A New Brazilian Friction-ResistantLow Alloy High Strength Steel – A Life Testing Approach

D. I. De Souza, G. P. Azevedo, R.Rocha

***Abstract*—**In this paper we will develop a sequential life test approach applied to a modified low alloy-high strength steel part used in highway overpasses in Brazil.We will consider two possible underlying sampling distributions: the Normal and theInverse Weibull models. The minimum life will be considered equal to zero. We will use the two underlying models to analyze a fatigue life test situation, comparing the results obtained from both.Since a major chemical component of this low alloy-high strength steel part has been changed, there is little information available about the possible values that the parameters of the corresponding Normal and Inverse Weibull underlying sampling distributions could have. To estimate the shape and the scale parameters of these two sampling models we will use a maximum likelihood approach for censored failure data. We will also develop a truncation mechanism for the Inverse Weibull and Normal models. We will provide rules to truncate a sequential life testing situation making one of the two possible decisions at the moment of truncation; that is, accept or reject the null hypothesis H_0. An example will develop the proposed truncated sequential life testing approach for the Inverse Weibull and Normal models.

***Keywords*—**Sequential Life Testing, Normal and Inverse Weibull Models, Maximum Likelihood Approach, Truncation Mechanism

I. INTRODUCTION

THE two-parameter Inverse Weibull distribution has a shape parameter β, which specifies the shape of the distribution, and a scale parameter θ, which represents the characteristic life of the distribution. Both parameters are positive.The Normal distribution has been widely used as a failure model. It has two parameters: a shape parameter σ and a scale parameterμ. The Inverse Weibull density function *f(t)*is given by.

$$f(t)=\frac{\beta}{\theta}\left(\frac{\theta}{t}\right)^{\beta+1} exp\left[-\left(\frac{\theta}{t}\right)^{\beta}\right]; \quad t\geq 0 \qquad (1)$$

Here, *t*representsthe time to failure of a component or part. The Normal density function*f(x)* is given by:

$$f(x)=\frac{1}{\sqrt{2\pi}\,\sigma}\, exp\left[\frac{(x-\mu)^2}{2\sigma^2}\right]; \quad x\geq 0 \qquad (2)$$

x is the time to failure of a component or part. The hypothesis testing situations was given by [1] and[2]:

a) In the Inverse Weibull Case:

1. For the scale parameter θ: H_0: $\theta \geq \theta_0$; H_1: $\theta < \theta_0$

The probability of accepting the null hypothesis H_0 will be set at $(1-\alpha)$ if $\theta = \theta_0$. Now, if $\theta = \theta_1$ where $\theta_1 < \theta_0$, then the probability of accepting H_0 will be set at a low level γ. H_1 represents the alternative hypothesis.

2. For the shape parameter β: H_0: $\beta \geq \beta_0$; H_1: $\beta<\beta_0$

The probability of accepting H_0will be set again at $(1-\alpha)$in the case of $\beta = \beta_0$. Now, if $\beta = \beta_1$, where $\beta_1<\beta_0$, then the probability of accepting H_0 will also be set at a low level γ.

b) In the Normal Case:

1. For the scale parameter μ: H_0: $\mu \geq \mu_0$; H_1: $\mu<\mu_0$

The probability of accepting H_0 will be set at $(1-\alpha)$when we have$\mu = \mu_0$. Now, if $\mu = \mu_1$, where $\mu_1<\mu_0$, then the probability of accepting H_0 will be set at a low level γ.

2. For the shape parameter σ: H_0: $\sigma\geq \sigma_0$; H_1: $\sigma<\sigma_0$

The probability of accepting H_0 will beset at $(1-\alpha)$ in the case of $\sigma = \sigma_0$. Now, if $\sigma = \sigma_1$ where $\sigma_1<\sigma_0$, then the probability of accepting H_0 will also be set at a low level γ.

II. METHODOLOGY

The development of a sequential test uses the likelihood ratio (*LR*) given by the following relationship proposed by [1]:

$$LR = L_{1;n}/L_{0;n}$$

The sequential probability ratio (*SPR*) will be given by:

$$SPR = L_{1;n}/L_{0;n}$$

Based on the paper from [3], for the Inverse Weibull case the (*SPR*) will be:

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$$SPR = \left(\frac{\beta_1}{\theta_0^{\beta_0}} \times \frac{\theta_1^{\beta_1}}{\beta_0}\right)^n \prod_{i=1}^{n} (t_i)^{\beta_0-\beta_1}$$

$$\exp\left[-\sum_{i=1}^{n}\left(\frac{\theta_1^{\beta_1}}{t_i^{\beta_1}} - \frac{\theta_0^{\beta_0}}{t_i^{\beta_0}}\right)\right]$$

In the Normal case, we will have:

$$SPR = \left(\frac{\sigma_0}{\sigma_1}\right)^n \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\left[\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2 - \left(\frac{x_i-\mu_0}{\sigma_0}\right)^2\right]\right\}$$

The continue region will become $A < SPR < B$, where:

$A = \gamma/(1-\alpha)$ and also $B = (1-\gamma)/\alpha$.

We will accept the null hypothesis H_0 if $SPR \geq B$ and we will reject H_0 if $SPR \leq A$.

Now, if $A < SPR < B$, we will take one more observation. Then, after some mathematical manipulation, we will have:

a) In the Inverse Weibull case:

$$n\ln\left(\frac{\beta_1}{\theta_0^{\beta_0}} \times \frac{\theta_1^{\beta_1}}{\beta_0}\right) - \ln\left[\frac{(1-\gamma)}{\alpha}\right] < W < n\ln\left(\frac{\beta_1}{\theta_0^{\beta_0}} \times \frac{\theta_1^{\beta_1}}{\beta_0}\right) + \ln\left[\frac{(1-\alpha)}{\gamma}\right] \quad (3)$$

$$W = \sum_{i=1}^{n}\left(\frac{\theta_1^{\beta_1}}{t_i^{\beta_1}} - \frac{\theta_0^{\beta_0}}{t_i^{\beta_0}}\right) + (\beta_1 - \beta_0)\sum_{i=1}^{n}\ln(t_i) \quad (4)$$

b) In the Normal case:

$$n\ln\left(\frac{\sigma_0}{\sigma_1}\right) - \ln\left[\frac{(1-\gamma)}{\alpha}\right] < N < n\ln\left(\frac{\sigma_0}{\sigma_1}\right) + \ln\left[\frac{(1-\alpha)}{\gamma}\right] \quad (5)$$

$$N = \frac{1}{2}\sum_{i=1}^{n}\left[\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2 - \left(\frac{x_i-\mu_0}{\sigma_0}\right)^2\right] \quad (6)$$

III. EXPECTED SAMPLE SIZE OF A SEQUENTIAL LIFE TESTING

According to [4], an approximate expression for the expected sample size E(n) of a sequential life testing will be given by:

$$E(n) = \frac{E(W_n^*)}{E(w)} \quad (7)$$

$$w = \ln\frac{f(t;\theta_1,\beta_1)}{f(t;\theta_o,\beta_o)} \quad (8)$$

$$E(W_n^*) \cong P(\theta,\beta)\ln(A) + [1-P(\theta,\beta)]\ln(B) \quad (9)$$

For the two-parameter Inverse Weibull sampling distribution, we will have:

$$E(w) = \ln\left(\frac{\beta_1}{\theta_0^{\beta_0}} \times \frac{\theta_1^{\beta_1}}{\beta_0}\right) + (\beta_0 - \beta_1)\,E[\ln(t)]$$

$$-\theta_1^{\beta_1}\,E\left(\frac{1}{t^{\beta_1}}\right) + \theta_0^{\beta_0}\,E\left(\frac{1}{t^{\beta_0}}\right) \quad (10)$$

$$E[\ln(t)] = \ln(\theta) - \frac{1}{\beta} \times \frac{g}{3} \times$$

$$\left\{\sum_{i=1}^{n+1}\left[\ln(U_i)e^{-U_i} \times (1, 2\ or\ 4)\right]\right\};\ U = \left(\frac{\theta}{t}\right)^\beta \quad (11)$$

To find the $E[\ln(t)]$ some numerical integration procedure (Simpson's 1/3 rule in this work) will have to be used. The solution of each component of (10) can be found in [3].

For the Normal sampling distribution, we will have:

$$E(w) = \ln(\sigma_0) - \ln(\sigma_1) + \frac{1}{2\sigma_1^2\sigma_0^2} \times$$

$$\left[\left(\sigma_0^2 - \sigma_1^2\right)\left(\sigma^2 + \mu^2\right) - 2\mu\left(\mu_1\sigma_0^2 - \mu_0\sigma_1^2\right) + \mu_1^2\sigma_0^2 - \mu_0^2\sigma_1^2\right] \quad (12)$$

When the decisions about the quantities θ_0, θ_1, β_0, β_1, α, γ and $P(\theta, \beta)$ are made, and after the $E(w)$ is calculated, the sequential test is totally defined.

IV. THE MAXIMUM LIKELIHOOD APPROACH

a) In the Inverse Weibull case:

The likelihood function $L(\beta;\ \theta)$ for the shape and scale parameters of an Inverse Weibull sampling distribution for censored Type II data (failure censored) will be given by:

$$L(\beta;\theta)=k!\left[\prod_{i=1}^{r} f(t_i)\right][1-F(t_r)]^{n-r}$$, or yet:

$$L(\beta;\theta)=k!\left[\prod_{i=1}^{r} f(t_i)\right][R(t_r)]^{n-r}\ ;\ t>0$$

With $f(t_i)=\dfrac{\beta}{\theta}\left(\dfrac{\theta}{t_i}\right)^{\beta+1} exp\left[-\left(\dfrac{\theta}{t_i}\right)^{\beta}\right]$ and with

$R(t_r)=exp\left[-\left(\dfrac{\theta}{t_r}\right)^{\beta}\right]$, we will have:

$$L(\beta;\theta)=k!\,\beta^r\,\theta^{\beta r}\left[\prod_{i=1}^{r}\frac{1}{(t_i)}\right]^{\beta+1} e^{-\sum_{i=1}^{r}(\theta/t_i)^{\beta}} \left[e^{-(\theta/t_r)^{\beta}}\right]^{n-r} \qquad (13)$$

The log likelihood function *ln [L(β; θ)]* will be given by:

$$ln[L(\beta;\theta)]=ln(k!)+r\,ln(\beta)+r\beta\,ln(\theta)-(\beta+1)\sum_{i=1}^{r}ln(t_i)-\sum_{i=1}^{r}\left(\frac{\theta}{t_i}\right)^{\beta}-(n-r)\left(\frac{\theta}{t_r}\right)^{\beta} \qquad (14)$$

To find the values of θ and β that maximizes the log likelihood function, we take the θ and β derivatives and make them equal to zero. Then, we will have:

$$\frac{dL}{d\theta}=\frac{r\beta}{\theta}-\beta\theta^{\beta-1}\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta}-(n-r)\beta\theta^{\beta-1}\left(\frac{1}{t_r}\right)^{\beta}=0 \qquad (15)$$

$$\frac{dL}{d\beta}=\frac{r}{\beta}+r\,ln(\theta)-\sum_{i=1}^{r}ln(t_i)-\sum_{i=1}^{r}\left(\frac{\theta}{t_i}\right)^{\beta} ln\left(\frac{\theta}{t_i}\right)-(n-r)\left(\frac{\theta}{t_r}\right)^{\beta} ln\left(\frac{\theta}{t_r}\right)=0 \qquad (16)$$

From (15) we obtain:

$$\theta=\left(r\Big/\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta}+(n-r)\left(\frac{1}{t_r}\right)^{\beta}\right)^{1/\beta} \qquad (17)$$

Notice that, when $\beta = 1$, (17) reduces to the maximum likelihood estimator for the inverse exponential distribution. Using (17) for θ in (16) and applying some algebra, (16) reduces to:

$$\frac{r}{\beta}-\sum_{i=1}^{r}ln(t_i)+\frac{r\times\left[\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta}ln(t_i)+(n-r)\left(\frac{1}{t_r}\right)^{\beta}ln(t_r)\right]}{\sum_{i=1}^{r}\left(\frac{1}{t_i}\right)^{\beta}+(n-r)\left(\frac{1}{t_r}\right)^{\beta}}=0 \qquad (18)$$

Equation (18) must be solved iteratively.

b) In the Normal case:

The likelihood function *L(μ; σ)* for the shape and scale parameters of a Normal sampling distribution for censored Type II data (failure censored) will be given by:

$$L(\mu;\sigma)=k!\left[\prod_{i=1}^{r} f(x_i)\right][1-F(x_r)]^{n-r}\ ;\ x>0$$

With $f(x_r)=\dfrac{1}{\sqrt{2\pi}\,\sigma}\,exp\left[\dfrac{(x_r-\mu)^2}{2\sigma^2}\right]$ and with

$F(x_r)=\int_{-\infty}^{x_r}\dfrac{1}{\sigma\sqrt{2\pi}}exp\left[-\dfrac{(\tau-\mu)^2}{2\sigma^2}\right]d\tau$; τ≥ 0, we have:

$$L(\mu;\sigma)=k!\left(\frac{1}{\sqrt{2\pi}}\right)^{r}\left(\frac{1}{\sigma}\right)^{r}\times exp\left[-\left(\frac{1}{2\sigma^2}\sum_{i=1}^{r}(x_i-\mu)^2\right)\right]\left[1-\int_{-\infty}^{x_r}\frac{1}{\sigma\sqrt{2\pi}}exp\left[-\frac{(\tau-\mu)^2}{2\sigma^2}\right]d\tau\right]^{n-r}$$

The log likelihood function *ln[L(μ; σ)]* will be given by:

$$ln[L(\mu;\sigma)]=ln(k)+r\,ln\left(\frac{1}{\sqrt{2\pi}}\right)+r\,ln\left(\frac{1}{\sigma}\right)-\frac{1}{2\sigma^2}\sum_{i=1}^{r}(x_i-\mu)^2+(n-r)\times ln\left(1-\int_{-\infty}^{x_r}\frac{1}{\sigma\sqrt{2\pi}}exp\left[-\frac{(\tau-\mu)^2}{2\sigma^2}\right]d\tau\right)$$

To find the values of μ and σ that maximizes the log likelihood function, we take the μ and σ derivatives and make them equal to zero. Then, we will have:

$$\frac{dL}{d\mu} = \frac{1}{\sigma^2}\sum_{i=1}^{r}\left(x_i - \mu\right) - \frac{\frac{(n-r)}{\sigma\sqrt{2\pi}} \times \frac{\left(x_r - \mu\right)}{\sigma^2} exp\left[-\frac{\left(x_r - \mu\right)^2}{2\sigma^2}\right]}{1 - \int_{-\infty}^{x_r}\frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(\tau-\mu)^2}{2\sigma^2}\right]d\tau} = 0 \quad (19)$$

$$\frac{dL}{d\sigma} = -\frac{r}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{r}\left(x_i - \mu\right)^2 - \frac{\frac{(n-r)}{\sigma^2\sqrt{2\pi}} \times exp\left[-\frac{\left(x_r - \mu\right)^2}{2\sigma^2}\right]\left[\frac{\left(x_r - \mu\right)^2}{\sigma^2} - 1\right]}{1 - \int_{-\infty}^{x_r}\frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(\tau-\mu)^2}{2\sigma^2}\right]d\tau} = 0 \quad (20)$$

Dividing (19) by (20), we get:

$$\frac{d\sigma}{d\mu} = \frac{\frac{1}{\sigma}\sum_{i=1}^{r}\left(x_i - \mu\right)}{-r + \frac{1}{\sigma^2}\sum_{i=1}^{r}\left(x_i - \mu\right)^2} - \frac{\left(x_r - \mu\right)}{\sigma \times \left[\frac{\left(x_r - \mu\right)^2}{\sigma^2} - 1\right]} = 0 \quad (21)$$

Equation (21) must be solved iteratively.

V. EXAMPLE

A low alloy-high strength steel product will be life tested. Since a major chemical component of this low alloy-high strength steel part has been changed, there is little information available about the possible values that the parameters of the corresponding Normal and Inverse Weibull underlying sampling distributions could have. To estimate the shape and the scale parameters of these two sampling models we will use a maximum likelihood approach for censored failure data. Some preliminarily life testing was performed in order to determine an estimated value for the parameters of the two sampling distributions. In this preliminary approach, a set of 15 items was life tested, with the testing being truncated at the moment of occurrence of the ninth failure. Table 1 shows the failures time data (cycles) from the preliminary life testing.

TABLE I
FAILURES TIME DATA (CYCLES)

2,251,930	2,780,470	2,934,330
3,154,093	3,322,329	3,568,961
3,781,710	4,023,048	4,517,904

Using the maximum likelihood estimator approach for the scale and shape parameters of the Inverse Weibull and Normal sampling distributions for censored Type II data (failure censored) we obtain the following values for these parameters:

For the Normal case:

$\mu = 3{,}370{,}530.556$ *cycles;* $\sigma = 709{,}115.7862$ *cycles*

For the Inverse Weibull case:

$\theta = 2{,}940{,}733$ *cycles;* $\beta = 4.807$

We will use these two underlying sampling models to life testing the low-alloy high strength steel product under analysis, comparing the results obtained from both models.

Initially, using the Normal sampling model, we elect the null hypothesis parameters to be$\mu_0 = 3{,}400{,}000$ *cycles;* $\sigma_0 = 710{,}000$ *cycles*; with $\alpha = 0.05$ and $\gamma = 0.10$ and choose the value of *3,000,000 cycles* for the alternative scale parameter μ_1 and the value of *650,000 cycles* for the alternative shape parameter σ_1. Then, using(5) and (6), we will have:

$$n\,ln\left(\frac{710{,}000}{650{,}000}\right) - ln\left[\frac{(1-0.10)}{0.05}\right] = n \times 0.0883 - 2.8904$$

$$n\,ln\left(\frac{710{,}000}{650{,}000}\right) + ln\left[\frac{(1-0.05)}{0.10}\right] = n \times 0.0883 + 2.2513$$

$$N = \frac{1}{2}\sum_{i=1}^{n}\left[\left(\frac{x_i - 3{,}000{,}000}{650{,}000}\right)^2 - \left(\frac{x_i - 3{,}400{,}000}{710{,}000}\right)^2\right] \quad (22)$$

The procedure is defined by the following rules:

1. If $N \geq n \times 0.0883 + 2.2513$, we will accept H_0.
2. If $N \leq n \times 0.0883 - 2.8904$, we will reject H_0.
3. If $n \times 0.0883 - 2.8904 < N < n \times 0.0883 + 2.2513$, we will take one more item.

Now using the Inverse Weibull sampling model, we elect the null hypothesis parameters to be equal to $\theta_0 = 2{,}950{,}000$ *cycles;*$\beta_0 = 4.8$; and choose the value of *2,600,000 cycles* for the alternative scale parameter θ_1 and the value of *4.0* for the alternative shape parameter β_1. It was decided that the value of α was *0.05* and γ was *0.10*. Then, using (3) and (4), we will have:

$$n\,ln\left(\frac{4.0}{(2{,}950{,}000)^{4.8}} \times \frac{(2{,}600{,}000)^{4.0}}{4.8}\right) - ln\left[\frac{(1-0.10)}{0.05}\right] =$$

$$-n \times 12.6053 - 2.8904$$

$$n\,ln\left(\frac{4.0}{(2{,}950{,}000)^{4.8}}\times\frac{(2{,}600{,}000)^{4.0}}{4.8}\right)+ln\left[\frac{(1-0.05)}{0.10}\right]=$$

$$-n\times 12.6053+2.2513$$

$$W=\sum_{i=1}^{n}\left(\frac{(2{,}600{,}000)^{4.0}}{t_i^{4.0}}-\frac{(2{,}950{,}000)^{4.8}}{t_i^{4.8}}\right)-$$

$$-0.8\sum_{i=1}^{n}ln(t_i)$$

Then, we get:

$$-n\times 12.6053-2.8904< W <-n\times 12.6053+2.2513$$

The procedure is defined by the following rules:

1. If $W\geq -n\times 12.6053+2.2513$, we will accept H_0.
2. If $W\leq -n\times 12.6053-2.8904$, we will reject H_0.
3. If $-n\times 12.6053-2.8904< W <-n\times 12.6053+2.2513$, we will take one more observation.

After a sequential test graph has been developed for this life-testing situation, a random sample is taken, item by item. After the analysis of the failure number five, the Inverse Weibull model made possible to make the decision to accept the null hypothesis H_0.The failure times obtained in this life testing(cycles to failure) were the following: *3,282,070; 2,038,658; 3,842,361; 4,441,792; 1,840,065*.

Table 2 shows the results of this test for theInverse Weibull model case.

TABLE II
SEQUENTIAL TEST RESULTS (CYCLES) FOR THE INVERSE WEIBULL MODEL

Unit Number	Lower Limit	Upper Limit	Value of N
1	–15.4957;	–10.3541;	–12.2087
2	–28.1011;	–22.9594;	–27.0778
3	–40.7064;	–35.5648;	–39.2786
4	–53.3118;	–48.1701;	–51.5467
5	–65.9171;	–60.7755;	–68.7379

In the Normal model case, even after the observation of fifteen times to failure, it was not possible to make the decision to accept or reject the null hypothesis H_0.

All the fifteen failure times obtained in this life testing(cycles to failure) were the following: *3,282,070; 2,038,658; 3,842,361; 4,441,792; 1,840,065; 4,388,466; 3,467,202; 2,807,120; 3,749,865; 2,985,436; 1,693,218; 2,432,809; 3,008,410; 2,246,590; 4,018,243.*

Table III shows the results of this test for the Normal model case.

TABLE III
SEQUENTIAL TEST RESULTS (CYCLES) FOR THE NORMAL MODEL

Unit Number	Lower Limit	Upper Limit	Value of N
1	–2.8021;	2.3396;	0.0804
2	–2.7138;	2.4279;	–0.664
3	–2.6255;	2.5162;	–0.018
4	–2.5372;	2.6044;	1.3651
5	–2.4489;	2.6928;	0.5437
6	–2.3606;	2.7810;	1.8561
7	–2.2723;	2.8693;	2.1099
8	–2.1840;	2.9576;	1.8053
9	–2.0957;	3.0459;	2.3493
10	–2.0074;	3.1342;	2.1791
11	–1.9191;	3.2225;	1.3106
12	–1.8309;	3.3108;	0.7635
13	–1.7426;	3.3991	0.6115
14	–1.6543;	3.4874	–0.036
15	–1.5660	3.5757	0.8116

Now, for the Normal model case, using (7) to (12), we can calculate the expected sample size $E(n)$ of this sequential life testing under analysis. So, with $\sigma=\sigma_0=710{,}000$ *cycles;* $\sigma_1=650{,}000$ *cycles;* $\mu=\mu_0=3{,}400{,}000$ *cycles;* $\mu_1=3{,}000{,}000$ *cycles;* $\alpha=0.05$; $\gamma=0.10$; and electing $P(\theta,\beta)$ to be 0.01, we will have:

$$E(w)=ln(\sigma_0)-ln(\sigma_1)+\frac{1}{2\sigma_1^2\sigma_0^2}\times$$

$$\left[\left(\sigma_0^2-\sigma_1^2\right)\left(\sigma^2+\mu^2\right)-2\mu\left(\mu_1\sigma_0^2-\mu_0\sigma_1^2\right)+\mu_1^2\sigma_0^2-\mu_0^2\sigma_1^2\right]$$

$$E(w)=0.088292607+2.3476134\times 10^{-24}\times$$

$$(9.8443056\times 10^{23}-5.1544\times 10^{23}+4.5369\times 10^{24}-4.8841\times 10^{24})$$

$$E(w)=0.0883+0.2856=0.3739$$

Then, we will have:

$$E(n)=\frac{P(\theta,\beta)\,ln\,A+[1-P(\theta,\beta)]\,ln\,B}{E(w)}=\frac{2.8390}{0.3739}$$

$$E(n)=7.593\cong 8 \text{ items}$$

Therefore, we could make a decision about accepting or rejecting the null hypothesis H_0 after the analysis of observation number 8.

VI A PROCEDURE FOR EARLY TRUNCATION

According to [1], when the truncation point is reached, a line partitioning the sequential graph can be drawn as shown in Fig. 1 below.

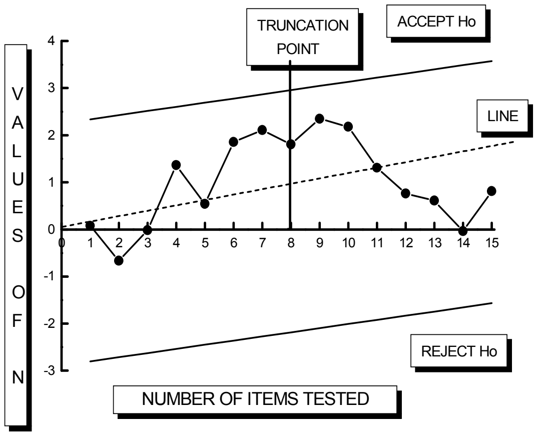

Fig. 1 A truncation procedure for the sequential testing Normal case

This line is drawn through the origin of the graph parallel to the accept and reject lines. The decision to accept or reject H_0 simply depends on which side of the line the final outcome lies. Obviously this procedure changes the levels of α and $\gamma$of the original test; however, the change is slight if the truncation point is not too small (less than four observations). As we can see in Fig. 1, the null hypothesis H_0 should be accepted since the final observation in the Normal model case (observation number 8) lies on the side of the line related to the acceptance of H_0.

VII CONCLUSIONS

The major advantage of a sequential life testing approach in relation to the fixed size approach is to keep the samples size small, with a resulting savings in cost. It happens that even with the use of a sequential life testing approach, sometimes the number of items necessary to reach a decision about accepting or rejecting a null hypothesis could be quite large [5]. Thus, the test must be truncated after a fixed time or number of observations. To estimate the shape and the scale parameters of the two sampling models Inverted Weibull and Normal we applied a maximum likelihood approach for censored failure data. We also developed a truncation mechanism for the Inverse Weibull and Normal models. We provided rules to truncate a sequential life testing situation making one of the two possible decisions at the moment of truncation; that is, accept or reject the null hypothesis $H_{0.}$

The sequential life testing approach developed in this paper shows that theInverse Weibull model could effectively represent the low alloy-high strength steel product being life-tested in the above example. In this Inverse Weibull model case, we were able to make a decision about accepting the null hypothesis H_0 after the analysis of observation number five. In the Normal model case, even after the observation of fifteen failure times, it was not possible to make the decision to accept or reject the null hypothesis H_0. So, the test needed to be truncated after a fixed number of observations (eight in this case). This fact shows the advantage of such a truncation mechanism to be used in a sequential life test approach.Therefore, the Inverted Weibull model has a better response in analyzing the modified low alloy-high strength steel part used in highway overpasses in Brazil.

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