International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:9, 2010

A Mahalanobis Distance-based Diversification and Nelder-Mead Simplex Intensification Search Scheme for Continuous Ant Colony Optimization

Sasadhar Bera, and Indrajit Mukherjee

***Abstract*—**Ant colony optimization (ACO) and its variants are applied extensively to resolve various continuous optimization problems. As per the various diversification and intensification schemes of ACO for continuous function optimization, researchers generally consider components of multidimensional state space to generate the new search point(s). However, diversifying to a new search space by updating only components of the multidimensional vector may not ensure that the new point is at a significant distance from the current solution. If a minimum distance is not ensured during diversification, then there is always a possibility that the search will end up with reaching only local optimum. Therefore, to overcome such situations, a Mahalanobis distance-based diversification with Nelder-Mead simplex-based search scheme for each ant is proposed for the ACO strategy. A comparative computational run results, based on nine nonlinear standard test problems, confirms that the performance of ACO is improved significantly with the integration of the proposed schemes in the ACO.

***Keywords*—**Ant Colony Optimization, Diversification Scheme, Intensification, Mahalanobis Distance, Nelder-Mead Simplex.

I. INTRODUCTION

ANT colony optimization (ACO) comes under the broad spectrum of metaheuristic search strategies [1]-[2]. Dorigo [3] first proposed the basic idea of ACO for discrete optimization problems. Later on, Dorigo et al. [4] claimed that ACO could also provide solutions for hard combinatorial optimization problems, such as assignment and job shop scheduling problem. With time, researchers also proposed different variants of ant colony to determine global optima for continuous multimodal problems. ACO has shown immense potential to handle higher dimensional state space problems with added constraint condition(s).

Sasadhar Bera is a research scholar from the Shailesh J. Mehta School of Management, Indian Institute of Technology Bombay, India, Powai 400076 (e-mail: sasadhar.bera@iitb.ac.in).

Indrajit Mukherjee, is with Shailesh J. Mehta School of Management, Indian Institute of Technology Bombay, India, Powai 400076 (phone: +91-22-25767742; fax: +91-22-25722872; e-mail: indrajitmukherjee@iitb.ac.in)

I Mukherjee thanks IRCC, IIT Bombay and Department of Science and Technology (DST), India for funding this ongoing research work.

ACO strategy works with a population of simple, independent, and asynchronous agents (or ants) which cooperate to find an improved solution(s) for a given optimization problem. It uses a pheromone model to probabilistically search the feasible space. The characteristic of ant system (AS) [4] is to update the pheromone value of each ant after iteration. A typical pseudo code for ACO is shown in Fig. 1.

Step 1: Initialization
- Initialize the pheromone trail

Step 2: Iteration
- For each ant *repeat*
- Construct solution using current pheromone trail
- Evaluate the solution constructed
- Update the pheromone trail
- Daemon Actions (optional)

- Until stopping criteria met

Fig. 1 Pseudo Code of Ant Colony Optimization

Stutzle and Hoos [5] proposed ‘max-min ant system’ (MMAS) and claimed its superiority over AS. In case of MMAS, only the best ant can update the pheromone trail information, and the value of the pheromone is considered to be bounded. Dorigo and Gambardella [6] proposed an ant colony system (ACS) for higher dimensional problems. Bilchev and Parmee [7] first proposed a conceptual framework for continuous ant colony optimization (CACO) problems. Wodrich and Bilchev [8] introduced a bi-level search procedure with a local and global search component for continuous function optimization problem. Mathur et al. [9] introduced a random walk and trail diffusion component in the CACO strategy to improve its performance. Their approach is based on global diversification based on inferior solutions, and local intensification using superior solution points. Crossover and mutation concept was introduced to produce a new region with added Gaussian noise. Dreo and Siarry [10] developed a new hierarchical algorithm so-called ‘continuous interacting ant colony algorithm’ (CIAC). The concept of hierarchy is described by a system where global level properties influence the local level properties. In case of

CIAC, the information passes through a population of agents, and uses two types of communication. This includes the spots of pheromone deposited in the search space and direct exchange of information between individuals. Socha and Dorigo [11] proposed an ant colony optimization strategy in real domain (ACO_R), where mixture of Gaussian kernel probability density function is used to improve the algorithm flexibility to handle multimodal functions. In this approach, the ant movement is not restricted to a finite set and a solution archive structure stores the values of variables and corresponding solution in the form of a table structure. Schluter et. al. [12] extended the ACO_R, so-called 'ACO_{mi}', for non-convex mixed integer problems. They use so-called 'oracle penalty strategy' to handle mixed integer and constraint conditions.

As per the literature review, all the ant strategies proposed use components of multidimensional state space for the diversification scheme. Generating neighbourhood by updating the components of vector space will not ensure that the new point(s) generated is at significant distance from the current point. Researchers also proposed grid and discrete points to generate neighbourhood in multidimensional continuous search space. With increase in dimensionality of search space, it becomes extremely important to ensure a minimum distance before the current point is diversified. In case of one dimensional problem it can be a minimum circle radius, and for two dimensional problems it can be a minimum radius sphere. This will ensure that the diversification does not create points too close to current point and avoid trapped to local optima. Based on the above mentioned criticality, for higher dimensional state space, a minimum statistical Mahalanobis distance [13] concept is proposed for the diversification scheme in case of higher dimensional problems. Local search in multiple dimensions by the ants, based on Nelder-Mead simplex (NM) [14], is also incorporated for improving the intensification scheme. The overall objective of this paper is to highlight the improvement in the existing ant colony strategy by incorporating above mentioned schemes and not to propose a new ant strategy.

The details of the initialization, diversification, intensification, and other intrinsic details of parameters are provided in section II. Section III provide the details of comparative study of a simple CACO strategy and CACO with Mahalanobis distance-based diversification and NM-based intensification scheme (CACO-MDS) using standard test case functions.

II. Continuous Ant Colony Strategy Based on Mahalanobis Distance-based DIVERSIFICATION and Multidirectional NM-based Intensification SCHEME [CACO-MDS]

A typical unconstrained form of nonlinear optimization problem is expressed as,

$$\underset{X}{\text{Minimize}}\ \mathrm{F(X)}, \tag{1}$$

subject to,

$$\alpha_i \le x_i \le \beta_i \qquad \forall \ i = 1, 2, \ldots, n, \tag{2}$$

where n is the number of independent variables. In other words, X is n dimensional and $\mathrm{F(X)}$ is the nonlinear function. α_i and β_i are lower bound and upper bound of x_i (or each element of X), respectively. The proposed CACO-MDS strategy for a typical unconstrained minimization problem is discussed in detail in subsection *A.*

A. Basic Characteristics of CACO-MDS

In CACO-MDS, diversifying to a region, which is at a significant distance from the current multidimensional point, is ensured based on Mahalanobis distance concept. Mahalanobis distance (MD) [13] is preferred for multidimensional problems as the Euclidean distance does not take into consideration the variances-covariance matrix of state space [13] for calculating the vector distance between two points. However, Mahalanobis distance considers the variance-covariance matrix and provides the exact vector distance. The expression for MD is

$$D = \sqrt{\left(X_1 - X_2\right)^T S^{-1}\left(X_1 - X_2\right)}, \tag{3}$$

where X_1 and X_2 are two n dimensional vector. S is the variance-covariance matrix of all multidimensional points considered for analysis. A typical expression for S is

$$S = \begin{bmatrix} s_{11} & s_{12} & . & . & s_{1n} \\ s_{21} & s_{22} & . & . & s_{2n} \\ . & . & . & . & . \\ s_{i1} & s_{ii} & . & . & s_{in} \\ . & . & . & . & . \\ s_{n1} & s_{n2} & . & . & s_{nn} \end{bmatrix}, \tag{4}$$

where s_{ii} is the variance of i^{th} variable and s_{ij} is the covariance between i^{th} and j^{th} variables ($\forall \ i \neq j$).

The inverse of the covariance matrix standardizes all the variables to the same variance so as to calculate the exact distance. Therefore, if one random variable has a larger variance than another, it will receive relatively lesser weight based on this concept. Similarly, two highly correlated variables do not contribute as much as another two variables that are less correlated. Fig 2 illustrates the elliptical distribution of the Mahalanobis distance for a two dimensional case situation, where maximum *MD* is considered as *1.3159* from the center point [here ($\overline{x_1}$, $\overline{x_2}$) is taken as (296.807, 284.1130)]. The variance-covariance matrix is taken as,

$$S = \begin{bmatrix} 11531 & -6320 \\ -6320 & 13695 \end{bmatrix}. \tag{5}$$

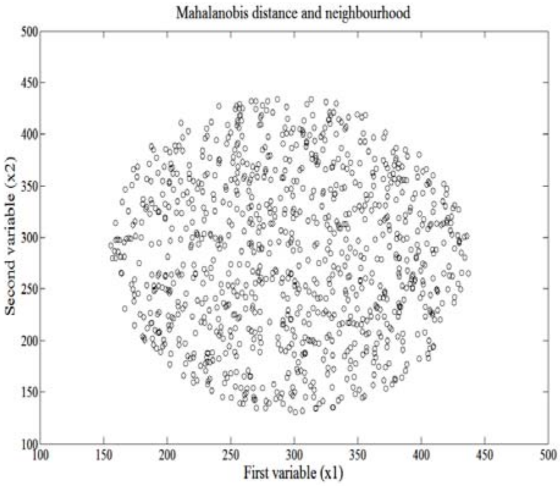

Fig. 2 An Elliptic Distribution of Mahalanobis Distance.

The intrinsic details of the proposed schemes are provided below.

1. Initialization of the Ants

Let us consider m number of total ants for a typical CACO. Initially, m number of random points will be generated in the n-dimensional state space. The mean vector ($\overline{X}$) and the variance-covariance matrix (S) of the m points are used for further analysis. The minimum MD value (say $D_{\min}$) from all the *m* points from $\overline{X}$ is also stored for reference. In this paper, the minimum vector distance that is considered to generate new points in the diversification scheme is taken as

$$D = 2 * D_{\min} \tag{6}$$

The multiple of $D_{\min}$ is taken as two based on initial trial runs. In addition, k number of nests is considered for this study. The first step of iteration is to calculate the number of ants for the r^{th} nest (where $r = 1, 2, \ldots, k$). This is based on the fitness values of k points, and expressed as

$$m_r = \frac{f_r}{\sum_{r=1}^{k} f_r}, \tag{7}$$

where, $r = 1, 2, \ldots, k$ and f_r is the fitness value of r^{th} nest. These nests are referred here as center point or c_r. m ants are distributed according to probability m_r. From each nest, $m * m_r$ number of ants are generated within a local search radius of d_r. d_r is expressed as

$$d_r = c_{ri} \pm Z \times \frac{\sqrt{s_{ii}}}{2} \times rand \tag{8}$$

where c_{ri} is i^{th} element of r^{th} center, Z is a multiplication factor, s_{ii} is i^{th} diagonal element of variance-covariance matrix (S), and $rand$ is a random number generator between *0* and *1*.

2. Movement of the Ants

Initially, allocated number of ants for the r^{th} center will generate new points to exploit the neighbourhood with a search radius of d_r and having Z value equal to one. All ants within d_r radius will use NM intensification search. It is to be noted that all the point generated from (8) will be within the ellipsoid. After the local search exploration, the best fitness value will be selected and compared with the existing center point or nest. If the best point in local search shows improvement than the existing center point, then the new point replaces the existing center. However, if there is no improvement in local search within d_r radius, the ants will again continue to search in greater radius with varied Z values. In case there is no improvement for a particular nest in local search, then all the ants are diversified to a distance of at least D. All the new points also need to satisfy the boundary constraint conditions [or $\alpha_i \le x_i \le \beta_i$]. The new ants will again start NM local search. If the diversification provides a better solution, then the center point will be shifted and updated. The ants of each center c_r uses similar ant movement scheme for updating the nest position. In the subsequent iteration, ants are distributed based on the average fitness value and according to a roulette wheel selection process. In other words, if at t^{th} iteration, $m * m_r$ ants go to center c_r, then after the completion of t^{th} iteration using NM, there will be $m * m_r$ best solutions. The average of the best solutions for the r^{th} center, say $\overline{F_r}$. Then, for the $(t+1)^{th}$ iteration, the proportion of ants (p_r) that goes to r^{th} center point is expressed as

$$p_r = \frac{\overline{F_r}}{\sum_{r=1}^{k} \overline{F_r}} \quad . \tag{9}$$

It is expected that all the search direction finally converges to the global optimum.

3. Pheromone Representation in CACO-MDS

In case of discrete optimization problem, movements of ants are restricted to countably finite number of feasible points. However, for a continuous optimization problem, the feasible points are countably infinite. In such situation, it becomes too difficult to represent the pheromone trail concept. In this paper, we have assumed that the trail information of each center or nest is its fitness value. Hence, a solution archive structure is used for k centers to store the independent variable and its corresponding fitness value. Therefore, pheromone trail in the form of a list for each center is maintained. Any improvement in fitness value at a particular center will lead to update the archive. The elite solution is the overall best solution among the k different centers.

This archive structure can be very useful to explore potential region(s) where trail deposition or fitness value is high. In case of no improvement, for say last p iteration, in anyone among k centers, a center of gravity (C_g) will be calculated. The expressions to calculate C_g is

$$C_g = \frac{1}{k}\sum_{r=1}^{k}(c_r)_{best}\ , \tag{10}$$

where $(c_r)_{best}$ is the best solution of the r^{th} archive and

$$(c_r)_{new} = C_g + \left(-1+2*rand\right)*\left(C_g-(c_r)_{best}\right), \tag{11}$$

where $(c_r)_{new}$ is new r^{th} center, and $rand$ is a random number generator between *0* and *1*. Therefore, k points are generated within C_g and $(c_r)_{best}$.

4. Radius Decrement Factor

The rate at which the search radius (d_r) is reduced is called radius decrement factor. In this context, if the rate of decrease is too fast, situations may arise in which ants can no longer escape local optima due to small radius size. Such situations are undesirable and small decrement is needed for efficient search. This will also ensure that the ant strategy does not terminate till the ants explore a reasonable search space. In this paper we adopted an exponential decay for the radius decrement rate. The expression of radius and MD distance for diversification at any $(t+1)^{th}$ iteration is given the following equations.

$$\left(d_r\right)_{t+1} = \left(d_r\right)_t\left(1-\left(\frac{t}{T}\right)^b\right), \text{ and} \tag{12}$$

$$\left(D\right)_{t+1} = \left(D\right)_t\left(1-\left(\frac{t}{T}\right)^b\right)\ , \tag{13}$$

where $\left(d_r\right)_t$ is local search radius, and $\left(D\right)_t$ diversification distance at iteration t, T is the maximum number of iterations, and b is positive constant that control the degree of nonlinearity. Fig. 3 illustrates the change in radius with respect to number of iteration.

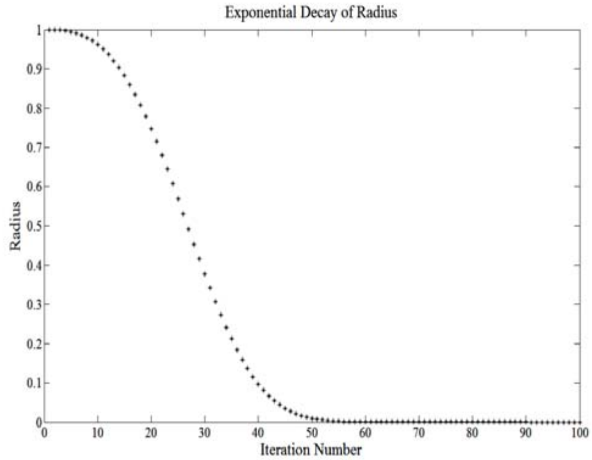

Fig. 3 Exponential decay of radius

The pseudo code of the CACO-MDS strategy with intensification and diversification is provided in Fig. 4.

```
while stopping criterion is not met do
 for r = 1:k      /* k : number of direction vectors */
  for Z = 1:3:7    /* Z : local search radius multiplication factor */
   for ants = 1:m_r   /* m_r : distributed ants for r^th direction vector */
    for i = 1: n          /* n: input dimension */
      Create new point: c_ri + Z × sqrt(s_ii)/2 × (-1+2*rand)
    endfor
    Find out new solutions by using Nelder-Mead simplex at each new point.
    Find out the minimum value point (called minPoint) among the solutions
   endfor
   if Fitness(minPoint) < Fitness(Center)
       break
   endif
  endfor
 /* Update of center and neighborhood points */
  if Fitness(minPoint) < Fitness(Center)
     newPoints = newSolutions
     newCenter = minPoint
  else
     Create m_r new points outside D distance. Find out new solution using Nelder-
     Mead simplex method. Find out diversified minimum value point (called min.
     diversified point).
        newPoints = newDiversifiedSolutions
        newCenter = minDiversifiedSolution
   end
 endfor
```

Fig. 4 Pseudo code of CACO-MDS for a typical minimization problem

III. Experimental setup and results

In this section, we present the experimental setup for evaluating the performance of CACO-MDS with basic CACO. The typical benchmark functions as available in open literatures are used to compare its performance. In this particular study, all the maximization functions are first converted to minimization problem by multiplying it with a minus one. However, the results and optimal values are reconverted as per the original maximization problem and reported. The test function used, their mathematical expression, ranges of the dependent variables, and corresponding global optimum solution is provided in the Appendix. A basic CACO strategy is proposed by Chen et al. [15]. We refer this as CACO-Chen in this paper. Chen et al. [15] also used an adaptive crossover and mutation for the intensification and diversification scheme. We will also refer CACO-MDS as strategy 1 and CACO-Chen as strategy 2. Performance of CACO-MDS and CACO-Chen is based on *30* simulations run results for each test function. A successful run is referred as if the following condition holds true.

$$|f - f^*| \leq \varepsilon_1 + \varepsilon_2 * abs(f^*) \;, \tag{14}$$

where f optimum value found by the strategy, f^* is known global optimum of the test function, ε_1 and ε_2 are two accuracy parameter (where ε_1 and ε_2 is considered as 10^{-4})

In this analysis, the total number of ants selected for each simulation run is *100*. The numbers of direction vector for generating nest is selected as *3*. The stopping criteria used are (i) if the difference between best and worst function evaluation is smaller than a pre-specified tolerance level, (ii) if $(D)_t$ reaches the specified limit, (iii) if pre-defined maximum number of iterations is reached, or (iv) if maximum number of function evaluations is achieved. All the program and simulation run are performed in Matlab 7.1 environment. The laptop configuration used to run the program codes is *1.60* GHz Intel dual core processor with *120* GB hard disk, and *1* GB RAM.

To compare the performance of each strategy the success rate, average value of the objective function and their sample standard deviation are summarized in Table 1 and Table 2. Table 1 clearly indicates that the success rate of CACO-MDS to determine global optimal solution is consistently higher than CACO-Chen for any of the test function selected for analysis. The sample standard deviation for CACO-MDS (in Table 2) is significantly lesser than CACO-Chen in all test situations. These can be attributed to the fact that CACO-Chen gets trapped into local optima. Whereas, CACO-MDS reaches the global optimum point or near to the point consistently.

Fig. 5 and Fig. 6 illustrate how CACO-MDS and CACO-Chen strategy reaches a best solution point in a typical run for the GR test function. It is also observed from the results and graph that CACO-MDS, as compared to CACO-Chen, is more stable and can reach global or near optimal points at a faster rate.

TABLE 1
SUCCESS RATES OF STRATEGIES FOR EACH TEST FUNCTION

Function	Search dimension	Strategy **	Success rate (%)
RC	2	1	100
		2	100
SF6	2	1	100
		2	0
AS	2	1	100
		2	0
MG6	4	1	100
		2	0
S4	4	1	100
		2	0
H64	6	1	100
		2	0
ACL	10	1	100
		2	0
RGN	10	1	83
		2	0
GR	10	1	77
		2	0

TABLE II
SUMMARY OF COMPUTATIONAL RUN RESULTS

Function	Strategy	Minimum.	Maximum	Average	Standard deviation
RC	1	0.39789	0.39789	0.39789	0.00000
	2	0.39789	0.39789	0.39789	0.00000
SF6	1	0.00000	0.00000	0.00000	0.00000
	2	0.00052	0.01026	0.00882	0.00243
AS	1	1.00000	1.00000	1.00000	0.00000
	2	0.06872	0.96540	0.51283	0.28074
MG6	1	0.00000	0.00000	0.00000	0.00000
	2	0.00798	284.148	11.3347	52.06211
S4	1	10.5364	10.5364	10.5364	0.00000
	2	0.68479	3.11001	1.27051	0.49171
H64	1	3.32237	3.32237	3.32237	0.00000
	2	0.00000	0.06305	0.00238	0.01151
ACL	1	0.00005	0.00008	0.00006	0.00001
	2	0.11624	0.26429	0.20382	0.03768
RGN	1	0.00000	0.99496	0.16583	0.37714
	2	0.99496	9.94959	3.74768	2.46061
GR	1	0.00000	0.01478	0.00227	0.00466
	2	0.07472	0.25807	0.18631	0.04229

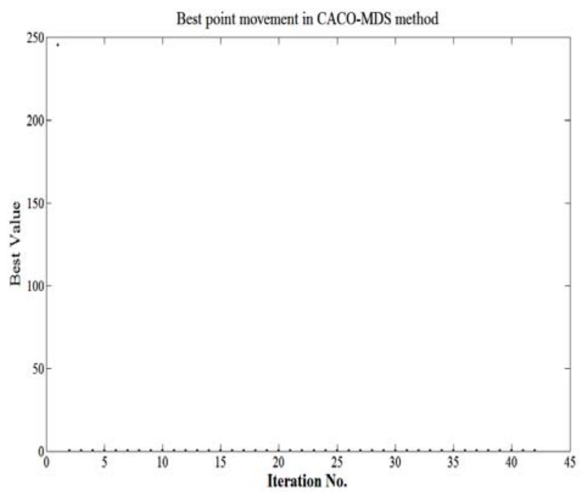

Fig. 5 Best point movement in CACO-MDS Method

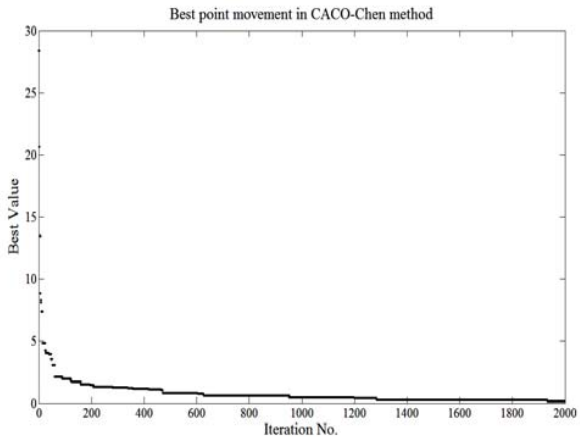

Fig. 6 Best point movement in CACO-Chen Method

IV. Conclusions

In this paper, a Mahalanobis distance based diversification scheme and Nelder-Mead simplex local search scheme is proposed and incorporated in a simple CACO strategy. The objective was to see the improvement in performance by incorporating the above mentioned schemes. The computational run results clearly indicate the improvement of solution quality, and also the success rate to converge to the global optimum using the above mentioned scheme in CACO. The highlighted points from the study are,

i) MD-based diversification scheme can ensures a minimum multivariate distance for a multidimensional problem and can avoid trap to local optima,
ii) Nelder-Mead simplex search by individual ant can make the strategy more intense in the neighbourhood state space,
iii) CACO-MDS has higher success rate to reach global optima as compared to CACO-Chen, and
iv) The lower value of standard deviation and solution quality confirms the consistency of CACO-MDS to reach global optima

The CACO-MDS can be adopted and tested in real life industrial optimization problem to determine near optimal conditions. In addition, the proposed intensification and diversification scheme can be introduced in other existing ant strategies (such as ACO_R, and CIAC) and the performance improvement (if any) can be studied. Future research can also be directed towards the best selection of ant parameters, optimal number of nests, and selecting optimal MD for varied problem situations.

Appendix

Branin RCOS (RC)
This function is expressed as

$F_{Min} = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1 + 8$, where the search domain is defined as $-5 \le x_1 \le 10$, and $0 \le x_2 \le 15$. It has no local optima, and has three different global minima. They are (π, 12.275), (π, 2.275), and (9.42478, 2.475), and the corresponding optimal value is *0.397887*.

Schaffer f6 (SF6)
This function is expressed as

$F_{Min} = 0.5 + \frac{\left(\sin\sqrt{x_1^2 + x_2^2}\right)^2 - 0.5}{\left(1 + 0.001*(x_1^2 + x_2^2)\right)^2}$, and the search space is $-100 \le x_1, x_2 \le 100$. It has several local optima, and the global optimum solution is zero, corresponding to X_{opt} = (0, 0).

Easom (AS)
This function can be mathematically expressed as

$F_{Max} = \cos(x_1)*\cos(x_2)*e^{-\left((x_1-\pi)^2+(x_2-\pi)^2\right)}$, where the search space is defined as $-100 \le x_1, x_2 \le 100$. The optimum solution is $X_{opt} = (\pi, \pi)$, and its corresponding solution is *1*.

Michalewicz's function number 6 (MG6)
The function is written as

$F_{min} = 100*\left(x_2 - x_1^2\right)^2 + \left(1 - x_1\right)^2 + 90*\left(x_4 - x_3^2\right)^2 + \left(1 - x_3\right)^2 + 10.1*\left(\left(x_2 - 1\right)^2 + \left(x_4 - 1\right)^2\right) + 19.8*\left(x_2 - 1\right)\left(x_4 - 1\right)$, and the multidimensional search space is given by $-10 \le x_i \le 10$. The global optimum point or X_{opt} is (1, 1, 1, 1) and corresponding solution is zero.

Shekel (S4)
This function is expressed as

$(F_{4,m})_{Max} = \sum_{i=1}^{m}\left[\sum_{j=1}^{4}(x_i - a_{ij})^2 + c_i\right]^{-1}$, and the search space is given by $-10 \le x_i \le 10$. Here, *m* is selected as *10*. The global optimum point or X_{opt} is (4, 4, 4, 4), and optimal solution is 10.5364. The value of a_{ij} and c_i are selected based on the following table.

i	a_{ij}				c_i
1	4	4	4	4	0.1
2	1	1	1	1	0.2
3	8	8	8	8	0.2
4	6	6	6	6	0.4
5	3	7	3	7	0.4
6	2	9	2	9	0.6
7	5	5	3	3	0.3
8	8	1	8	1	0.7
9	6	2	6	2	0.5
10	7	3.6	7	3.6	0.5

Hartmann (H64)

This function is expressed as

$(F_{6,4})_{Max} = \sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right]$, where search space is -10 ≤ x_j ≤ 10. It has four maxima. The optimum point or X_{opt} is given by (0.20169, 0.150011, 0.476874, 0.275332, 0.311652, 0.6573), and the corresponding solution is 3.3224. The selected values of c_i, a_{ij} and p_{ij} values are given in the following tables.

i	a_{ij}						c_i
1	10.0	3.00	17.00	3.50	1.70	8.00	1.0
2	0.05	10.00	17.00	0.10	8.00	14.00	1.2
3	3.00	3.50	1.70	10.00	17.00	8.00	3.0
4	17.00	8.00	0.05	10.00	0.10	14.00	3.2

i	p_{ij}					
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

Ackley (ACL)

The function is given by

$F_{Min} = -20 * e^{\left(-0.2*\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right)} - e^{\left(\frac{1}{n} * \sum_{i=1}^{n} \cos(2 p x_i)\right)} +$ 20 + e, and the search space is -30 ≤ x_i ≤ 30.

There are several local optima and the global optimal point is (0, 0,. . . ,0_n). The global optimal solution for this function is zero.

Rastrigin (RGN)

This function is given by

$F_{Min} = 10*n + \sum_{i=1}^{n}\left\{x_i^2 - 10*\cos(2\pi x_i)\right\}$, where the search space is -5.12 ≤ x_i ≤ 5.12. It has several local optima and the optimal point is (0,0,. . . ,0_n) with its solution as zero.

Griewank (GR)

The function is expressed as

$F_{Min} = \sum_{i=1}^{n}\frac{x_i^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$, and the search space is defined as -500 ≤ x_i ≤ 500. It has several local optima. The global optimum point is given by X_{opt} = (0,0,. . . ,0_n), and its corresponding solution is *0*.

Acknowledgment

I Mukherjee thanks Soumen Kumar Manna for his initial effort to develop a basic Matlab code of an ant colony optimization.

References

[1] I. Mukherjee, and P.K. Ray, “A review of Optimization Techniques in Metal Cutting Processes,” *Computers and Industrial Enginering*, Vol. 50, pp. 15-34, 2006.

[2] I. Mukherjee, and P.K. Ray, “Artificial Neural Network and Metaheuristic Strategies: Emerging Tools for Metal Cutting Process Optimization,” in *Handbook on Computational Intelligence in Manufacturing and Production Management*, Idea Group Inc (IGI), 2008 , pp 366-397, Ch. XIX.

[3] M. Dorigo, “Optimization, Learning and Natural Algorithms (in Italian),” *Ph.D. thesis*, ipartimento di Elettronica, Politecnico di Milano, Italy, 1992.

[4] M. Dorigo, V. Maniezzo, and A. Colorni, “Ant System: Optimization by a Colony of Cooperating Agents,” *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 26, no. 1, Part B, 1996, pp. 29–41.

[5] T. Stutzle, and H. H Hoos, “MAX–MIN, Ant System,” *Future Generation Computer Systems*, Vol. 16, no. 8, pp. 889–914, 2000.

[6] M. Dorigo, and L. M. Gambardella, “Ant Colony System: A cooperative learning approach to the traveling salesman problem,” *IEEE Transactions on Evolutionary Computation*, Vol. 1, no. 1, pp. 53–66, 1997.

[7] G. Bilchev, and I. C. Parmee, “The Ant Colony Metaphor for Searching Continuous Design Spaces,” *Lecture Notes in Computer Science*, Volume 993, pp. 25-39, 1995.

[8] M. Wodrich, and G. Bilchev, “Cooperative distributed search: the ant’s way.” *Control Cybernetics*, Vol. 26, no. 3, pp. 413-446, 1997.

[9] M. Mathur, , S.B. Karale, S Priye, V.K. Jyaraman, and B.D. Kulkarni, “Ant colony approach to continuous function optimization,” *Industrial and Engineering Chemistry Research*, Volume 39, pp. 3814–3822, 2000.

[10] J. Dreo, and P. Siarry, “Continuous interacting ant colony algorithm based on dense hierarchy,” *Future Generation Computer Systems*, Vol. 20, pp. 841-856, 2004.

[11] K. Socha, and M. Dorigo, “Ant colony optimization for continuous domains,” *European Journal of Operational Research*, Vol. 185, pp. 1155–1173, 2008.

[12] M. Schluter, J. A. Egea, and J. R Banga, “Extended ant colony optimization for non-convex mixed integer nonlinear programming,” *Computers & Operations Research*, Vol. 36, pp. 2217-2229, 2009.

[13] P.C. Mahalanobis, “On the generalized distance in statistics,” in *Proc. of the National Institute of Science*, India, Vol. 2, no. 1, 1936, pp. 49-55.

[14] J. A. Nelder, and R. Mead, “A Simplex Method for Function Minimization,” *Computer Journal*, Vol. 7, pp. 308-313, 1965.

[15] L. Chen, J. Shen, L. Qin, and J. Fan, “A Method for Solving Optimization Problem in Continuous Space Using Improved Ant Colony Algorithm,” *Lecture Notes in Computer Science*, Vol. 3327, pp. 61-70, 2004.

Sasadhar Bera was born in Hooghly, West Bengal, India. He did his bachelor’s degree in Mechanical Engineering from NIT Durgapur, West Bengal, and master’s in quality, reliability, and operations research from Indian Statistical Institute, Kollkata. His major field of study is quality engineering and management. His research work appeared in pear reviewed conference proceedings. He is currently pursuing his research in the area of multiple response quality characteristics optimization.

Indrajit Mukherjee was born in Kolkata, West Bengal, India. He did his graduation in mechanical engineering from Jalpaiguri Govt. Engg. College, West Bengal, and completed his Masters degree in quality, reliability, and operations research from the Indian Statistical Institute, Kollkata. He is working as Assistant Professor in Shailesh J. Mehta School of Management, IIT Bombay, and his major field of study is operations management. His research papers appeared in various pear reviewed national and international journals and conference proceedings. His current research interests are in the domain of multiple response optimization, modelling variability in supply chain, and service quality issues.