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A New Approach For Ranking Of Generalized Trapezoidal Fuzzy Numbers

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Abstract—Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting etc. Fuzzy numbers must be ranked before an action is taken by a decision maker. In this paper, with the help of several counter examples it is proved that ranking method proposed by Chen and Chen (Expert Systems with Applications 36 (2009) 6833-6842) is incorrect. The main aim of this paper is to propose a new approach for the ranking of generalized trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. It is shown that proposed ranking function satisfies all the reasonable properties of fuzzy quantities proposed by Wang and Kerre (Fuzzy Sets and Systems 118 (2001) 375-385).

Keywords—Ranking function, Generalized trapezoidal fuzzy numbers

I. Introduction

FUZZY set theory [1] is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by $\geq$ or $\leq$, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $\Re : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory.

The method for ranking was first proposed by Jain [2]. Yager [3] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1]. In Kaufmann and Gupta [4], an approach is presented for the ranking of fuzzy numbers. Campos and Gonzalez [5] proposed a subjective approach for ranking fuzzy numbers. Liou and Wang [6] developed a ranking method based on integral value index. Cheng [7] presented a method for ranking fuzzy numbers by using the distance method. Kwang and Lee [8] considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Modarres and Nezhad [9] proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao [10] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [11] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [12] and Wang and Lee [13] also used the centroid concept in developing their ranking index. Chen and Chen [14] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [15] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-levels of trapezoidal fuzzy numbers. Chen and Chen [16] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads.

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In this paper, with the help of several counter examples it is shown that ranking method proposed by Chen and Chen [16] is incorrect. The main aim of this paper is to propose a new approach for the ranking of generalized trapezoidal fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

This paper is organized as follows: In section II, some basic definitions, arithmetic operations and ranking function are reviewed. In section III, the shortcomings of Chen and Chen approach [16] is discussed. In section IV, a new approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. In section V, the correct ordering of fuzzy numbers are obtained and also it is shown that the proposed ranking function satisfies all the reasonable properties of fuzzy quantities. The conclusion is discussed in section VI.

II. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

A. Basic Definitions

In this section some basic definitions are reviewed.

Definition 1. [4] The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that

the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A.
The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x));$
$x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2. [4] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:
1. $\mu_{\tilde{A}} : R \longrightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b,c]$, where $a < b < c < d$.

Definition 3. [4] A fuzzy number $\tilde{A} = (a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a < x < b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

Definition 4. [16] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:
1. $\mu_{\tilde{A}} : R \longrightarrow [0,w]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = w$, for all $x \in [b,c]$, where $0 < w \leq 1$.

Definition 5. [17] A fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$ is said to be a L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} wL(\frac{b-x}{b-a}), & \text{for } a < x < b \\ w & \text{for } b \leq x \leq c \\ wR(\frac{x-c}{d-c}) & \text{for } c < x < d. \end{cases}$$

where L and R are reference functions.

Definition 6. [17] A L-R type generalized fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{(x-a)}{(b-a)}, & a < x < b \\ w & b \leq x \leq c \\ w\frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

B. Arithmetic operations

In this subsection, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [16].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; min\,(w_1, w_2))$
(ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2; min\,(w_1, w_2))$
(iii) $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1) & \lambda < 0. \end{cases}$

C. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [2], $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\tilde{A} \succ \tilde{B}$ iff $\Re(\tilde{A}) > \Re(\tilde{B})$
(ii) $\tilde{A} \prec \tilde{B}$ iff $\Re(\tilde{A}) < \Re(\tilde{B})$
(iii) $\tilde{A} \sim \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$

Remark 1. [18] For all fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ we have

(i) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{C}$
(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} \succ \tilde{B} \ominus \tilde{C}$
(iii) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$
(iv) $\tilde{A} \succ \tilde{B}, \tilde{C} \succ \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{D}$

III. Shortcomings of Chen and Chen approach

In this section, the shortcomings of Chen and Chen approach [16], on the basis of reasonable properties of fuzzy quantities [18] and on the basis of height of fuzzy numbers, are pointed out

A. On the basis of reasonable properties of fuzzy quantities

Let $\tilde{A}$ and $\tilde{B}$ be any two fuzzy numbers then

$\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$ (Using remark 1)
i.e., $\Re(\tilde{A}) > \Re(\tilde{B}) \Rightarrow \Re(\tilde{A} \ominus \tilde{B}) > \Re(\tilde{B} \ominus \tilde{B})$

In this subsection, several examples are choosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property, $\tilde{A} \succ \tilde{B} \Rightarrow (\tilde{A} \ominus \tilde{B}) \succ (\tilde{B} \ominus \tilde{B})$, for the ordering of fuzzy quantities i.e., according to Chen Chen approach $\tilde{A} \succ \tilde{B} \not\Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$, which is contradiction according to Wang and Kerre [18].

Example 1. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \not\Rightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

Example 2. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \not\Rightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

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Example 3. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \succ \tilde{B}$ but $\tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$ i.e., $\tilde{A} \succ \tilde{B} \nRightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.

Example 4. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

B. *On the basis of height of fuzzy numbers*

In this subsection, it is proved that, in some cases, Chen and Chen approach [16] states that the ranking of fuzzy numbers depends upon height of fuzzy numbers while in several cases the ranking does not depend upon the height of fuzzy numbers.

Let $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (a_1, a_2, a_3, a_4; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen [16] there may be two cases

Case (i) If $(a_1 + a_2 + a_3 + a_4) \neq 0$ then
$$\begin{cases} \tilde{A} \prec \tilde{B}, & \text{if } w_1 < w_2 \\ \tilde{A} \succ \tilde{B}, & \text{if } w_1 > w_2 \\ \tilde{A} \sim \tilde{B}, & \text{if } w_1 = w_2. \end{cases}$$
Case (ii) If $(a_1 + a_2 + a_3 + a_4) = 0$ then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

According to Chen and Chen [16] in first case ranking of fuzzy numbers depends upon height and in second case ranking does not depend upon the height which is contradiction.

Example 5. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \prec \tilde{B}$ if $w_1 < w_2$, $\tilde{A} \succ \tilde{B}$ if $w_1 > w_2$ and $\tilde{A} \sim \tilde{B}$ if $w_1 = w_2$.

Example 6. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized trapezoidal fuzzy numbers then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

IV. Proposed Approach

In this section, a new approach is proposed for the ranking of generalized trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A} \succ \tilde{B}$ if RM$(\tilde{A})$ >RM$(\tilde{B})$
(ii) $\tilde{A} \prec \tilde{B}$ if RM$(\tilde{A})$ <RM$(\tilde{B})$
(iii) $\tilde{A} \sim \tilde{B}$ if RM$(\tilde{A})$ =RM$(\tilde{B})$ (1)

A. *Method to find values of RM$(\tilde{A})$ and RM$(\tilde{B})$*

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then use the following steps to find the values of RM$(\tilde{A})$ and RM$(\tilde{B})$

Step 1 Find $w = min(w_1, w_2)$

Step 2 Find $\Re(\tilde{A}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_1 + \frac{(b_1 - a_1)}{w}x$, $R^{-1}(x) = c_1 + \frac{(b_1 - c_1)}{w}x$ $\Rightarrow \Re(\tilde{A}) = \frac{w(a_1+b_1+c_1+d_1)}{4}$ and $\Re(\tilde{B}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_2 + \frac{(b_2 - a_2)}{w}x$, $R^{-1}(x) = c_2 + \frac{(b_2 - c_2)}{w}x \Rightarrow \Re(\tilde{B}) = \frac{w(a_2+b_2+c_2+d_2)}{4}$.

Step 3 If $\Re(\tilde{A}) \neq \Re(\tilde{B})$ then RM$(\tilde{A}) = \Re(\tilde{A})$ and RM $(\tilde{B}) = \Re(\tilde{B})$
otherwise RM$(\tilde{A})$ =mode$(\tilde{A}) = \frac{1}{2}\int_0^w b_1 dx + \frac{1}{2}\int_0^w c_1 dx = \frac{w(b_1+c_1)}{2}$ and RM$(\tilde{B})$ =mode$(\tilde{B}) = \frac{1}{2}\int_0^w b_2 dx + \frac{1}{2}\int_0^w c_2 dx = \frac{w(b_2+c_2)}{2}$

Remark 2 The arithmetic operations between two fuzzy numbers are obtained using the α- cut method [4] and the maximum value of α, that will be common for both fuzzy numbers, will be obtained by finding the minimum value of the height of the fuzzy numbers due to which, in step 1, $min(w_1, w_2) = w$ is considered.

V. Results and discussion

In this section, the correct ordering of fuzzy numbers, discussed in section 3, are obtained. Also, in the Table 1, it is shown that proposed ranking function satisfies all the reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

Example 7. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(1, 1) = 1$
Step 2 $\Re(\tilde{A}) = 0.3$ and $\Re(\tilde{B}) = 0.3$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0.3$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0.3$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 8. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(0.8, 1) = 0.8$
Step 2 $\Re(\tilde{A}) = 0.24$ and $\Re(\tilde{B}) = 0.24$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0.24$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0.24$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 9. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized fuzzy numbers
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = -0.175$ and $\Re(\tilde{B}) = -0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A}) = \Re(\tilde{A})$ and RM$(\tilde{B}) = \Re(\tilde{B})$. Now RM$(\tilde{A})$ < RM$(\tilde{B}) \Rightarrow \tilde{A} \prec \tilde{B}$.

Example 10. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = 0.175$ and $\Re(\tilde{B}) = 0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A}) = \Re(\tilde{A})$ and RM$(\tilde{B}) = \Re(\tilde{B})$. Now RM$(\tilde{A}) >$ RM$(\tilde{B}) \Rightarrow \tilde{A} \succ \tilde{B}$.

Example 11. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers.
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = w$ and $\Re(\tilde{B}) = w$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = w$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = w$. Now RM$(\tilde{A}$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 12. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized fuzzy numbers then
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = 0$ and $\Re(\tilde{B}) = 0$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

A. Validation of the results

In the above examples it can be easily check that

(i) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \sim \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$ = RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \succ$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(iii) $\tilde{A} \prec \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \prec$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

B. Validation of the proposed ranking function

For the validation of the proposed ranking function, in Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

TABLE I
FULFILMENT OF THE AXIOMS FOR THE ORDERING IN THE FIRST AND SECOND CLASS [18]

Index	A_1	A_2	A_3	A_4	A'_4	A_5	A_6	A'_6	A_7
Y_1	Y	Y	Y	Y	Y	Y	N	N	N
Y_2	Y	Y	Y	Y	Y	Y	Y	Y	N
Y_3	Y	Y	Y	N	N	Y	N	N	N
Y_4	Y	Y	Y	Y	Y	Y	N	N	N
C	Y	Y	Y	N	N	Y	N	N	N
FR	Y	Y	Y	Y	Y	Y	Y	Y	N
CL	Y	Y	Y	Y	Y	Y	Y	Y	N
LW^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_1^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_2^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
K	Y	Y	Y	N	N	N	N	N	N
W	Y	Y	Y	Y	N	N	N	N	N
J^k	Y	Y	Y	Y	Y	N	N	N	N
CH^k	Y	Y	Y	Y	Y	N	N	N	N
KP^k	Y	Y	Y	Y	Y	N	N	N	N
Proposed Approach	Y	Y	Y	Y	Y	Y	Y	Y	N

VI. Conclusion

In this paper, the shortcomings of Chen and Chen [16] approach are pointed out and a new ranking approach is proposed for finding the correct ordering of generalized trapezoidal fuzzy numbers. It is shown that proposed ranking function satisfies all the reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

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