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A Mixed Expert Evaluation System and Dynamic Interval-Valued Hesitant Fuzzy Selection Approach

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Abstract**—**In the last decades, concerns about the environmental issues lead to professional and academic efforts on green supplier selection problems. In this sake, one of the main issues in evaluating the green supplier selection problems, which could increase the uncertainty, is the preferences of the experts' judgments about the candidate green suppliers. Therefore, preparing an expert system to evaluate the problem based on the historical data and the experts' knowledge can be sensible. This study provides an expert evaluation system to assess the candidate green suppliers under selected criteria in a multi-period approach. In addition, a ranking approach under interval-valued hesitant fuzzy set (IVHFS) environment is proposed to select the most appropriate green supplier in planning horizon. In the proposed ranking approach, the IVHFS and the last aggregation approach are considered to margin the errors and to prevent data loss, respectively. Hence, a comparative analysis is provided based on an illustrative example to show the feasibility of the proposed approach.

Keywords**—**Green supplier selection, expert system, ranking approach, interval-valued hesitant fuzzy setting.

I. Introduction

ASSESSMENT and choosing the appropriate green supplier based on economic and environmental criteria is an inevitable manner of a group decision-making process. Thus, different information and conflicting criteria may be respected to select the most suitable candidate green supplier in an imprecise situation. In this sake, green supplier selection problem as a multi-criteria decision making problem could be the main issue for many companies. In this respect, some researchers focused on this interesting topic based on group decision analysis under precise/imprecise information.

Herein, Handfield et al. [1] used the AHP (Analytical Hierarchy Process) method to evaluate the significance of various environmental traits and specify the relative performance of the candidate suppliers under the traits. Yang and Wu [2] proposed a multi-level grey entropy synthetic evaluation approach to avoid the lower weight factor which could lead to more powerful evaluation method. Hsu and Hu [3] presented a multi-criteria decision model to determine the hazardous substance management criteria and then utilized an ANP (Analytic Network Process) method for solve the green supplier selection problem. Feyzioğlu and Büyüközkan [4] prepared a multi-criteria assessment approach based on Choquet integral operators for evaluating the performance of candidate green suppliers. Tsui and Wen [5] proposed a hybrid multi-criteria group decision making (MCGDM) approach based on AHP, entropy, ELECTRE III (Elimination and Choice Translating Reality III) and the linear assessment methods to assist the manufacturing companies for selecting the best green supplier.

In many real cases, group decision-making problems under complex conditions increase the uncertainty in which experts assign their preferences judgments based on imprecise information. Hence, evaluating the candidate green suppliers under precise environment is difficult and should be defined under imprecisely/uncertainty environment. Accordingly, the fuzzy set theory and its developments are known as an appropriate tool to deal with uncertainty. Herein, some authors utilized this theory to cope with uncertain situations for evaluating the candidate green suppliers. In this respect, Büyüközkan and Çifçi [6] extended a novel approach by using the fuzzy ANP method based on incomplete preference relations under the multi-person decision making scheme. Datta et al. [7] implemented a VIKOR method under the interval-valued fuzzy setting environment to assess the best candidate green supplier. Kannan et al. [8] presented an integrated TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) and AHP method to determine the importance of selected criteria according to the preferences of the experts' judgments for solving the green supplier selection problem. In their study, a multi-objective programming model is extended to assign the optimum order among them. Sepehriar et al. [9] presented a new group decision making method by using the trapezoidal fuzzy information for assessing the supplier selection problems. In addition, Khamseh and Mahmoodi [10] proposed an integrated TOPSIS-TODIM method based on triangular fuzzy time function to solve the green supplier selection problems. Chen [11] developed an outranking approach based on the preferences of the experts' judgments under the interval type-2 fuzzy set environment to determine the best candidate supplier. Cao et al. [12] presented a new approach based on optimization model under intuitionistic fuzzy set environment to specify the subjective and objective weights, respectively. Then, a TOPSIS method is developed based on intuitionistic fuzzy set to rank the candidate green suppliers. Kannan et al. [13] proposed a multi-criteria decision making method based on axiomatic design and fuzzy set theory to assess the candidate green suppliers. Celik et al. [14] as well as [15], [16] focused on ELECTRE methods by considering the interval type-2 fuzzy information to evaluate the candidate green logistics service providers.

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Survey of supplier selection as well as green supplier selection literature indicates that evaluating the problems are provided based on the experts' opinions. In this respect, preparing an expert system based on historical data and experts' knowledge to evaluate the green supplier selection is more interesting because reduction of the experts' opinions in the procedure of group decision making problems could decrease the uncertainty. To address the issue, poor attention is provided to assess decision making problems based on expert system [17]-[19]. However, this paper elaborated an expert system to evaluate green supplier selection problems called expert evaluation system. In addition, a novel ranking approach is manipulated based on group decision analysis and the IVHFS theory to select the most suitable green supplier. IVHFS could help the experts by assigning some interval-values membership degrees for a candidate green supplier along the conflicted criteria under a set to margin the errors. Moreover, the last aggregation approach is considered in the procedure of the proposed ranking method to avoid the loss of data.

For the sake of clarity, the basic concepts and operators about the dynamic IVHFS are presented in Section II. In Section III, an expert system is provided to evaluate the green supplier selection; then, a ranking approach is presented to select the most suitable green supplier. In addition, computational experiment is provided in Section IV, to show the feasibility of the presented approach. Finally, conclusions and future directions are prepared in Section V.

II. Preliminaries

In this section, the basic concept and operations on dynamic IVHFS are defined. Furthermore, some operations, which are required for the proposed approach, are developed.

Definition 1 [20]. Let t as a time variable, then $\tilde{h}(t) = \cup_{\tilde{\gamma}(t)\in\tilde{h}(t)} \left\{\left[\tilde{\gamma}^L(t), \tilde{\gamma}^U(t)\right]\right\}$ is an IVHFV, where $0 \le \tilde{\gamma}^L(t) \le \tilde{\gamma}^U(t) \le 1$. If $t = t_1, t_2, ..., t_p$, is defined for an IVHFV then $\tilde{h}(t_p) = \{\tilde{h}(t_1), \tilde{h}(t_2), ..., \tilde{h}(t_p)\}$ $\forall p = 1, 2, ..., P$ expressed as p IVHFEs which collected at p different periods.

Definition 2 [20]. Let $\tilde{h}(t_1)$ and $\tilde{h}(t_2)$ as two IVHFVs, then the following relations are defined:

$$\lambda \tilde{h}(t_1) = \cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1)} \left\{\left[1-\left(1-\tilde{\gamma}^L(t_1)\right)^{\lambda}, 1-\left(1-\tilde{\gamma}^U(t_1)\right)^{\lambda}\right]\right\} \quad (1)$$

$$\left(\tilde{h}(t_1)\right)^{\lambda} = \cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1)} \left\{\left[\left(\tilde{\gamma}^L(t_1)\right)^{\lambda}, \left(\tilde{\gamma}^U(t_1)\right)^{\lambda}\right]\right\} \quad (2)$$

$$\tilde{h}(t_1) \oplus \tilde{h}(t_2) = \cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1), \tilde{\gamma}(t_2)\in\tilde{h}(t_2)} \left\{\left[\tilde{\gamma}^L(t_1)+\tilde{\gamma}^L(t_2)-\tilde{\gamma}^L(t_1)\tilde{\gamma}^L(t_2), \tilde{\gamma}^U(t_1)+\tilde{\gamma}^U(t_2)-\tilde{\gamma}^U(t_1)\tilde{\gamma}^U(t_2)\right]\right\} \quad (3)$$

$$\tilde{h}(t_1) \otimes \tilde{h}(t_2) = \cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1), \tilde{\gamma}(t_2)\in\tilde{h}(t_2)} \left\{\left[\tilde{\gamma}^L(t_1)\tilde{\gamma}^L(t_2), \tilde{\gamma}^U(t_1)\tilde{\gamma}^U(t_2)\right]\right\} \quad (4)$$

Definition 3. Let $E = \{\tilde{h}(t_1), \tilde{h}(t_2), ..., \tilde{h}(t_P)\}$ be a collection of IVHFVs. Then, the following extended relations are obtained based on Definition 2:

$$\bigoplus_{p=1}^{P} \tilde{h}(t_p) = \cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1), \tilde{\gamma}(t_2)\in\tilde{h}(t_2), ..., \tilde{\gamma}(t_p)\in\tilde{h}(t_p)} \left\{\left[1-\prod_{p=1}^{P}(1-\tilde{\gamma}^L(t_p)), 1-\prod_{p=1}^{P}(1-\tilde{\gamma}^U(t_p))\right]\right\} \quad (5)$$

$$\tilde{h}(t_1)\otimes\tilde{h}(t_2)\otimes....\otimes\tilde{h}(t_P)=\cup_{\tilde{\gamma}(t_1)\in\tilde{h}(t_1),\tilde{\gamma}(t_2)\in\tilde{h}(t_2),...,\tilde{\gamma}(t_P)\in\tilde{h}(t_P)}\left\{\left[\prod_{p=1}^{P}\tilde{\gamma}^L(t_p),\prod_{i=1}^{P}\tilde{\gamma}^U(t_p)\right]\right\} \tag{6}$$

Definition 4 [20]. A dynamic interval-valued hesitant fuzzy weighted geometric (DIVHFWG) relation is defined as:

$$DIVHFWG_{w(t)}\left(\tilde{h}(t_1),\tilde{h}(t_2),...,\tilde{h}(t_p)\right)=\overset{P}{\underset{p=1}{\oplus}}\tilde{h}(t_p)^{w(t_p)}$$
$$=\cup_{\tilde{\gamma}(t_P)\in\tilde{h}(t_P),p=1,2,...,P}\left\{\left[\prod_{p=1}^{P}(\tilde{\gamma}^L(t_p))^{w(t_p)},\prod_{p=1}^{P}(\tilde{\gamma}^U(t_p))^{w(t_p)}\right]\right\} \tag{7}$$

where $w(t)=(w(t_1),w(t_2),...,w(t_p))^T$ are the weight vector of the time series $t=t_1,t_2,...,t_p$ and $w(t)>0,\sum_{p=1}^{P}w(t_p)=1$.

Definition 5. The dynamic interval-valued hesitant fuzzy geometric (DIVHFG) operator could be defined based on Definition 4, as:

$$DIVHFG\left(\tilde{h}(t_1),\tilde{h}(t_2),...,\tilde{h}(t_p)\right)=\overset{P}{\underset{p=1}{\oplus}}\tilde{h}(t_p)^{\frac{1}{P}}$$
$$=\cup_{\tilde{\gamma}(t_p)\in\tilde{h}(t_p),p=1,2,...,P}\left\{\left[\prod_{p=1}^{P}(\tilde{\gamma}^L(t_p))^{\frac{1}{P}},\prod_{p=1}^{P}(\tilde{\gamma}^U(t_p))^{\frac{1}{P}}\right]\right\} \tag{8}$$

Definition 6. The dynamic interval-valued hesitant fuzzy Euclidean (DIVHFE) distance measure and the dynamic interval-valued hesitant fuzzy hamming (DIVHFH) distance measure are defined, respectively as:

$$d\left(\tilde{h}(t_1),\tilde{h}(t_2)\right)_{DIVHFE}=\sqrt{\frac{1}{2l_{x_i}}\sum_{\lambda=1}^{l_{x_i}}\left(\left|\tilde{h}^{\sigma(\lambda)L}(t_1)-\tilde{h}^{\sigma(\lambda)L}(t_2)\right|^2+\left|\tilde{h}^{\sigma(\lambda)U}(t_1)-\tilde{h}^{\sigma(\lambda)U}(t_2)\right|^2\right)} \tag{9}$$

$$d\left(\tilde{h}(t_1),\tilde{h}(t_2)\right)_{DIVHFH}=\frac{1}{2l_{x_i}}\sum_{\lambda=1}^{l_{x_i}}\left(\left|\tilde{h}^{\sigma(\lambda)L}(t_1)-\tilde{h}^{\sigma(\lambda)L}(t_2)\right|+\left|\tilde{h}^{\sigma(\lambda)U}(t_1)-\tilde{h}^{\sigma(\lambda)U}(t_2)\right|\right) \tag{10}$$

where $\tilde{h}(t_1)$ and $\tilde{h}(t_2)$ are IVHFVs which indicates as $\tilde{h}^{\sigma(\lambda)}(t_1)=\left[\tilde{h}^{\sigma(\lambda)L}(t_1),\tilde{h}^{\sigma(\lambda)U}(t_1)\right]$, $\tilde{h}^{\sigma(\lambda)}(t_2)=\left[\tilde{h}^{\sigma(\lambda)L}(t_2),\tilde{h}^{\sigma(\lambda)U}(t_2)\right]$ respectively; and $\tilde{h}^{\sigma(\lambda)}(t_1),\tilde{h}^{\sigma(\lambda)}(t_2)$ are the λth largest intervals in $\tilde{h}(t_1)$ and $\tilde{h}(t_2)$, respectively.

Definition 7. The dynamic interval-valued hesitant fuzzy decision matrix $\left(\tilde{D}=\left(\tilde{h}_{ij}(t_p)\right)_{m\times n}\right)$ could be normalized $\left(\tilde{N}=\left(\tilde{f}_{ij}(t_p)\right)_{m\times n}\right)$ based on:

$$\tilde{f}_{ij}\left(t_p\right)=\begin{cases}\left\{\left[\tilde{\gamma}_{ij}^{L}\left(t_p\right),\tilde{\gamma}_{ij}^{U}\left(t_p\right)\right]\right\} & \text{For positive criteria}\\ \left\{\left[1-\tilde{\gamma}_{ij}^{L}\left(t_p\right),1-\tilde{\gamma}_{ij}^{U}\left(t_p\right)\right]\right\} & \text{For negative criteria}\end{cases} \tag{11}$$
$$\forall i=1,2,...,m;\ j=1,2,...,n,\ p=1,2,...,P$$

III. Proposed Approach

In this section, an evaluation module is presented based on the expert system; next, obtained results from the proposed expert evaluation system which is dynamic interval-valued hesitant fuzzy decision matrix is considered as the input data of the ranking module. In this sake, the relative importance of each criterion is determined based on preferences experts' opinions. Then the candidates are ranked based on a new ranking method under uncertainty. Hereupon, Fig. 1 represented the structure of the proposed approach.

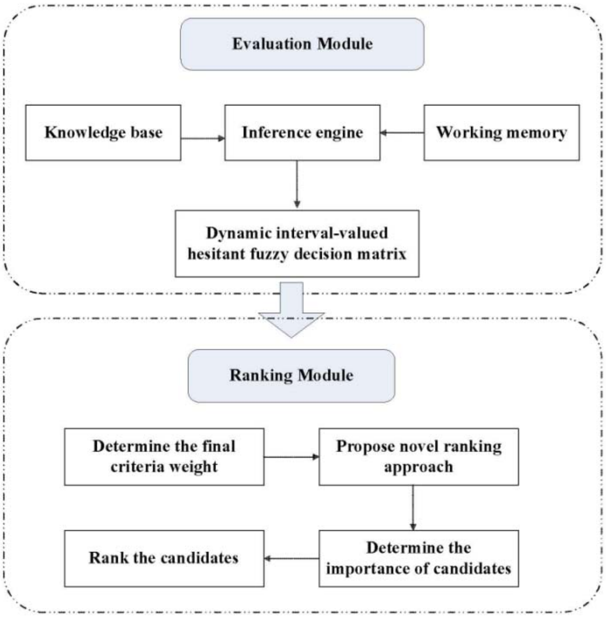

Fig. 1 Hierarchical structure of the proposed approach

A. Proposed Evaluation Approach

In this section, an expert system approach is proposed to evaluate the green supplier selection problem. To address the issue, a group of experts $\left(E_k, k=1,2,...,K\right)$ is established to judge the candidate green suppliers $\left(S_i, i=1,2,....,m\right)$ under conflicted criteria $\left(C_i, j=1,2,....,n\right)$ in each period $\left(t_p, p=1,2,....,P\right)$. Accordingly, the evaluation of the candidate green suppliers is determined based on the historical data and the preferences experts' judgments under the proposed expert evaluation system. Thus, the interval-valued hesitant fuzzy group decision matrix is obtained from the proposed expert evaluation module for each period based on linguistic terms, which are converted to interval-valued hesitant fuzzy elements (IVHFEs). In this sake, the linguistic terms for assessment the criteria weights and the rating of candidates are converted to IVHFEs based on Tables I and II, respectively.

TABLE I
Linguistic Variables for Rating the Importance of Criteria

Linguistic variables	Interval-valued hesitant fuzzy elements
Very important (VI)	[0.90,0.90]
Important (I)	[0.75, 0.80]
Medium (M)	[0.50, 0.55]
Unimportant (UI)	[0.35, 0.40]
Very unimportant (VUI)	[0.10,0.10]

TABLE II
Linguistic Variables for Rating the Potential Candidates

Linguistic variables	Interval-valued hesitant fuzzy elements
Extremely good (EG)	[1.00,1.00]
Very very good (VVG)	[0.90,0.90]
Very good (VG)	[0.80, 0.90]
Good (G)	[0.70, 0.80]
Medium good (MG)	[0.60, 0.70]
Fair (F)	[0.50, 0.60]
Medium bad (MB)	[0.40, 0.50]
Bad (B)	[0.25, 0.40]
Very bad (VB)	[0.10, 0.25]
Very very bad (VVB)	[0.10,0.10]

Hence, an evaluation module based on expert evaluation system is proposed to design the production system for the green supplier selection problem. To elucidate on, an expert system based on rule-based approach and regarding to the aforementioned statements is elaborated. In this respect, a meta-rule is defined to determine the framework of the evaluation module, and then some rule-sets based on the involved parameters are established to satisfy the conflicted criterions in meta-rule for assessment of the candidate green suppliers. In the other words, when the value of a positive criterion among the involved parameters is low; then, the candidate green supplier is bad vs. when the value of a negative criterion among the involved parameters is high; then the candidate green supplier is bad, too. Hence, the manipulated rule-based approach is coded by MATLAB R2013a software and all results are obtained on a 3 GHz computer with 4 GB RAM. However, the candidates are evaluated based on the following conflicted criteria:

- Cost (C_1);
- Quality (C_2);
- Capability of supplier (C_3);
- Environmental competency (C_4).

In this respect, the manipulated rule-based approach is expressed to evaluate the candidate green suppliers in brief module as follows:

- **Candidate green supplier evaluation rules set**

Rule C; Candidate green supplier is bad in cost criterion

If the purchasing price of the candidate green supplier is high
AND the product cost of the candidate green supplier is high
AND the cost of component disposal of the candidate green supplier is high
AND the logistics cost of the candidate green supplier is high
THEN the candidate green supplier in cost criterion is bad

Rule Q; Candidate green supplier is bad in quality criterion

If rejection ratio of the candidate green supplier is high
AND the quality assurance of the candidate green supplier is low
AND the process capability of the candidate green supplier is low
THEN the candidate green supplier in quality criterion is bad

Rule CS; Candidate green supplier is bad in capability of supplier criterion

If lead time of the candidate green supplier is high
AND the technology level of the candidate green supplier is low
AND the supplying capability of the candidate green supplier is low
AND the capability of product development of the candidate green supplier is low
THEN the candidate green supplier in capability of supplier criterion is bad

Rule EC; Candidate green supplier is bad in environmental competency criterion

If environment protection system certification of the candidate green supplier is low
AND the air emissions of the candidate green supplier is high
AND the waste water of the candidate green supplier is high
AND the pollution reduction capability of the candidate green supplier is low
AND the recycle capability of the candidate green supplier is low
THEN the candidate green supplier in environmental competency criterion is bad

Therefore, the dynamic interval-valued hesitant fuzzy group decision matrix is obtained based on the aforementioned rules set regarding to linguistic variables. Then, the linguistic variables are converted to IVHFEs according to Tables I and II.

B. Proposed Ranking Approach

In this section, a novel ranking approach is elaborated under dynamic IVHFS environment to choose the most suitable candidate green supplier. IVHFS could help the experts by assigning some interval-values membership degrees for a candidate green supplier versus the conflicted criteria under a set to margin the errors. In addition, the last aggregation approach is considered in the process of the proposed ranking method to prevent the loss of data. Therefore, the novel proposed ranking approach is provided under following steps:

Step 1: The dynamic interval-valued hesitant fuzzy group decision matrix (DIVHF-GDM) is obtained from the proposed evaluation module, which represented as:

$$G^{p} = \begin{matrix} \\ A_1 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} C_1 & \cdots & C_n \\ \left\{\left[\mu_{11}^{L1p},\mu_{11}^{U1p}\right],\left[\mu_{11}^{L2p},\mu_{11}^{U2p}\right],\ldots,\left[\mu_{11}^{Lkp},\mu_{11}^{Ukp}\right]\right\} & \cdots & \left\{\left[\mu_{1n}^{L1p},\mu_{1n}^{U1p}\right],\left[\mu_{1n}^{L2p},\mu_{1n}^{U2p}\right],\ldots,\left[\mu_{1n}^{Lkp},\mu_{1n}^{Ukp}\right]\right\} \\ \vdots & \ddots & \vdots \\ \left\{\left[\mu_{m1}^{L1p},\mu_{m1}^{U1p}\right],\left[\mu_{m1}^{L2p},\mu_{m1}^{U2p}\right],\ldots,\left[\mu_{m1}^{Lkp},\mu_{m1}^{Ukp}\right]\right\} & \cdots & \left\{\left[\mu_{mn}^{L1p},\mu_{mn}^{U1p}\right],\left[\mu_{mn}^{L2p},\mu_{mn}^{U2p}\right],\ldots,\left[\mu_{mn}^{Lkp},\mu_{mn}^{Ukp}\right]\right\} \end{bmatrix}_{m\times n} \quad (12)$$

$$\forall p = 1,2,\ldots,P$$

where G^p is the DIVHF-GDM in period p and $\left[\mu_{mn}^{Lk},\mu_{mn}^{Uk}\right]$ represented the opinion of kth expert for mth candidate green supplier under the nth criterion based on the IVHFS.

Step 2: Normalize the DIVHF-GDM regarding to the Definition 7.

Step 3: The criteria weights are expressed based on the preferences of the experts' judgments for each period. The final weight of each criterion in planning horizon is computed as:

$$\varpi_j^{Np} = DIVHFG\left(\varpi_{1j}^{p}, \varpi_{2j}^{p}, \ldots, \varpi_{Kj}^{p}\right) = \overset{K}{\underset{k=1}{\oplus}} \left(\varpi_{kj}^{p}\right)^{\frac{1}{K}} = \bigcup_{\left[\varpi_{kj}^{Lp}, \varpi_{kj}^{Up}\right] \in \varpi_{kj}^{p}, k=1,2,\ldots,K} \left\{ \frac{\prod_{k=1}^{K} (\varpi_{kj}^{Lp})^{\frac{1}{K}} + \prod_{k=1}^{K} (\varpi_{kj}^{Up})^{\frac{1}{K}}}{\sum_{j=1}^{n} \left(\prod_{k=1}^{K} (\varpi_{kj}^{Lp})^{\frac{1}{K}} + \prod_{k=1}^{K} (\varpi_{kj}^{Up})^{\frac{1}{K}} \right)} \right\} \quad \forall j, p \tag{13}$$

where ϖ_{kj}^{p} is the relative importance of jth criterion which determined by kth expert in period p, and ϖ_j^{Np} is the normalized weight of jth criterion in period p.

Step 4: Construct the weighted normalized DIVHF-GDM $\left(T_{k(ij)}^{p} = \left[T_{k(ij)}^{Lp}, T_{k(ij)}^{Up}\right]_{m \times n}\right)$ for each expert based on the criteria weights in each period.

$$T_k^{p} = \begin{matrix} & C_1 & \cdots & C_n \\ A_1 & \varpi_1^{Np}\left[\mu_{11}^{Lkp}, \mu_{11}^{Ukp}\right] & \cdots & \varpi_n^{Np}\left[\mu_{1n}^{Lkp}, \mu_{1n}^{Ukp}\right] \\ \vdots & \vdots & \ddots & \vdots \\ A_m & \varpi_1^{Np}\left[\mu_{m1}^{Lkp}, \mu_{m1}^{Ukp}\right] & \cdots & \varpi_n^{Np}\left[\mu_{mn}^{Lkp}, \mu_{mn}^{Ukp}\right] \end{matrix}_{m \times n} \quad \forall k, p \tag{14}$$

Step 5: Define dynamic interval-valued hesitant fuzzy positive ideal solution (DIVHF-PIS) and the dynamic interval-valued hesitant fuzzy negative ideal solution (DIVHF-NIS) as below:

$$A_{jk}^{*p} = \left\{h_{1k}^{*p}, h_{2k}^{*p}, \ldots, h_{nk}^{*p}\right\} \tag{15}$$

$$h_{jk}^{*p} = \begin{cases} \left\{x_j, \max_i \left\langle T_{k(ij)}^{p} \right\rangle\right\}, & \forall J, k, p \\ \left\{x_j, \min_i \left\langle T_{k(ij)}^{p} \right\rangle\right\}, & \forall J', k, p \end{cases} \tag{16}$$

$$A_{jk}^{-p} = \left\{h_{1k}^{-p}, h_{2k}^{-p}, \ldots, h_{nk}^{-p}\right\} \tag{17}$$

$$h_{jk}^{-p} = \begin{cases} \left\{x_j, \min_i \left\langle T_{k(ij)}^{p} \right\rangle\right\}, & \forall J, k, p \\ \left\{x_j, \max_i \left\langle T_{k(ij)}^{p} \right\rangle\right\}, & \forall J', k, p \end{cases} \tag{18}$$

where J is a set of benefit criteria and J' is a set of cost criteria.

Step 6: Determine the distance values between the weighted normalized DIVHF-GDM (T_k^{p}) and DIVHF-PIS ($\Im_i^{*kp}$), and DIVHF-NIS ($\Im_i^{-kp}$) as:

$$\Im_i^{*kp} = \sum_{j=1}^{n} \sqrt{\frac{1}{2l_{x_i}} \sum_{\lambda=1}^{l_{x_i}} \left(\left| T_{k(ij)}^{Lp} - A_{jk}^{*Lp\sigma(\lambda)} \right|^2 + \left| T_{k(ij)}^{Up} - A_{jk}^{*Up\sigma(\lambda)} \right|^2 \right)} \quad \forall i, k, p \tag{19}$$

$$\Im_i^{-kp} = \sum_{j=1}^{n} \sqrt{\frac{1}{2l_{x_i}} \sum_{\lambda=1}^{l_{x_i}} \left(\left| T_{k(ij)}^{Lp} - A_{jk}^{-Lp\sigma(\lambda)} \right|^2 + \left| T_{k(ij)}^{Up} - A_{jk}^{-Up\sigma(\lambda)} \right|^2 \right)} \quad \forall i, k, p \tag{20}$$

Step 7: Specify the closeness coefficient to determine the relative importance of each candidate green supplier in planning horizon ($\wp_i$).

$$\wp_i = \frac{\prod_{p=1}^{P}\prod_{k=1}^{K}\left(\Im_i^{-kp}\right)^{\frac{1}{KP}}}{\prod_{p=1}^{P}\prod_{k=1}^{K}\left(\Im_i^{*kp}\right)^{\frac{1}{KP}}+\prod_{p=1}^{P}\prod_{k=1}^{K}\left(\Im_i^{-kp}\right)^{\frac{1}{KP}}} \quad \forall i \tag{21}$$

Step 8: Rank the candidate green suppliers by decreasing sorting of closeness coefficient in planning horizon.

IV. Illustrative Example

In this section, an illustrative example is provided to show the capability of the proposed approach. In addition, a comparative analysis is determined to indicate the feasibility of the proposed evaluation and the ranking module. In this regard, three candidate green suppliers (S_1, S_2, S_3) are evaluated based on the opinions of three experts (E_1, E_2, E_3) under four criteria (C_1, C_2, C_3, C_4) in two periods (t_1, t_2). The dynamic interval-valued hesitant fuzzy group decision matrix and the relative significance of each criterion are obtained based on the elaborated expert evaluation system and represented in Tables III and IV, respectively.

TABLE III
THE OBTAINED GREEN SUPPLIER EVALUATION FROM EXPERT EVALUATION SYSTEM

Criteria	Candidates	Experts E_1	E_2	E_3
First period (t_1)				
C_1	S_1	[0.80, 0.90]	[0.80, 0.90]	[0.70, 0.80]
	S_2	[0.50, 0.60]	[0.60, 0.70]	[0.60, 0.70]
	S_3	[0.70, 0.80]	[0.80, 0.90]	[0.70, 0.80]
C_2	S_1	[0.60, 0.70]	[0.70, 0.80]	[0.70, 0.80]
	S_2	[0.70, 0.80]	[0.80, 0.90]	[0.80, 0.90]
	S_3	[0.60, 0.70]	[0.50, 0.60]	[0.50, 0.60]
C_3	S_1	[0.50, 0.60]	[0.50, 0.60]	[0.40, 0.50]
	S_2	[0.70, 0.80]	[0.60, 0.70]	[0.70, 0.80]
	S_3	[0.60, 0.70]	[0.60, 0.70]	[0.50, 0.60]
C_4	S_1	[0.80, 0.90]	[0.80, 0.90]	[0.70, 0.80]
	S_2	[0.70, 0.80]	[0.80, 0.90]	[0.90,0.90]
	S_3	[0.70, 0.80]	[0.70, 0.80]	[0.60, 0.70]
Second period (t_2)				
C_1	S_1	[0.80, 0.90]	[0.90,0.90]	[0.80, 0.90]
	S_2	[0.60, 0.70]	[0.70, 0.80]	[0.70, 0.80]
	S_3	[0.80, 0.90]	[0.80, 0.90]	[0.70, 0.80]
C_2	S_1	[0.70, 0.80]	[0.70, 0.80]	[0.80, 0.90]
	S_2	[0.90,0.90]	[0.80, 0.90]	[0.80, 0.90]
	S_3	[0.70, 0.80]	[0.80, 0.90]	[0.60, 0.70]
C_3	S_1	[0.50, 0.60]	[0.60, 0.70]	[0.50, 0.60]
	S_2	[0.70, 0.80]	[0.60, 0.70]	[0.80, 0.90]
	S_3	[0.70, 0.80]	[0.60, 0.70]	[0.60, 0.70]
C_4	S_1	[0.70, 0.80]	[0.70, 0.80]	[0.80, 0.90]
	S_2	[0.90,0.90]	[0.80, 0.90]	[0.80, 0.90]
	S_3	[0.80, 0.90]	[0.60, 0.70]	[0.60, 0.70]

TABLE IV
THE OBTAINED CRITERIA IMPORTANCE BASED ON EXPERT EVALUATION SYSTEM

Criteria	Periods	Experts E_1	E_2	E_3
C_1	t_1	[0.75, 0.80]	[0.90,0.90]	[0.90,0.90]
	t_2	[0.50, 0.55]	[0.75, 0.80]	[0.90,0.90]
C_2	t_1	[0.75, 0.80]	[0.50, 0.55]	[0.50, 0.55]
	t_2	[0.50, 0.55]	[0.50, 0.55]	[0.75, 0.80]
C_3	t_1	[0.90,0.90]	[0.90,0.90]	[0.75, 0.80]
	t_2	[0.75, 0.80]	[0.75, 0.80]	[0.75, 0.80]
C_4	t_1	[0.50, 0.55]	[0.50, 0.55]	[0.75, 0.80]
	t_2	[0.75, 0.80]	[0.75, 0.80]	[0.50, 0.55]

Herein, the proposed ranking approach is considered to rank the candidate green suppliers. In this sake, the DIVHF-GDM is normalized based on Definition 7 (Step 2). In addition, the normalized weight of each criterion is determined based on (13) and represented in Table V. Then, the weighted normalized DIVHF-GDM is founded based on (14). Hence, the DIVHF-PIS and DIVHF-NIS are specified regarding to (15)-(18). In this respect, the distance values between the weighted normalized DIVHF-GDM and DIVHF-PIS, and DIVHF-NIS are computed by using (19) and (20). Results are given in Tables VI and VII, respectively.

TABLE V
THE CRITERIA WEIGHT REGARDING TO EXPERTS' OPINIONS

Criteria	Periods	Aggregated judgments for criteria weights	ϖ_j^{Np}
C_1	t_1	[0.84693, 0.86535]	0.29442
	t_2	[0.69623, 0.73434]	0.25835
C_2	t_1	[0.57235, 0.62316]	0.20557
	t_2	[0.57235, 0.62316]	0.21590
C_3	t_1	[0.84693, 0.86535]	0.29442
	t_2	[0.75000, 0.80000]	0.27991
C_4	t_1	[0.57235, 0.62316]	0.20557
	t_2	[0.65518, 0.70607]	0.24583

TABLE VI
THE DISTANCE VALUES BETWEEN THE WEIGHTED NORMALIZED DIVHF-GDM AND DIVHF-PIS

t_1	Experts E_1	E_2	E_3
S_1	0.07944	0.05000	0.14138
S_2	0.07944	0.04655	0.02944
S_3	0.07055	0.10304	0.20241
t_2	Experts E_1	E_2	E_3
S_1	0.12899	0.04617	0.08397
S_2	0.05167	0.04084	0.02583
S_3	0.05152	0.06743	0.17416

TABLE VII
THE DISTANCE VALUES BETWEEN THE WEIGHTED NORMALIZED DIVHF-GDM AND DIVHF-NIS

t_1	Experts		
	E_1	E_2	E_3
S_1	0.07944	0.10822	0.09111
S_2	0.07944	0.11167	0.20241
S_3	0.08833	0.05889	0.02944
t_2	Experts		
	E_1	E_2	E_3
S_1	0.05167	0.06543	0.11818
S_2	0.12899	0.07076	0.17632
S_3	0.13224	0.04743	0.02799

Finally, the relative importance of each candidate green supplier in the planning horizon is determined based on proposed closeness coefficient index (21). In this sake, the candidate green suppliers are ranked by decreasing sorting of their closeness coefficient value. In addition, the Peng and Wang method [20] is implemented to our illustrative example for comparing the obtained ranking results to indicate the verifying of the proposed approach. The results show that both methods achieved the same ranking results and selected the second green supplier as the most suitable candidate. The aforementioned results are given in Table VIII.

TABLE VIII
THE CLOSENESS COEFFICIENT OF EACH CANDIDATE AND COMPARATIVE ANALYSIS

Candidate green suppliers	$\wp_i$	Ranked by proposed approach	Ranked by Peng and Wang [20]' method
S_1	0.50451	2	2
S_2	0.73716	1	1
S_3	0.35815	3	3

As indicated in Tables VIII, both methods selected the second green supplier as the best candidate and specified the third green supplier as the worst candidate. In this respect, both ranking methods are tailored based on dynamic IVHFS, but the proposed approach of this study considered the weight of each criterion in the procedure of the proposed raking method to decrease the errors. In this respect, a group of experts is founded to evaluate the importance of the criteria and assessment the candidate green suppliers versus the conflicted criteria. In addition, the last aggregation approach is considered in the procedure of the proposed ranking approach to lead a precise solution by preventing the loss of data.

V. CONCLUSIONS AND FUTURE DIRECTIONS

Green supplier selection problems are the main issue for companies to enhance their economic and environmental performances. To address this issue, choosing the best green supplier as a group decision-making problem could play an important role for these companies. In this sake, this paper presented an expert system to evaluate the green supplier selection problem named expert evaluation system to decrease the uncertainty of experts' judgments by considering the knowledge of expert and historical data, simultaneously. Then, a new interval-valued hesitant fuzzy ranking method is prepared under dynamic environment to indicate the best and worst green supplier. The proposed ranking approach is elaborated based on last aggregation approach to avoid the loss of data. In this respect, the preferences of the experts' judgments are aggregated in the last step, which could lead to prevention of the data loss. Finally, an illustrative example is provided to show the applicability of the proposed modules. In this sake, the obtained ranking results in compared with a proposed method in the recent literature show that the same ranking results which is represented the feasibility of the proposed approach. For future direction, the hierarchical structure for defining the criteria is more interesting to evaluate the green supplier selection problems appropriately. In addition, determining the weights of each expert and each criterion based on novel approaches could enhance the proposed ranking method. Consequently, considering the experts and criteria weights in the procedure of the proposed ranking approach could lead to precise solution.

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