A New Class $\Gamma^2\left(M,\phi,\Delta^{\mu}_{\gamma},p\right)^F$ of The Double Difference Sequences of Fuzzy Numbers

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Abstract—The double difference sequence space $\Gamma^2\left(M,\phi,\Delta^{\mu}_{\gamma},p\right)^F$ of fuzzy numbers for both $1 \le p < \infty$ and $0 < p < 1$, is introduced. Some general properties of this sequence space are studied. Some inclusion relations involving this sequence space are obtained.

Keywords—Orlicz function, solid space, metric space, completeness.

I. Introduction

THE concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [11] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations of fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming.

Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$is said to be convergent if and only if the double sequence $(S_{mn})$is convergent, where

$S_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m,n = 1,2,3,...)$ (see[1]).

We denote w^2 as the class of all complex double sequences (x_{mn}). A sequence $x = (x_{mn})$is said to be double analytic if

$$sup_{mn} |x_{mn}|^{1/m+n} < \infty.$$

The vector space of all prime sense double analytic sequences are usually denoted by Λ^2. A sequence $x = (x_{mn})$ is called double entire sequence if

$$|x_{mn}|^{\ 1/m+n} \to 0 \text{ as } m,n \to \infty.$$

The vector space of all prime sense double entire sequences are usually denoted by Γ^2. The space Λ^2 as well as Γ^2 is a metric space with the metric

$$d(x,y) = sup_{mn}\left\{|x_{mn} - y_{mn}|^{1/m+n} : m,n : 1,2,3,...\right\}, \tag{1}$$

forall $x = \{x_{mn}\}$ and $y = \{y_{mn}\}$ in Γ^2.

Let (x_{mn}) be a double sequence of real or complex numbers. Then the series $\sum_{m,n=1}^{\infty} x_{mn}$is called a double series. The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (s_{mn}) is convergent, where $s_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m,n = 1,2,3,...)$ (see[1]). A sequence $x = (x_{mn})$ is said to be double analytic if $sup_{mn} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences is usually denoted by Λ^2. A sequence $x = (x_{mn})$ is called double entire sequence if $|x_{mn}|^{\ 1/m+n} \to 0$ as $m,n \to \infty$. The vector space of double entire sequences is usually denoted by Γ^2. Consider a double sequence $x = (x_{ij})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\ _{ij}$ for all $m,n \in \aleph$, where

$$_{mn} = \begin{pmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 0, & 0, & ...1, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \end{pmatrix}$$ with 1 in the $(m,n)^{th}$

position and zero other wise. An FK-space(or a metric space)X is said to have AK property if $(\ _{mn})$ is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$. We need the following inequality in the sequel of the paper:

Lemma 1: For $a,b, \ge 0$ and $0 < p < 1$, we have

$$(a+b)^p \le a^p + b^p$$

Some initial works on double sequence spaces is found in Bromwich[3]. Later on it was investigated by Hardy [5], Moricz [7], Moricz and Rhoades [8], Basarir and Solankan [2], Tripathy [9], Colak and Turkmenoglu [4], Turkmenoglu [10], and many others.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [6] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c, c_0$ and $\ _\infty$, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \aleph$. Here w, c, c_0 and $\ _\infty$ denote the classes of all, convergent,null and bounded sclar valued single sequences respectively. The above spaces are Banach spaces, normed by,

$$\|x\| = |x_1| + sup_{k\ge1} |\Delta x_k|$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta) = \left\{x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z\right\}$$

where $Z = \Lambda^2$ and Γ^2. respetively. $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m,n \in \aleph$. Further generalized this notion and introduced the following notion. For $m,n \ge 1$,

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$Z\left(\Delta_{\gamma}^{\mu}\right) = \left\{x = (x_{mn} : \left(\Delta_{\gamma}^{\mu} x_{mn}\right) \in Z\right\}$ for $Z = \Lambda^2$ and Γ^2,where $\Delta_{\gamma}^{\mu} x_{mn} = \Delta\Delta_{\gamma}^{\mu^{m-1}} x_{mn} = \Delta_{\gamma}^{\mu^{m-1}} x_{mn} - \Delta_{\gamma}^{\mu^{m-1}} x_{mn+1} - \Delta_{\gamma}^{\mu^{m-1}} x_{m+1n} + \Delta_{\gamma}^{\mu^{m-1}} x_{m+1n+1}$.

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x + y) \leq M(x) + M(y)$, then this function is called modulus function.

Remark 1: An Orlicz function satisfies the inequality $M(\ \ x) \leq \ \ M(x)$ for all with $0 < \ \ < 1$.

In this article are introduce the space $\Gamma^2\left(M, \ \ , \Delta_{\gamma}^{\mu}, p\right)$ of sequences of fuzzy numbers defined by Orlicz function.

Throughout the article $(w^2)^F, (\Lambda^2)^F$ and $(\Gamma^2)^F$ represent the classes of all, double analytic and double entire sequences of fuzzy numbers respectively.

II. Definitions and Preliminaries

Let D be the set of all bounded intervals $A = \left[\underline{A}, \overline{A}\right]$ on the real line R. For $A, B \in D$, define $A \leq B$ if and only if $\underline{A} \leq \underline{B}$ and $\overline{A} \leq \overline{B}$, $d(A, B) = max\left\{\underline{A} - \underline{B}, \overline{A} - \overline{B}\right\}$.

Then it can be easily see that d defines a metric on D and (D, d) is complete metric space.

A fuzzy number is fuzzy subset of the real line R which is bounded, convex and normal. Let $L(R)$ denote the set of all fuzzy numbers which are upper semi continuous and have compact support, i.e if $X \in L(R)$ then for any $\ \ \in [0, 1]$, X^{α} is compact where

$$X^{(\alpha)} = \begin{cases} t : X(t) \geq & \text{if } 0 < \ \ \leq 1, \\ t : X(t) > 0, & \text{if } \ \ = 0 \end{cases}$$

For each $0 < \ \ \leq 1$, the $\ \ -$ level set X^{α} is a nonempty compact subset of R. The linear structure of $L(R)$ includes addition $X + Y$ and scalar multiplication $\ \ X$, (a scalar)in terms of $\ \ -$ level sets, by $[X + Y]^{\alpha} = [X]^{\alpha} + [Y]^{\alpha}$ and $[\ \ X]^{\alpha} = \ \ [X]^{\alpha}$. for each $0 \leq \ \ \leq 1$.

The additive identity andmultiplicative identity of $L(R)$ are denoted by $\overline{0}$ and $\overline{1}$ respectively. The zero sequence of fuzzy numbers is denoted by $\overline{\ \ }$.

Define a map $\overline{d} : L(R) \times L(R) \to R$ by $\overline{d}(X, Y) = sup_{0 \leq \alpha \leq 1} d(X^{\alpha}, Y^{\alpha})$.

For $X, Y \in L(R)$ define $X \leq Y$ if and only if $X^{\alpha} \leq Y^{\alpha}$ for any $\ \ \in [0, 1]$. It is known that $\left(L(R), \overline{d}\right)$ is a complete metric space.

A sequence $X = (X_{mn})$ of fuzzy numbers is a function X from the set $\aleph$ of natural numbers into $L(R)$. The fuzzy number X_{mn} denotes the value of the function at $m, n \in \aleph$ and is called the $(m, n)^{th}$ term of the sequence.

A sequence E is said to be solid if $(y_{mn}) \in E$, whenever $(x_{mn}) \in E$ and $|y_{mn}| \leq |x_{mn}|$, for all $m, n \in \aleph$.

A sequence E is said to be monotone if E contains the canonical pre-images of all its step spaces.

Lemma 2: A sequence space E is monotone whenever it is solid.

Let $\Im_{sq}$ be the class of all subsets of $\aleph$ those do not contain more than (sq) number of elements. Throughout $\{\ \ _{sq}\}$ is a non-decreasing sequence of positive real numbers such that $mn \ \ _{m+1,n+1} \leq (m + 1)(n + 1) \ \ _{mn}$ for all $m, n \in \aleph$. The space

$\Gamma^2(\ \ ) = (x_{mn}) \in w^2; sup_{sq \geq 1, \sigma \in \phi_{sq}} \frac{1}{\phi_{sq}} \sum_{mn \in \sigma} |x_{mn}|^{1/m+n} \to 0 \ as\ m, n \to \infty$

Lindenstrauss and Tzafriri [12] used the idea of Orlicz function to construct Orlicz sequence space

$$\ \ _M = \left\{x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty,\ for\, some \ \ > 0\right\},$$

where w denotes all real or complex sequences.

The space $\ \ _M$ with the norm

$$\|x\| = inf\left\{\ \ > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p (1 \leq p < \infty)$, the spaces $\ \ _M$ coincide with the classical sequence space $\ \ _p$. In this article we introduce the following difference sequence space $\Gamma^2\left(M, \ \ , \Delta_{\gamma}^{\mu}, p\right)^F =$

$x = (x_{mn}) : \frac{1}{\phi_{sq}} \sum_{mn \in \sigma} \left\{M\left(\frac{\overline{d}\left(\Delta_{\gamma}^{\mu} X_{mn} - \Delta_{\gamma}^{\mu} Y_{mn}, \overline{0}\right)}{\rho}\right)\right\}^{p/m+n} \to 0 \ as\ m, n \to \infty,\ for\ some \ \ > 0$ for $0 < p < \infty$.

III. Main Results

A. Proposition

If $\overline{d}$ is a translation invariant metric on $L(R)$ then (i)$\overline{d}(X + Y, 0) \leq \overline{d}(X, 0) + \overline{d}(Y, 0)$ (ii)$\overline{d}(\ \ X, 0) \leq |\ \ |\overline{d}(X, 0), |\ \ | > 1$.

B. Proposition

The sequence space $\Gamma^2\left(M, \ \ , \Delta_{\gamma}^{\mu}, p\right)^F$ is a complete metric space under the metric $d(X, Y) = \sum_{m,n=1}^{k,\ell} \overline{d}(X_{mn}, Y_{mn}) + inf(\ \ > 0 : sup_{s,q \geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}}$
$(\sum_{mn \in \sigma} M\left(\frac{\overline{d}\left(\Delta_{\gamma}^{\mu} X_{mn} - \Delta_{\gamma}^{\mu} Y_{mn}, \overline{0}\right)}{\rho}\right))^{p/m+n} \leq 1)$
for $X, Y \in \Gamma^2\left(M, \ \ , \Delta_{\gamma}^{\mu}, p\right)^F$, $\ \ \geq 1, \mu \geq 1$ and $1 \leq p < \infty$ are the sequence of sequence of fuzzy numbers.

Proof: Let $\left\{X^{(i)}\right\}$ be a cauchy sequence in $\Gamma^2\left(M, \ \ , \Delta_{\gamma}^{\mu}, p\right)^F$. Then given any $\ \ > 0$ there exists a positive integer $\aleph$ depending on such that $d\left(X^{(i)}, X^{(j)}\right) <$ for all $p \geq \aleph$ and for all $q \geq \aleph$. Hence
$\sum_{m,n=1}^{k,\ell} \overline{d}\left(X_{mn}^{(i)}, X_{mn}^{(j)}\right) +$
$inf(\ \ > 0 : sup_{s,q \geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}}$
$\sum_{mn \in \sigma} M\left(\frac{\overline{d}\left(\Delta_{\gamma}^{\mu} X_{mn}^{(i)} - \Delta^{\mu} Y_{mn}^{(j)}, \overline{0}\right)}{\rho}\right))^{p/m+n} \leq 1)$
$<$ for all $i, j \geq \aleph$
which implies that, $\sum_{m,n=1}^{j,k} \overline{d}\left(X_{mn}^{(i)}, X_{mn}^{(j)}\right) <$ for all $i, j \geq \aleph$, and finally we get $\overline{d}\left(X_{mn}^{(i)}, X_{mn}^{(j)}\right) <$ for all

$i, j \geq \aleph$. Consequently $\left\{X^{(i)}_{mn}\right\}$ is a Cauchy sequence in the metric space $L(R)$. But $L(R)$ is complete. So, $X^{(i)}_{mn} \rightarrow X_{mn}$ as $i \rightarrow \infty$. Hence there exists a positive integer i_0 such that $inf(\ > 0 : sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}}$ $\sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X^{(i)}_{mn} - \Delta^\mu_\gamma Y^{(j)}_{mn}, \bar{0}\right)}{\rho}\right))^{p/m+n} \leq 1 < \ $ for all $i \geq i_0$. Now

$\sum^{k,\ell}_{m,n=1} \bar{d}\left(X^{(i)}_{mn}, X^{(j)}_{mn}\right) +$ $inf(\ > 0 : sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}}$ $\sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X^{(i)}_{mn} - \Delta^\mu_\gamma Y^{(j)}_{mn}, \bar{0}\right)}{\rho}\right))^{p/m+n} \leq 1 \leq \ + \ = 2 \ .$ That is $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$. This completes the proof.

C. Proposition

The sequence space $\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$ is not solid in general, for $0 < p < \infty$.

Proof: The result follows from the following example.

Example:Let $\ = 3, \mu = 2, p = 2$. Let $X_{mn} = \overline{mn}$ for all $m, n \in \aleph$ and $\ _{sq} = \ \cdot \ $ for all $s, q \in \aleph$. Let $M(x) = x$, for all $x \in [0, \infty)$. Then, we have $\bar{d}\left(\Delta^2_3 X_{mn}, \bar{0}\right) = 0$, for all $m, n \in \aleph$. Hence we have

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\xi}$ $\sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^2_3 X_{mn}, \bar{0}\right)}{\rho}\right)^{2/m+n} \rightarrow 0 \ as \ m, n \rightarrow \infty, \ for \ some \ > 0$

which implies that, $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^2_3, 2\right)^F$. Consider the sequence (X_{mn}) of scalars defined by $\ _{mn}$=1 for m, n is even and 0 otherwise. Which implies that $\ _{mn} X_{mn}$= $\overline{mn}$ for m, n is even, $\bar{0}$ otherwise, which implies that

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\xi} \sum_{mn\in\sigma}(\bar{d}\left(M\left(\frac{\left(\Delta^2_3 X_{mn}, \bar{0}\right)}{\rho}\right)\right))^{2/m+n} \not\rightarrow 0 as m, n \rightarrow \infty, for any fixed \ > 0$

which shows that $(\ X_{mn}) \notin \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$. Hence $\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$ is not solid in general, for $0 < p < \infty$. This completes the proof.

D. Proposition

$\Gamma^2\left(M, \ , \Delta^\mu_\gamma\right)^F \subseteqq \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, for all $1 \leq p < \infty$.

Proof: Let $X \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma\right)^F$, then we have,

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}}$ $\sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)^{1/m+n} \rightarrow 0 \ as$ $m, n \rightarrow \infty, \ for \ any \ fixed \ > 0.$

Hence, for each fixed s, q and $\ \in \Im_{sq}$, we have, for $\ > 0$;

$\left\{\frac{1}{\phi_{sq}} \sum_{mn\in\sigma} \left\{M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{1/m+n}\right\} \leq \left\{\frac{1}{\phi_{sq}} \sum_{mn\in\sigma} \left\{M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)^p\right\}^{1/p(m+n)}\right\}$

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}} \sum_{mn\in\sigma} \left(M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)^p\right\}^{1/p(m+n)} \rightarrow 0 \ as \ m, n \rightarrow \infty.$

Which implies that, $X \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, for $1 \leq p < \infty$. This completes the proof.

E. Proposition

$\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F \subseteqq \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, if and only if $sup_{sq\geq 1}\left(\frac{\phi_{sq}}{\chi_{sq}}\right) < \infty$, for $0 < p < \infty$.

Proof: First, suppose that $sup_{sq\geq 1}\left(\frac{\phi_{sq}}{\chi_{sq}}\right) = K < \infty$, then we have, $\ _{sq} \leq K \ _{sq}$. Now, if $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, then

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}} \sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)^{p/(m+n)} \rightarrow 0 \ as \ m, n \rightarrow \infty.$

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{K\chi_{sq}} \sum_{mn\in\sigma} M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)^{p/(m+n)} \rightarrow 0 \ as \ m, n \rightarrow \infty.$

(i.e) $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)$. Hence $\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F \subseteq \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$.

Conversely, suppose that $\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F \subseteq \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$. We should prove that $sup_{s,q\geq 1}\left(\frac{\phi_{sq}}{\chi_{sq}}\right) = sup_{s,q\geq 1} \ _{sq} < \infty$. Suppose that $sup_{s,q\geq 1}(\ _{sq}) = \infty$. Then there exists a subsequence $(\ _{s_i q_j})$ of $(\ _{sq})$ such that $lim_{i,j\rightarrow\infty}(\ _{s_i q_j}) = \infty$. Then for $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, we have,

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\chi_{sq}} \sum_{mn\in\sigma} \left\{M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{p/(m+n)} \geq$ $sup_{s,q\geq 1 \sigma \in \Im_{sq}} \left(\frac{\eta_{s_i q_j}}{\phi_{s_i q_j}}\right) \sum_{mn\in\sigma} \left\{M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{p/(m+n)} = \infty$

(i.e)$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\chi_{sq}} \sum_{mn\in\sigma} \left\{M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{p/(m+n)} = \infty$, which implies that $(X_{mn}) \notin \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, a contradiction. This completes the proof.

Corollary:$\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F = \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$, if and only if $sup_{s,q\geq 1}(\ _{sq}) < \infty$ and $sup_{s,q\geq 1}(\ ^{-1}_{sq}) < \infty$, where $\ _{sq} = \frac{\phi_{sq}}{\chi_{sq}}$, for $0 < p < \infty$.

F. Proposition

$\ ^2_p\left(M, \Delta^\mu_\gamma\right)^F \subseteq \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F \subseteq \Lambda^2\left(M, \Delta^\mu_\gamma\right)^F$, for $1 \leq p < \infty$.

Proof: By taking $M(x) = X^p$, for $1 \leq p < \infty$ and $\ _{sq} = 1$ for all $s, q \in \aleph$, we get that $\Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F = \ ^2_p\left(M, \Delta^\mu_\gamma\right)^F$. So, the first inclusion is proved. Next; suppose that, $(X_{mn}) \in \Gamma^2\left(M, \ , \Delta^\mu_\gamma, p\right)^F$. This implies that

$sup_{s,q\geq 1 \sigma \in \Im_{sq}} \frac{1}{\phi_{sq}} \sum_{mn\in\sigma} \left\{\left(M\left(\frac{\bar{d}\left(\Delta^\mu_\gamma X_{mn}, \bar{0}\right)}{\rho}\right)\right)^p\right\}^{1/p(m+n)} \rightarrow$ 0 as $m, n \rightarrow \infty$.

For $s, q = 1$; then there exists a positive integer K such that

$$\left\{ M\left(\frac{\bar{d}\left(\Delta_\gamma^\mu X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{1/(m+n)} \leq K \quad _1, m, n \in \quad ,$$

which implies that

$$sup_{s,q\geq 1}\left\{ M\left(\frac{\bar{d}\left(\Delta_\gamma^\mu X_{mn}, \bar{0}\right)}{\rho}\right)\right\}^{1/(m+n)} < \infty.$$

Thus we have that $(X_{mn}) \in \Lambda^2 \left(M, \Delta_\gamma^\mu\right)^F$. This completes the proof.

IV. Conclusion

Inclusion relations and general properties involving the double difference sequence spaces are obtained and also $\Gamma^2 \left(M, \ \ , \Delta_\gamma^\mu, p\right)^F$ of fuzzy numbers for both $1 \leq p < \infty$ and $0 < p < 1$, is introduced.

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