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A Mathematical Model Approach Regarding the Children's Height Development with Fractional Calculus

Nisa Özge Önal, Kamil Karaçuha, Göksu Hazar Erdinç, Banu Bahar Karaçuha, Ertuğrul Karaçuha

Abstract**—**The study aims to use a mathematical approach with the fractional calculus which is developed to have the ability to continuously analyze the factors related to the children's height development. Until now, tracking the development of the child is getting more important and meaningful. Knowing and determining the factors related to the physical development of the child any desired time would provide better, reliable and accurate results for childcare. In this frame, 7 groups for height percentile curve (3th, 10th, 25th, 50th, 75th, 90th, and 97th) of Turkey are used. By using discrete height data of 0-18 years old children and the least squares method, a continuous curve is developed valid for any time interval. By doing so, in any desired instant, it is possible to find the percentage and location of the child in Percentage Chart. Here, with the help of the fractional calculus theory, a mathematical model is developed. The outcomes of the proposed approach are quite promising compared to the linear and the polynomial method. The approach also yields to predict the expected values of children in the sense of height.

Keywords**—**Children growth percentile, children physical development, fractional calculus, linear and polynomial model.

I. Introduction

ONE of the most reliable indicators for assessing the health status of a child is weight and height measurements for each age. In almost all developed countries, children are assessed according to their country's local growth standards. The current reference values of Turkey that can be used to evaluate the growth of Turkish children are determined [1]. Previously, curves and tables with values for the ages between 0 and 17 were formed with measurements made in boys and girls born between 1950 and 1960 [1]. World Health Organization (WHO) published generalized growth percentile curves [2]. Smoothing curves are obtained with box-cox transformation and LMS (lambda-mu-sigma) methods that developed [3]-[7]. In 2017, a web application is developed for Turkish children with same model [8].

A continuous curve from discrete data using the fractional calculus theory is obtained in this study. Fractional Calculus that is a discipline of applied mathematics tackles fractional and complex orders in derivatives and integrals [9]. Their applications are mostly used in science, engineering, mathematics, economics, and other fields [10]-[14]. The fractional derivative provides pretty good insight for the memory and hereditary of a process or phenomena. So that, we advanced a mathematical approach by the help of fractional calculus theory which analyses and examines the factors continuously related to the height of the children. Dataset contains values of 7 groups for height in percentage chart of Turkey. A continuous curve, valid for any time interval, is obtained by using discrete height data of 0-18 years old children with the developed fractional calculus model. Therefore, it became possible to find the percentage and location of the children in percentage chart. In this paper, the theory, numerical results of the developed theory and comparison with the other modeling methods such as linear and polynomial methods are presented.

Önal N.Ö. is with the Informatics Institute, Istanbul Technical University, Istanbul 34467, Turkey (corresponding author, phone: +90 212 285 7391; e-mail: onal16@ itu.edu.tr).

Karaçuha K., Erdinç G.H., and Karaçuha B. B. are with the Informatics Institute, Istanbul Technical University, Istanbul 34467, Turkey (phone: +90 212 285 7391; e-mail: karacuha17@itu.edu.tr, erdincg@itu.edu.tr, b.baharkaracuha@gmail.com).

Karaçuha E. is with the Informatics Institute, Istanbul Technical University, Istanbul 34467, Turkey (phone: +90 212 285 6325; e-mail: karacuhae@ itu.edu.tr).

II. Methodology

The fractional derivative of f(x) is equal to the expression given in (2). Derivative order is α and $\alpha \epsilon$ (0, 1).

$$\mathfrak{D}_x^{\alpha} f(x) = \frac{d^{\alpha} f(x)}{dx^{\alpha}} = \sum_{n=1}^{\infty} a_n n x^{n-1} \quad (1)$$

Here, f(x) corresponds to the data of children's height with respect to time which is denoted as x in (1). After taking the Laplace transform of (1), (2) is achieved.

$$f(x) \cong f(0) + \sum_{n=1}^{N} \frac{a_n \Gamma(n+1) x^{\alpha+n-1}}{\Gamma(\alpha+n)} \quad (2)$$

We have a dataset to make regression on it.

$$P_K = [\, p_0 \, p_1 \dots p_K]$$
$$x_K = [\, x_0 \, x_1 \dots x_K]$$

The error between the value p_i and $f(x_i)$ is shown as ϵ_i in (3). In the least squares method, the purpose is to minimize the square of the total error contributing from each data point.

$$(\epsilon_i)^2 = \left(p_i - f(x_i)\right)^2 \quad (3)$$

Total error's square is given in (4):

$$\epsilon_T^2 = \sum_{i=0}^{K} (\epsilon_i)^2 \quad (6)$$

By using the least mean square error approach, System of Linear Algebraic Equations (SLAE) is achieved. SLAE can be denoted as given in (5);

$$[A]_{N+1xN+1}[\Omega]_{N+1x1} = [B]_{N+1x1} \quad (6)$$

where,

$$A = \begin{bmatrix} k+1 & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{\alpha} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{\alpha+1} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K} x_i^{\alpha+n-1} \\ \sum_{i=0}^{K} x_i^{\alpha} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+1} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K} x_i^{2\alpha+n-1} \\ \sum_{i=0}^{K} x_i^{\alpha+1} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha+1} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+2} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K} x_i^{2\alpha+n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{K} x_i^{\alpha+n-1} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha+n-1} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+n} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K} x_i^{2(\alpha+n-1)} \end{bmatrix}$$

$$[\Omega] = [f(0) \quad a_1 \quad a_2 \quad \ldots \quad a_n]^T$$

$$[B] = [\sum_{i=0}^{K} P_i \quad \sum_{i=0}^{K} P_i x_i^{\alpha} \quad \sum_{i=0}^{K} P_i x_i^{\alpha+1} \quad \ldots \quad \sum_{i=0}^{K} P_i x_i^{\alpha+n-1}]^T$$

Here, T is the matrix transpose. Unknown coefficients in the vector Ω can be found by (6).

$$[\Omega]_{N-1x1} = [A]^{-1}_{N-1xN-1}[B]_{N-1x1} \quad (6)$$

where $[A]^{-1}$ is the inverse of [A] matrix.

III. Dataset

The dataset of children growth percentile includes 7 groups (3-10-25-50-75-85-97 percentiles for body height) for boys and girls. In this study, we use height values of 0-18 years old children indicated in Percentage Chart of Turkey [1].

IV. Results

In this study, children height values modeled with Linear, Polynomial, and Fractional Models. In those models are used for comparing different N exponent values up to 3, 4, and 5 (N = 3, 4, 5) in (2).

We obtained the model results from Linear, Polynomial, and Fractional Models, respectively. In order to compare the results of the models, we make use of the Mean Absolute Percentage Error (MAPE) [15]. Equation (7) shows that the formulation of the MAPE. The results of the age versus height are shown in Table I.

$$MAPE = \frac{1}{k}\sum_{i=1}^{k}\left|\frac{v(i)-\tilde{v}(i)}{v(i)}\right| \times 100 \quad (7)$$

where $v(i)$ is the real value and $\tilde{v}(i)$ is the predicted value.

The Polynomial and Fractional Models are equal to each other as mathematically when the fractional order (α-alpha) value in the Fractional Model is equal to one. Thus, the results that when alpha value is equal to one can be the same both in Polynomial Model and Fractional Model. For the fractional model, alpha (α) value is changing from 0.001 to 1 with the step size 0.001. In this study, alpha values are selected according to the minimum MAPE value.

Table I indicates the MAPE results of the age versus height and alpha values in the Fractional Model according to the methods of Linear, Polynomial, and Fractional Models. Considering Table I, when the truncation number N is increased in (2), MAPE ratio in both the Polynomial and the Fractional Models are decreased as expected.

The average of the total MAPE (AMAPE) is calculated with (8):

$$AMAPE = \frac{\sum MAPE}{M} \quad (8)$$

where, M represents the number of values, which is 14.

When the results of the Fractional calculus model analysis are compared to the Linear and Polynomial Model analysis, we observe that the fractional model is more successful. The outcomes of the proposed approach are better than the linear and the polynomial methods. By this way, the method gives a better chance to predict the expected height of the child for the future.

A ratio of MAPE results of each model is calculated in order to compare the Polynomial Model and Fractional Model. We divide each Polynomial Model's MAPE value by the corresponding Fractional Model's MAPE value. This transaction is applied for all the N values where N is equal to 3, 4, and 5 respectively. In addition to this, after doing each calculation, the maximum and the minimum values are chosen as the limits. According to the limits, MAPE results evaluated by Polynomial Model are at least 2.01 times and at most 3.95 times greater than MAPE results evaluated by the Fractional Model.

By applying (2), for N=3, the least AMAPE is found as 1.26 in the Fractional Model. When N is equal to 4 and 5, AMAPE values are found as 0.49 and 0.43 in the Fractional Model, respectively. Their approximate ratios in order are 3.22 and 2.74.

While truncation number is equal to 3 (N = 3), Fig. 1 demonstrates 3 percentile age versus height graphs in the fractional, polynomial, and linear models. Fractional Model has the minimum error for modeling discrete age versus body height values. A continuous curve valid was developed with the Fractional Model.

Fig. 2 illustrates 3 percentile age versus height graphs using the fractional, polynomial, and linear model for N=4.

Fig. 3 demonstrates the 3 percentile age versus height graphs using the fractional, polynomial, and linear model for N=5.

TABLE I
RESULTS OF AGE - BODY HEIGHT TO LINEAR, POLYNOMIAL, AND FRACTIONAL MODEL FROM 3 TO 5 EXPONENT NUMBERS

RESULTS		N=3			N=4			N=5		
AGE - HEIGHT	Linear Model	Polynomial Model	Fractional Model		Polynomial Model	Fractional Model		Polynomial Model	Fractional Model	
	MAPE	MAPE	MAPE	Alfa	MAPE	MAPE	Alfa	MAPE	MAPE	Alfa
3 % -Boy	5.402163228	2.947443454	0.879511614	0.506	1.450408	0.570475167	0.657	1.387619	0.43146	0.547
10 % -Boy	5.407534646	3.01494031	0.963707668	0.495	1.413681	0.534720874	0.661	1.337368	0.387715	0.56
25 % -Boy	5.464773802	3.060094876	1.034929851	0.483	1.40315	0.491194116	0.66	1.317915	0.363516	0.563
50 % -Boy	5.559953069	3.110350759	1.128587385	0.474	1.373834	0.440018959	0.665	1.272034	0.344825	0.587
75 % -Boy	5.632404553	3.163640075	1.22741669	0.462	1.391571	0.422563403	0.666	1.246001	0.354468	0.575
90 % -Boy	5.69813391	3.212386377	1.318764065	0.455	1.408876	0.435351278	0.671	1.224827	0.373018	0.617
97 % -Boy	5.776556168	3.272317243	1.40963534	0.447	1.422935	0.453567536	0.674	1.230354	0.393108	0.63
3 % -Girl	6.823285327	3.291558273	1.473083602	0.411	1.763813	0.605919002	0.62	1.344039	0.604952	0.622
10 % -Girl	6.9437678	3.218607376	1.431390254	0.416	1.736411	0.536266756	0.639	1.249269	0.549924	0.636
25 % -Girl	7.063383601	3.173419071	1.407654414	0.421	1.725992	0.475034474	0.64	1.149177	0.50914	0.7
50 % -Girl	7.193639107	3.0837981	1.357629583	0.427	1.719019	0.434119083	0.639	1.056893	0.463723	0.713
75 % -Girl	7.323473266	3.036228472	1.3501217	0.432	1.745766	0.446222807	0.637	0.983734	0.432988	0.715
90 % -Girl	7.450645847	2.991379294	1.337844927	0.438	1.749846	0.459484184	0.638	0.924253	0.429191	0.721
97 % -Girl	7.553575607	2.952629006	1.317973151	0.441	1.779546	0.486929947	0.636	0.882858	0.437864	0.742
$AMAPE$ (M=14)	6.378092137	3.109199477	1.2598750174		1.577489142	0.485133399		1.18616721	0.4339922	

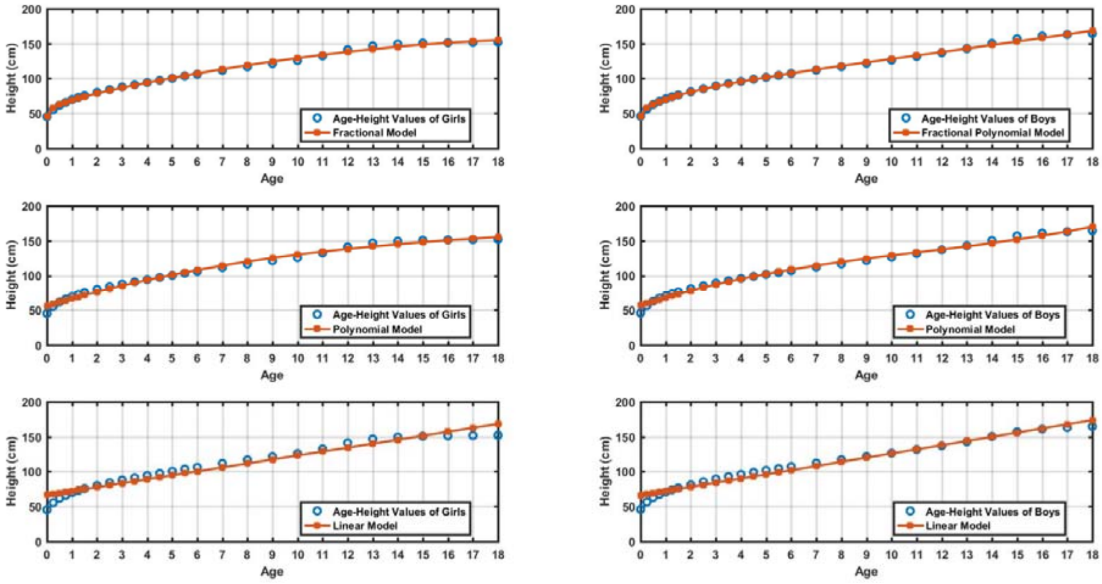

Fig. 1 3 percentile age - height boy and girl for N=3

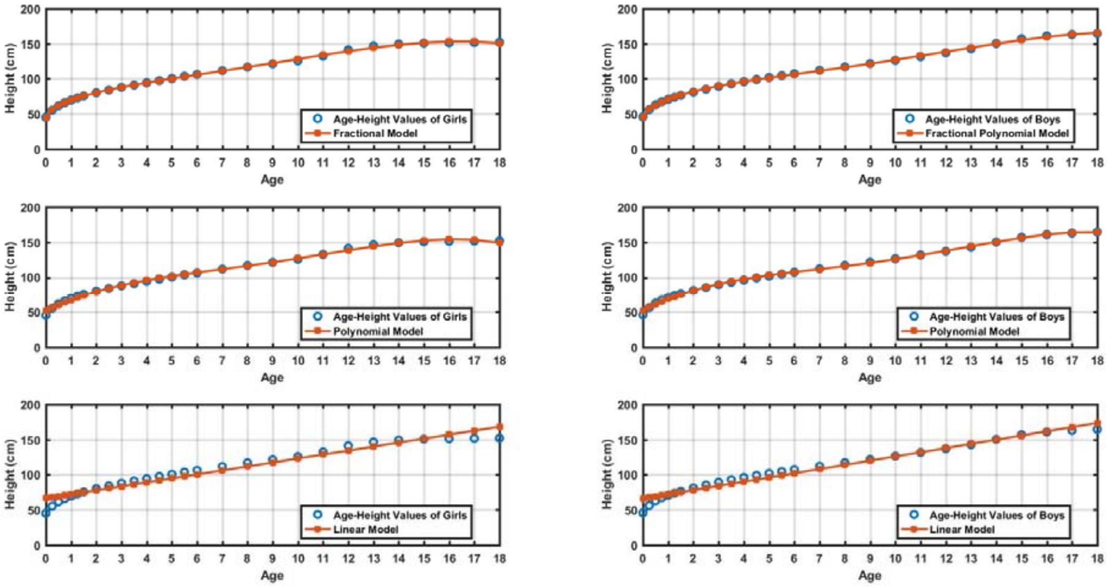

Fig. 2 3 percentile age - height boy and girl for N=4

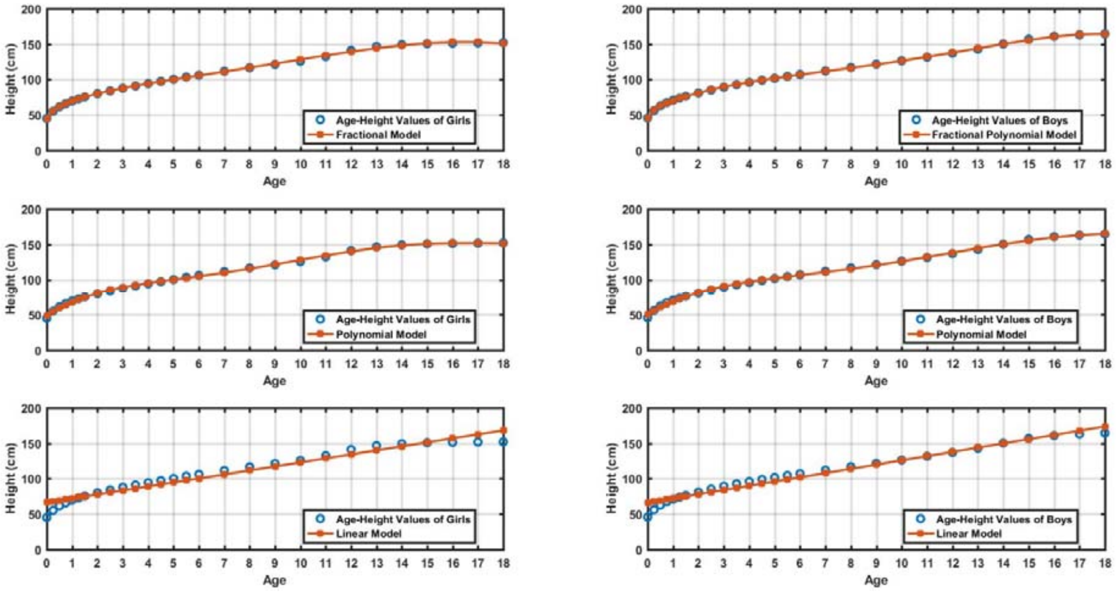

Fig. 3 3 percentile age - height boy and girl for N=5

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