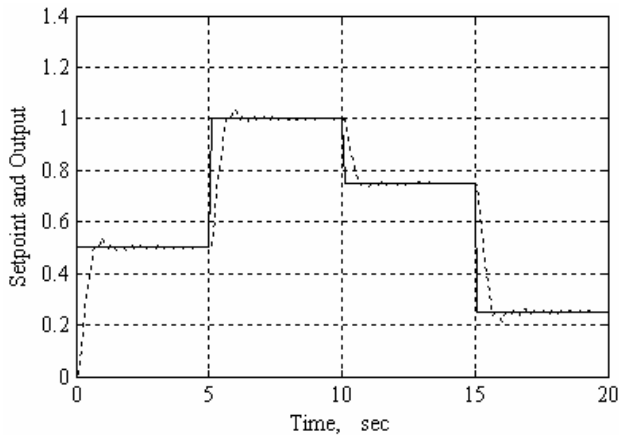$$P = M = 5, \quad \beta = 0.05, \quad \alpha = 0.05$$

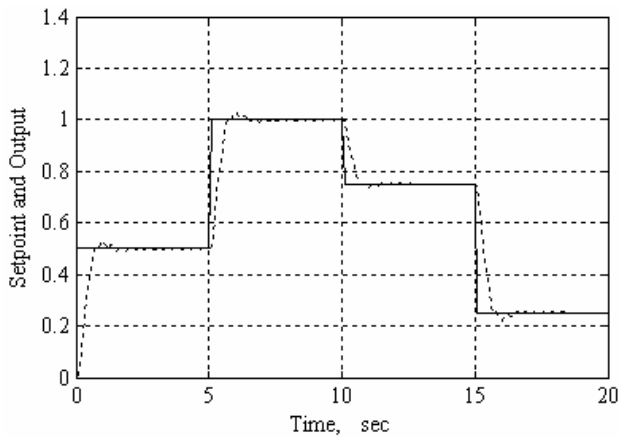Fig. 1 Simulation results for the process in (9)

Fig. 2 Simulation results for the process in (10)

V. CONCLUSION

We propose a new method of choosing weighting matrices in order to penalize the prediction error in the early stages of the prediction horizon. It seems that this choice control rapid variations of the prediction error in some special unstable process. Among general advantages of the proposed method, the followings may be of considerable ones.

1. Since the constraints don't exit in the optimizing problem, the problems such as offset and feasibility are avoided.

2. The optimal value of α and β can be determined offline, and then the controller can be used in online form.

3. In studying different processes, it becomes appear that by selecting aforementioned optimized parameters; the system can be controlled with minimum control and predictive horizon.

4. Since α and β are determined offline, in the practice one can use the broad and strong searching methods such as simplex search or genetic algorithm.

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