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A New Approach of Fuzzy Methods for Evaluating of Hydrological Data

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Abstract**—**The main criteria of designing in the most hydraulic constructions essentially are based on runoff or discharge of water. Two of those important criteria are runoff and return period. Mostly, these measures are calculated or estimated by stochastic data. Another feature in hydrological data is their impreciseness. Therefore, in order to deal with uncertainty and impreciseness, based on Buckley's estimation method, a new fuzzy method of evaluating hydrological measures are developed. The method introduces triangular shape fuzzy numbers for different measures in which both of the uncertainty and impreciseness concepts are considered. Besides, since another important consideration in most of the hydrological studies is comparison of a measure during different months or years, a new fuzzy method which is consistent with special form of proposed fuzzy numbers, is also developed. Finally, to illustrate the methods more explicitly, the two algorithms are tested on one simple example and a real case study.

Keywords**—** Fuzzy Discharge, Fuzzy estimation, Fuzzy ranking method, Hydrological data

I. Introduction

In most of hydraulic constructions (e.g. reservoir dams, water retention structures, flood mitigation structures, and storage tank) or hydraulic plans (e.g. designing of spillways or waterways) studies are done due to runoff evaluation. It means that most of the measures that are calculated are due to runoff [23]. Discharge (Q) and return period (R.P.) are examples of those measures.

Generally, hydrological data are associated with two features: uncertainty and impreciseness. These two features weaken the reliability of data to be used for calculating different measures. In the literature on bad effects of uncertainty, statistical methods and calculations are done, such as calculating point or interval estimations [16]. On the other hand, to handle bad effects of impreciseness, the fuzzy concept proposed by Zadeh [25], is used.

Following Zadeh's introduction of the fuzzy concept and expansion of its capabilities, it has been utilized in different studies and has become an essential technique in different fields such as environment and natural sciences and engineering fields. This concept has also been developed in hydrological studies related to stream flow forecasting, runoff-induced sediment transport from bare soil surfaces, water quality modeling [15], rainfall-runoff forecasting [11] and some other related hydrological issues.

Bankert et al. [3] believed that using fuzzy concept in neural network for the state of applying parameter with fuzzy expression would increase learning. Fontae et al. [9] showed that the fuzzy concept can be used as a multi criteria decision-making tool on the basis of uncertain data, which are applied with a fuzzy membership function in water resources studies. Bardossy et al. [4] modeled the flow of water in unsaturated zone of soil by fuzzy technique.

In the recent decade, to model time series and forecast hydrological variables, a lot of models based on fuzzy logic approaches and neuro-fuzzy modeling (i.e. a combination of artificial neural network (ANN) and fuzzy logic approaches) has been carried out by many researchers. These developments are the result of a complete scheme of both neural networks knowledge and fuzzy logic [12] and the combination can overcome the shortcomings of both techniques. Tayfur et al. [22] have considered the sediment loads from bare soil surfaces by utilizing the rainfall intensity and slope data as input variables and, by using fuzzy-logic algorithm. They have modeled the sediment load which is transported by flow-discharge. Nayak et al. [17] applied Sugeno fuzzy inference systems based on artificial neural networks and adaptive network-based fuzzy for discharge modeling of an Indian River. Jacquin et al. [10] has used long period precipitation forecasting in water resources management in the arid and semi-arid region. Rahimi et al. [18] used a comparison of fuzzy sets theory, normal kriging and kriging fuzzy to conclude that: kriging fuzzy technique is better than kriging method for estimation of rainfall spatial distribution. By using fuzzy multi criteria decision-making technique and with regards to different factors and FDM (fuzzy decision making) software, the priority of the water in inter-basin transfer project has been carried out in Karun water transfer tunnel of Iran [19]. Abolpour et al. [1] to obtain optimal values of the decision variables and improve water allocation in river basin combined ANFIS with fuzzy reinforcement learning and created a new algorithm which is called ANFRL (Adaptive Neural Fuzzy Reinforcement Learning).

The other fuzzy methods including fuzzy clustering [20], [21] or fuzzy regression [14] are also considered in the papers.

Buckley & Eslami, [5], [6], using $(1-\beta)100\%$ confidence intervals for a parameter as a family of $\alpha-cuts$, introduced triangular shaped fuzzy numbers for different parameters. In this paper, their approach is applied to finding

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fuzzy estimates of two hydrological data (i.e. runoff and R.P.). These fuzzy numbers considered both of the bad features of stochastic data (i.e. uncertainty and impreciseness). The idea which used in creating a fuzzy comparing method that compares discharges of different months has also taken from Buckley's study [7]. He introduced and used his new comparing method to carry out a hypothesis testing by comparing fuzzy statistic with fuzzy critical value in a hypothesis testing. However, some differences are considered in our method in comparison with Buckley's method.

II. Materials and Methodologies

In this research, the evaluation of two important data of hydrological studies which are discharge and return period is developed. These data are considered to be of utmost importance in most of hydraulic constructions like water retention structures or hydraulic plans such as designing of spillways or waterways. However, since they are calculated by taking samples of uncertain data, experts are encountered with two unpredictable features: 1) uncertainty and 2) impreciseness.

In classical hydrologic studies, to resolve uncertainty, statistical concepts like confidence intervals are used. Confidence intervals of these data are defined as follows:

$$\{\overline{Q} - t_{\alpha/2}\frac{S}{\sqrt{n}}, \overline{Q} + t_{\alpha/2}\frac{S}{\sqrt{n}}\} \quad \text{For } 0<\alpha<1. \qquad (1)$$

$$\{Q_\varphi - t_{\alpha/2,(n-1)} * \frac{S*\sqrt{Z^2{}_\varphi + 2}}{\sqrt{2*n}}, Q_\varphi + t_{\alpha/2,(n-1)} * \frac{S*\sqrt{Z^2{}_\varphi + 2}}{\sqrt{2*n}}\}$$

$$\text{For, } 0<\alpha<1 \qquad (2)$$

Where

$$Q_\varphi = \overline{Q} + S * Z_\varphi \quad 0<\varphi<1, \qquad (3)$$

and φ denotes exceedance probability.

The first confidence interval stands for mean of discharge [23] and the second one stands for return period [25]. $\overline{Q}$ in both of the confidence intervals shows the point estimate of mean of discharge and Q_φ shows the point estimate of the return period.

Another unpredicted feature of sample data is their impreciseness or vagueness. As it was mentioned, to face this feature, the fuzzy concept proposed by Zadeh [24] is utilized. In this paper a new method based on Buckley's fuzzy method [5]-[6] is developed to create fuzzy numbers for different measures. Our method uses confidence intervals of each measure as the $\alpha - cuts$ of its fuzzy number. This method leads to triangular fuzzy shape numbers for each measure which consider both unpredictable features of sample data simultaneously. In spite of that, since comparing of measures during different months is a typical hydrologic assessment, a new fuzzy comparing method which is again based on Buckley's [7] is introduced. To show the performance of the methods more explicitly, they are executed on two examples; a simple example, and a real world example that assess mentioned measures on the data that are obtained from two rivers of Qazvin province of Iran. The data are presented in table 1 in the Appendix after their outliers were modified.

Since the second example is a real case study, determining whether data have normal distribution or not is important in our assessment. For testing normality, there are three options for hypothesis testing:

1) Anderson-Darling test, which is an ECDF (empirical cumulative distribution function) based test.

2) Ryan-Joiner test [8] which is a correlation based test.

3) Kolmogorov-Smirnov test [13] which is an ECDF based test.

The Anderson-Darling and Ryan-Joiner tests have similar power of detecting non-normality. On the other hand the Kolmogorov-Smirnov test has low power [2] for discussions of these tests for normality. Therefore Anderson-Darling test is picked up for testing normality in the paper. The common null hypothesis for these three tests is H_0: data follow a normal distribution. If α level is more than the p-value of the test, H_0 is rejected. The graphs of the method are plotted in Minitab software (Ver. 14) and significant level (α) is considered as 0.05.

III. Fuzzy Concept and Buckley's Estimation Approach

In what follows, with modification, fuzzy estimation based on Buckley's approach [5], [6] is presented. First, some notations and definitions are introduced. A fuzzy number (N) is derived from a fuzzy set by two features:

(1) Normality: the membership function of different point of the set must be one at least in one point.

(2) Convexity: the set must be convex.

Considering theses two features, one can create a different type of fuzzy numbers with different shapes. In this paper, triangular shape fuzzy numbers are obtained as outputs.

There are different methods for creating a fuzzy number from an exact number. The method which is used in this paper is creating left and right function for modeling a fuzzy number. These functions are typically called L-R functions. The left function should have an increasing form and the right function should have a decreasing form. Using theses L-R function, this paper creates a triangular shaped fuzzy number (N) which considers two main features of fuzzy numbers as follows:

(1) N(x) =1 for exactly one x $\in$ R (Normality feature).

(2) For $\alpha \in (0,1]$, the $\alpha - cut$ of N is a closed and bounded interval, which denote by :

$N_\alpha = [n_1(\alpha), n_2(\alpha)]$, .Where $n_1(\alpha)$ is increasing (Left function) and $n_2(\alpha)$ is decreasing (Right function) continuous functions (Convexity feature).

It should be mentioned that $\alpha - cut$ refers to the cut of the membership function of the fuzzy number for a specific degree of membership function (α).

Consider X is a random variable with p. d .f (p .m .f) $f(x;\theta)$ for single parameter θ. Unknown θ must be estimated from a random sample $X_1, X_2, X_3, ..., X_n$. Assume y = U($X_1, X_2, X_3, ..., X_n$) is a statistic used to estimate θ. Having the values of these random variables (by sampling), e.g. $X_i = x_i$, for $1 \le i \le n$, one can obtain a point estimate $\hat{\theta} = y = u(x_1, x_2, x_3, ..., x_n)$ for θ. Since it is not logical to expect θ (as a random parameter) be exactly equal to this point estimate, a $(1-\beta)100\%$ confidence interval is also created for θ.

In this paper, a $(1-\beta)100\%$ confidence interval for θ denotes by $[\theta_1(\beta), \theta_2(\beta)]$, for $0 < \beta < 1$. Therefore the interval $\theta_1 = [\hat{\theta}, \hat{\theta}]$ is 0% confidence interval for θ and $\theta_0 = \ddot{\theta}$ is a 100% confidence interval for θ, where $\ddot{\theta}$ is the whole parameter space. Then a family of $(1-\beta)100\%$ confidence intervals for θ, where $0 \le \beta \le 1$ is obtained. Placing these confidence intervals, one on top of the other, we have a triangular shaped fuzzy number θ whose $\alpha - cuts$ are the following confidence intervals:

$\theta_\alpha = [\theta_1(\alpha), \theta_2(\alpha)]$ For $0 < \alpha < 1 : \theta_0 = \ddot{\theta}$ and $\theta_1 = [\hat{\theta}, \hat{\theta}]$.

Hence, we use more information about θ rather than a point estimate, or just a single interval estimate. It is easy to generalize Buckley's method in the case where θ is a vector of parameters [5], [6].

It should be mentioned that this paper uses β here as significant level because α is reserved for denoting $\alpha - cuts$ of fuzzy numbers. The rest of the section computes $(1-\beta)100\%$ confidence intervals for discharge and return period. Section 4 uses these intervals as $\alpha - cuts$ of fuzzy estimators of their corresponding fuzzy numbers.

III-I. Cuts of a Fuzzy Estimate for Mean of Discharge (Q)

According to the statistical methods, the confidence interval for mean of population with normal distribution and unknown mean and variance is defined as follows:

$$\{\hat{\mu} - t_{\alpha/2}\frac{S}{\sqrt{n}}, \hat{\mu} + t_{\alpha/2}\frac{S}{\sqrt{n}}\} \quad \text{For } 0<\alpha<1 \qquad (4)$$

Since in hydrological studies discharge is always unknown, its mean and variance should be estimated by means of taking samples. Consequently, its confidence interval is defined as follows:

$$\theta_\alpha^1 : \{\overline{Q} - t_{\alpha/2}\frac{S}{\sqrt{n}}, \overline{Q} + t_{\alpha/2}\frac{S}{\sqrt{n}}\} \quad \text{For } 0<\alpha<1 \qquad (5)$$

III-II. Cuts of a Fuzzy Estimate for Return Period (R.P.)

Now according to what was mentioned in this section and previous section (equation 2), we can define $\alpha - cuts$ for R.P. as follows:

$$\theta_\alpha^2 : \{Q_\varphi - t_{\alpha/2,(n-1)} * \frac{S * \sqrt{Z^2{}_\varphi + 2}}{\sqrt{2*n}}, Q_\varphi + t_{\alpha/2,(n-1)} * \frac{S * \sqrt{Z^2{}_\varphi + 2}}{\sqrt{2*n}}\}$$

For $0<\alpha<1$. (6)

Where $Q_\varphi = \overline{Q} + S * Z_\varphi$ $0<\varphi<1$ and φ denotes exceedance probability.

IV. A New Algorithm to Estimate Fuzzy μ_Q and R.P.

Let θ_α^i for $\alpha \in (0,1)$ and *i*=1, 2 be as in section 3. In this section Buckley's approach is extended to create a new algorithm to find fuzzy estimates for μ_Q and return period.

IV-I. Algorithm of Fuzzy Estimation of Hydrological Measures (FEHM)

(1) Let θ_0^i = R $^+$, θ_1^i = $[\hat{\theta}^i, \hat{\theta}^i]$ for *i*=1, 2; $\hat{\theta}^1 = \mu_Q$ and $\hat{\theta}^2 = R.P.$, where R $^+$ is the set of all positive real numbers (This step creates confidence intervals of measures).

(2) Place θ_α^i; $0 \le \alpha \le 1$, one on top of the other , to produce a triangular shaped fuzzy number $\hat{\theta}^i$ for *i*=1,2 ; where $\hat{\theta}^1 = \mu_Q$ and $\hat{\theta}^2 = R.P.$ (This step creates fuzzy numbers of measures by means of their confidence intervals).

Since μ_Q and R.P are generally unknown and must be estimated from observations, the uncertainty due to sampling variability is unavoidable. Therefore, this paper introduces FEHM algorithm to guard against uncertainty in order to get close to the real value of the measures. In what follows, performance of FEHM algorithm is illustrated by two examples. It should be mentioned that in the following sections of this paper, μ_Q or mean discharge of each month is only called discharge.

IV-II. Examples

As mentioned above, to assess the performance of the methods, two examples are given. The first one is a simple example which taken from Ziaee [25] and the second one is a real world case study.

IV-II-I. EXAMPLE 1. Consider that discharge observations of 45 years (1921-1965) of a river are summarized as follows:

$$\overline{Q} = \frac{\sum Q_i}{45} = 6.90 m^3/\sec \quad (7)$$

$$S^2 = \frac{(\sum Q^2 - 450\overline{Q}^2)}{44} = 5.414 \quad (8)$$

$$Q_{0.9} = 6.9 + 2.32 * Z_{0.9} = 3.92 \quad \text{Where } \varphi = 0.9 \quad (9)$$

Now, according to what was defined in (5) and (6), we can compute $\alpha - cuts$ of $\hat{\theta}^i$ for i=1,2 as follows:

$$\theta_\alpha^1: \{6.9 - t_{\alpha/2}\frac{2.32}{\sqrt{45}}, 6.9 + t_{\alpha/2}\frac{2.32}{\sqrt{45}}\} \text{ For } 0<\alpha<1 \quad (10)$$

θ_α^2:

$$\{3.92 - t_{\alpha/2,(n-1)} * \frac{2.32*\sqrt{1.28^2+2}}{\sqrt{2*45}}, 3.92 + t_{\alpha/2,(n-1)} * \frac{2.32*\sqrt{1.28^2+2}}{\sqrt{2*45}}\}$$

For $0<\alpha<1$. (11)

In the first step of FEHM algorithm, we can obtain θ_0^i and θ_1^i for i=1, 2 (for each i separately). During the second step, by placing $\theta_0^i, \theta_\alpha^i$ for $\alpha \in (0,1)$, and θ_1^i for i=1, 2 (for each i separately), which are calculated by the first step and (5) and (6) respectively, one on top of the other, we can obtain fuzzy estimates for mean and T respectively. The graphs of their membership functions are shown in Fig.1 and are coded by Matlab (Ver. 2010) software.

Note that in classical method, as it was shown in (7), (8) and (9) one can find estimates $\overline{Q} = 6.90$, $S^2 = 5.414$ and $Q_{0.9} = 3.92$. We would never expect these precise point estimates to be exactly equal to the parameters value, so we often compute $(1-\beta)100\%$ confidence intervals for our parameters. The fuzzy estimate obtained by the FEHM algorithm contains more information than a point or interval estimate, in the sense that the fuzzy estimate contains point estimates and $(1-\beta)100\%$ confidence intervals for all at once for $\beta \in [0,1)$, which is very useful for a practitioner.

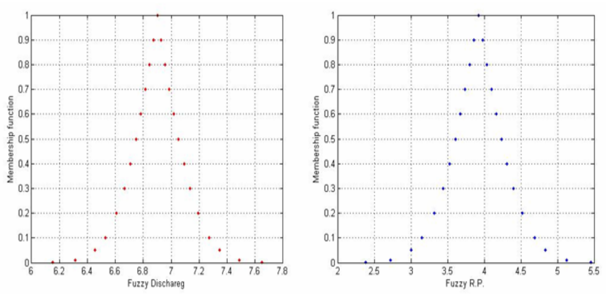

Fig.1 Fuzzy membership functions of discharge and return period.

From the fuzzy estimate, one can conclude that the classical estimate $Q_{0.9} = 3.92$ belongs to the fuzzy estimate $Q_{0.9}$ with grade of membership equal to 1. Clearly, fuzzy set of $Q_{0.9}$ contains more elements other than "3.92" with corresponding grades of membership. For example, one can say that $Q_{0.9} = 3.861$ belongs to the $Q_{0.9}$ with grade of membership $Q(3.861) = 0.9$ (Fig.1).

IV-II-II. EXAMPLE 2. This example addresses the above-mentioned fuzzy estimation method on a real world case study to create new fuzzy measures and develop use of fuzzy concept in hydrological studies more extensively. Hydrological measures (data) are taken from regional water department of Qazvin. Discharge observations of the two rivers are represented in Table 1 in the Appendix. Two columns of rivers represent the discharges of two different months of each river during the different years. First month is Esfand (from 20th of Feb to 20th of Mar) and the second month is Farvardin (from 21th of Mar to 20th Apr). For clarifying, the first month is denoted by Month1 and second month is denoted by Month2 in the following assessment reports.

However, since there were outlier data among observations that interrupt calculations, before using these data in the paper, they needed some modifications. Therefore, observations in the Appendix are modified data. Amount of observations of each group is at least for 25 years that can support usage of normal distribution for most of estimated methods or formulas of hydrological studies that are obtained from normal distribution. Nevertheless, for more assurance of this usage, a normality test called Anderson-Darling test with % 95 confidences (or %0.05 of significant level) is used and run for each group of observations in Minitab software (Ver. 14).

Fig.2 to Fig.5 summarizes outputs of Anderson-Darling test for four groups of observations and Fig.6 to Fig.7 summarizes fuzzy measures of two rivers.

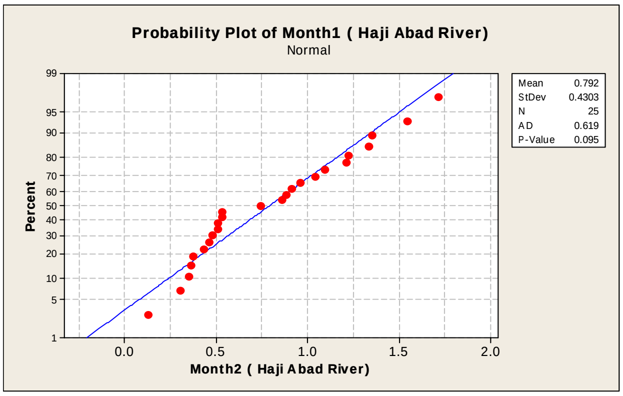

Fig.2 Anderson Darling normal test for Haji Arab river's Data (Month 1)

IV-III. Interpreting the Results

The output of the test is the plot of normal probabilities versus the data. Most evidently in the extremes, or distribution tails the data depart from the fitted line. In all figures (which are doing Anderson-Darling test graphically), the Anderson-Darling test's p-value indicates at a significant level of %0.05 there is evidence that the data of four groups can follow a normal distribution. Because in all groups α < p-value.

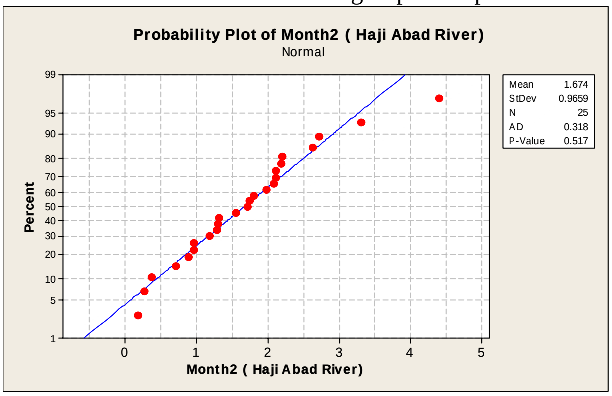

Fig.3 Anderson Darling normal test for Haji Arab river's Data (Month 2)

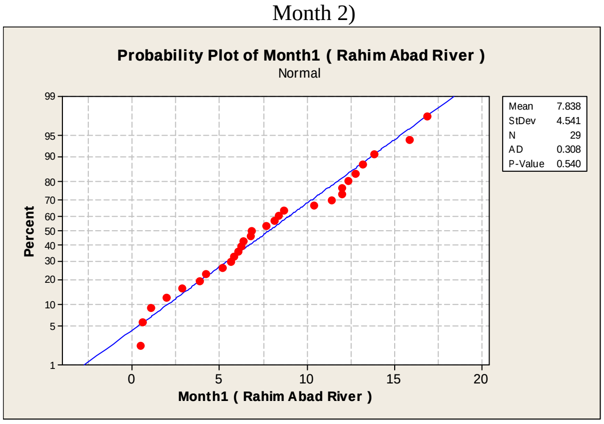

Fig.4 Anderson Darling normal test for Rahim Abad river's Data (Month 1)

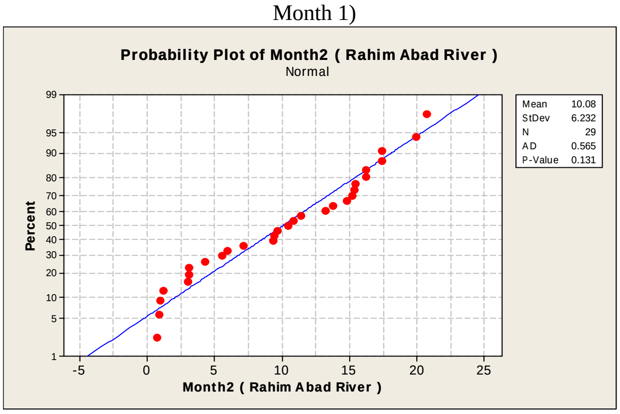

Fig.5 Anderson Darling normal test for Rahim Abad river's Data (Month 2)

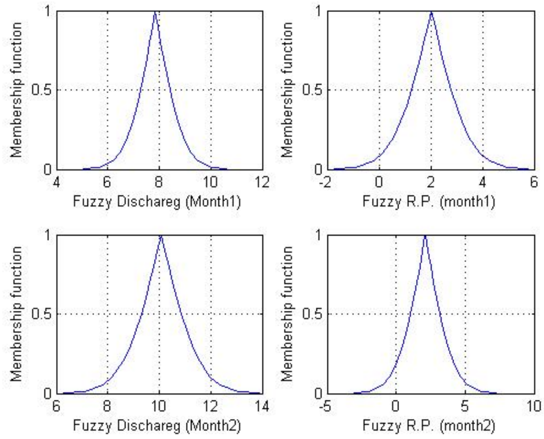

Fig.6 Fuzzy discharge and fuzzy return priod of Rahim Abad River

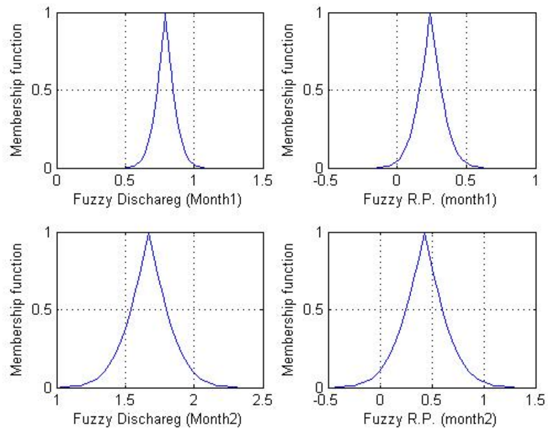

Fig.7 Fuzzy discharge and fuzzy return period of Haji Arab River

V. New Method for Ranking Fuzzy Discharge of Different Months

An important typical assessment in hydrological studies is ranking (comparing) outputs of different measures. For instance one may want to compare discharges of two or more months together to rank them. These types of assessments can be easily done in an exact (Non Fuzzy) environment like what we've been doing in classical studies. However, in a fuzzy assessment like the one this research is studying, comparing is done among two or more fuzzy numbers to rank them. Consequently we need a fuzzy method for doing such ranking. On the other hand, this paper introduces especial form of fuzzy hydrological measures; therefore it suggests a compatible fuzzy ranking method for its users.

V-I. Algorithm of Fuzzy Ranking of Hydrological Measures (FROM)

This algorithm has two main steps:

(1) Linear Estimating of nonlinear fuzzy numbers.
(2) Comparing of estimated linear numbers.

V-I-I. Illustration of first step of FROM algorithm

Generally, using mentioned fuzzy method of creating fuzzy numbers, we have non linear triangular shaped fuzzy number, like in Fig.1. But in order to simplify the calculating, the paper uses estimated linear fuzzy numbers like those ones you see in Fig.8 and Fig.9. Membership functions of these linear functions are given as (12) to (15) in respect to the order of the figures from up to down. It means equation 12 is related to Rahim Abad's first month (Month1) and equation 15 is related to the Haji Arab' seconds month (Month2).

The left function of each of these linear functions is made by two left points: 1) median point and 2) the point in the left extreme. On the other hand, the right function of each of these linear functions is made by two right points: 1) median point and 2) the point in the right extreme.

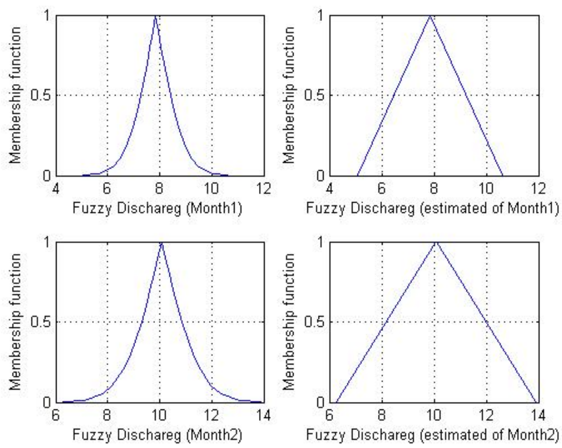

Fig.8 Fuzzy discharges and their linear estimated numbers of Rahim Abad River

$$\begin{cases} \frac{x-5.065}{7.84-5.065} & 5.065 \le x < 7.84 \\ 1 & x = 7.84 \\ \frac{10.615-x}{10.615-7.84} & 7.84 < x \le 10.615 \end{cases} \quad (12)$$

$$\begin{cases} \frac{x-6.273}{10.08-6.273} & 6.273 \le x < 10.08 \\ 1 & x = 10.08 \\ \frac{13.887-x}{13.887-10.08} & 10.08 < x \le 13.887 \end{cases} \quad (13)$$

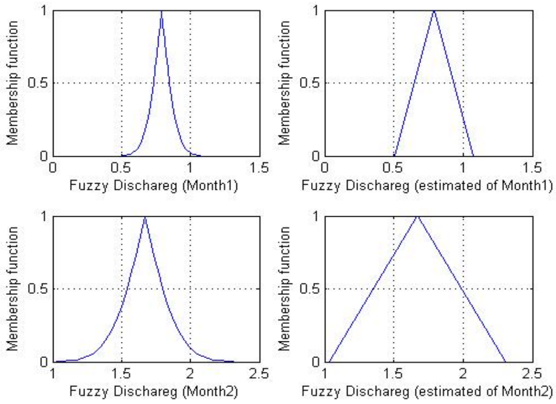

Fig.9 Fuzzy discharges and their linear estimated numbers of Haji Arab River

$$\begin{cases} \frac{x-0.507}{0.79-0.507} & 0.79 \le x < 7.84 \\ 1 & x = 0.79 \\ \frac{1.073-x}{1.073-0.79} & 0.79 < x \le 1.073 \end{cases} \quad (14)$$

$$\begin{cases} \frac{x-1.032}{1.67-1.032} & 1.032 \le x < 1.67 \\ 1 & x = 1.67 \\ \frac{2.038-x}{2.038-1.67} & 1.67 < x \le 2.038 \end{cases} \quad (15)$$

You can see in all linear figures the estimation is pessimistic and causes more fuzziness. Therefore the comparing method which is introduced in following section is strict.

V-I-II. Illustration of second step of FROM algorithm

The performance of this step is illustrated graphically. As seen in Fig.10, the vertex of fuzzy discharge 1 is at $x = d$ and the vertex of fuzzy discharge 2 is at $x = c$. A.D. represents the total area under the graph of fuzzy discharge 1, and A.R. is the area under the graph of fuzzy discharge 2, but to the right side of the vertical line through $x = c$. We choose a value of $\gamma \in (0,1)$ and our decision rule is: if $\frac{A.R.}{A.D.} \ge \gamma$, then the left figure (fuzzy discharge 1) is greater, otherwise the right figure (fuzzy discharge 2) is greater. Lets in this paper choose $\gamma = 0.4$. Surely, $\gamma \ge 0.5$ is not acceptable. Notice that in Fig.10 we get $\frac{A.R.}{A.D.} \ge 0.5$ when $x = d$ lies to the right of $x = c$.

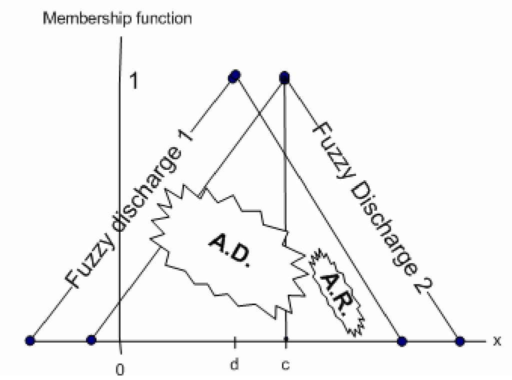

Fig.10 Fuzzy comparison of two different fuzzy discharges

Related comparison figures of two rivers are shown in Fig.11 (summarizes linear shapes of Fig.6 and Fig.7) and Fig.12 (summarizes linear shapes of Fig.8 and Fig.9).
In both figures the left figure is for Month1 and the right figure is for Month2.

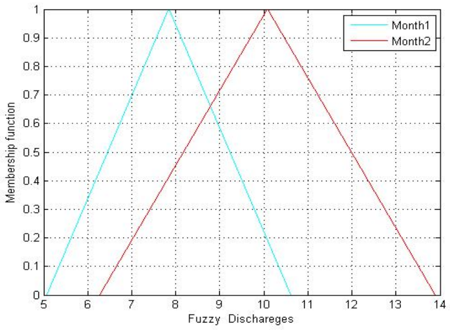

Fig.11 Comparison of linear estimated mean discharges (for two months) of Rahim Abad River

To illustrate the ranking method more explicitly in Fig.13 and Fig.11 is plotted like Fig.10. Final calculations are done in (16) and (17).

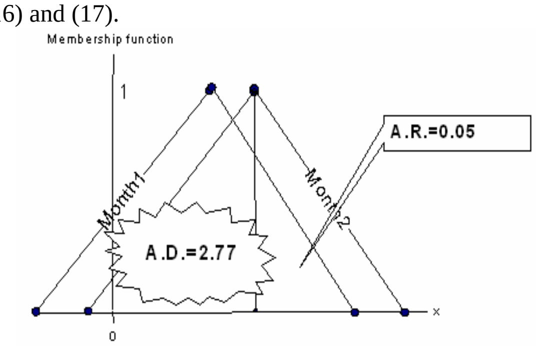

Fig.13. Comparison of linear estimated mean discharges (for two months) of Rahim Abad River

$$\frac{A.R.}{A.D.} = \frac{0.05}{2.77} \approx 0.02 < 0.4 \qquad (16)$$

$$\frac{A.R.}{A.D.} = \frac{0}{0.142} = 0 < 0.4 \qquad (17)$$

Equation 16 shows that in Rahim Abad River mean discharge of Month2 is greater than mean discharge of Month1. Equation 17 shows that result of Haji Arab River is as the same of Rahim Abad River. Since the ranking method is pessimistic and in both rivers Month2 has a greater mean discharge than Month1, therefore we can expand results to crisp calculations as follows; In a crisp ranking for both rivers, Month2 has a greater mean discharge than Month1.

VI. Conclusion

Since mean discharge and return period (R.P.) are estimated using sample data, it is necessary to obtain confidence intervals rather than simple point estimates to estimate them. To find fuzzy estimates for mean discharge and return period, a new algorithm named FEHM algorithm, based on Buckley's approach, is introduced. The final results of the proposed algorithm contain not only point estimates, but also interval estimates and, hence, provide more information for its users. The proposed method also considered uncertainty and impreciseness simultaneously. The fuzzy method is also performed on real world observations. Since outputs of the paper have a special form of fuzzy numbers, a new compatible method named FROM algorithm is introduced in order to rank fuzzy estimated discharges of different months, and the real world numerical example is used to illustrate the performance of the new comparing method. Future researches can use these methods for different stochastic parameters of hydrologic models to study them fuzzily. Moreover, these fuzzy numbers and other fuzzy applications can be used to do all calculations of a hydrologic study fuzzily. In addition, other techniques of making fuzzy numbers can be developed for model's parameter to test the pros and cons of techniques on real world studies.

Appendix

Table 1 Discharge observation of the case study of the paper, for two months during the years

	Haji arab River		Rahim Abad River	
Years	Month1	Month2	Month1	Month2
1	0.88	2.62	6.07	15.35
2	0.53	1.71	5.82	10.81
3	1.04	2.18	8.15	17.38
4	1.35	3.30	6.75	11.39
5	1.21	2.20	12.37	16.21
6	0.48	0.95	6.20	9.66
7	0.36	1.18	13.85	15.40
8	0.46	2.71	15.86	14.80
9	1.33	2.11	16.85	10.42
10	0.91	1.55	7.66	9.29
11	0.74	1.30	6.82	13.76

12	1.71	2.10	8.38	19.93
13	0.43	0.96	10.39	9.37
14	1.22	2.08	13.17	15.21
15	0.30	0.27	11.99	16.20
16	0.35	1.28	12.01	20.70
17	0.51	0.36	6.35	4.30
18	0.13	0.17	11.40	17.41
19	0.96	1.80	3.86	3.09
20	0.86	1.97	2.83	5.54
21	1.54	1.74	5.60	3.10
22	0.53	0.71	4.20	0.68
23	0.51	4.39	1.97	13.22
24	1.09	0.89	0.47	3.02
25	0.37	1.31	12.78	1.20
26			1.07	0.92
27			8.64	7.14
28			5.18	0.86
29			0.60	5.90

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