

# A Location Routing Model for the Logistic System in the Mining Collection Centers of the Northern Region of Boyacá-Colombia

Erika Ruíz, Luis Amaya, Diego Carreño

**Abstract**—The main objective of this study is to design a mathematical model for the logistics of mining collection centers in the northern region of the department of Boyacá (Colombia), determining the structure that facilitates the flow of products along the supply chain. In order to achieve this, it is necessary to define a suitable design of the distribution network, taking into account the products, customer's characteristics and the availability of information. Likewise, some other aspects must be defined, such as number and capacity of collection centers to establish, routes that must be taken to deliver products to the customers, among others. This research will use one of the operation research problems, which is used in the design of distribution networks known as Location Routing Problem (LRP).

**Keywords**—Location routing problem, logistic, mining collection, model.

## I. INTRODUCTION

A supply chain consists of all parts directly or indirectly involved in customer satisfaction [1]. The objective of a supply chain must be to maximize the total value generated. The value generated by a supply chain is the difference between the final product value for the customer and the cost in which the chain incurs to meet his/her request.

The supply chain design consists in determining the structure that facilitates the flow of goods and/or services from beginning to end. This is characterized by being one of the decisions having a major impact on organizations, as it has an influence on costs: transport, installation and operation. Additionally, it affects performance and efficiency of the systems, and the customer service level [2]. In this context, one of the problems of the Operation Research used in conjunction for the design of distribution networks and Vehicle Routing Problem is the LRP. The purpose of this model is to determine the number, capacity and location of installations in order to serve the customers, and at the same time, to find the optimal number of vehicles and routes that guarantee goods delivery [3]. As a NP-Hard problem, LRP has been solved through different heuristic methods, such as: neural networks, particle intelligence, evolutionary computation, artificial immune systems and fuzzy systems. Likewise, hybrid methods have been developed known as memetic algorithm [4].

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The main objective of this research is to explore the application of LRP, specifically in the network design for coal distribution in the Colombian mining sector. There are different LRP variants. In this case, LRP will be taken with capacity restrictions over the deposits known as CRLP and will be designed the algorithm of the metaheuristics of Particle Swarm Optimization, as a proposal for its solution. In order to achieve this, the methodology will be carried out according to the phases indicated in a research study of operations presented by [5]. Likewise, the information is collected for modelling supply and demand nodes of the problem through the characterization of logistic management of collecting centers, using information from National Planning Department, and others governmental entities.

This article is organized as follows: The next section presents a brief literature revision about the location-routing problem, giving solution to the research problem; then a mathematical model is presented, which is formulated using binary integer linear programming, the algorithm design and its application to the Colombian mining sector; and finally, conclusions are presented.

## II. LITERATURE REVIEW

The distribution network arises from the interaction between a set of customers, who order a number of products to a set of fixed installations with a specific assigned capacity [1]. A distribution network design must guarantee the efficiency in the goods delivery process, as well as, guarantee competitiveness of the organizations, ensuring service fulfilment, using the least resources. Likewise, network design varies according to the criteria taken into account for the decisions-making, the products flow from the place of storage to the customer location, as well as, the existence or not of intermediate installations, finding the following designs: direct distribution without a stock, direct distribution with a warehouse, decentralized distribution, distribution according to customer, according to the product [6]-[10].

LRP forms part of the analysis of locations of installations, characterized by being centred on the interdependence between location decisions and vehicle routing decisions [11]. The problem consists of opening one or more deposits and designing for each open deposit a number of routes, so that the total demand of customers does not exceed the deposit capacity. Each route must start and end in the same deposit and the total number of utilized vehicles is a variable decision. Likewise, the set of routes must serve all customers and

minimize the overall cost, which comprises fixed costs of deposits opening, fixed costs of vehicles use and costs of established routes [12], [13].

The beginning of LRP dates back from the Travelling Salesman Problem (TSP), as a particular case for the location of an installation with only one vehicle, which is known as Traveling Salesman Location Problem (TSLP) [14]. In 1977, [15] formulates the first routing and location deterministic problem, in which vehicles leave from an origin and meet a client to collect the load, delivering it to another client and finally going back to the origin. Likewise, the problem has been analysed for those cases when groups are created, not for the customer demand, but for the vehicles capacity, so that each group of customers is analysed using the TSP, and then, a unique deposit is located, assimilating each travel as a customer [16].

Afterwards, probability concept was introduced to LRP, where routing and location, dynamic and stochastic problems arise, considering that, along the route, vehicles could meet some clients which do not require the service. The probabilistic TSLP was introduced to the literature, as a problem associated to this particular case [15].

Finally, the most recent research of the routing and location problem is focused on incorporating the variables of vehicle routing problem, as it is the case for CVRP, in which the assigned vehicles to each deposit have a definite capacity [12] and other variables, such as: Open LRP (OLRP) [16]-[18].

Based upon the foregoing, it is concluded that studies carried out using the LRP and all its variants pursue a common objective, which consists of facilitating decisions-making regarding to the design of distribution networks, complying with the organizational policies in order to become more competitive.

### III. MATHEMATICAL MODEL FOR LOCATION OF COLLECTION CENTERS

The Province of Valderrama has always been characterized by the mining activity, accounting for 60% of activities in the department of Boyacá. This region satisfies not only the other departments within the country but also the other countries. Thus, Boyacá has become one of the major exporters of minerals in the country [19]. Because the region contributes to the national GDP significantly, it is necessary to document and apply new logistic models that satisfy international, national and local needs in order to strengthen each characteristic of the region, enhancing the productivity, which will influence on the National Export Indexes.

This work studies the LRP model with restriction of capacity over the deposits CLRP. The proposed mathematical model determinates the location of installations or collection points, in which different blends of coal, coming from the mines, will be collected and storage and from where products will be delivered to the clients, so that, decisions-making will be facilitated in order to determinate the design of the enterprise distribution network.

The network will be designed taking into account a previous characterization of this supply chain, considering the

following factors: mines and customers location, blend of different types of coal to distribute, demand behaviour, routing costs, vehicles capacity and installation costs. The objective function minimizes the overall cost of the network design, which includes: variable cost of transportation and fixed cost of collection points establishment.

The designed network for the investigated problem can be defined as follows:

The network  $G(I, K, A)$  is a complete graph that represents the distribution network, where  $I = \{1, 2, \dots, N\}$  is the set of collection centers or demand nodes,  $K = \{1, 2, \dots, M\}$  is the set of candidates locations. The set  $A = \{(i, j) : i, j \in N\}$  is the set of arches that represent road connections among nodes. Each arch  $(i, j)$  has an associated transportation cost, this cost is proportional to the distance between nodes, likewise, two transport modalities are considered: own fleet or rented fleet. Each node of mines  $k \in N_0$  has an associated capacity  $[(CD)]_k$  (number of tons that can be dispatched in a homogeneous fleet). Each customer node has a demand  $d_i$  that represents the number of tons of coal in a specific blend required by the customer.  $d_i$  obeys a demand aggregation focus and the establishment cost for each collection centre corresponding to the rental cost of the property. On the other hand, the probabilistic demand of customers is considered, and under this premise, the demand is only known at the time when the vehicle visits the customer. The mathematical model CLRP that describes the aforementioned problem is presented as follows:

Let  $i$  be the index used to identify the coal blend, where  $i \in I$ ,  $j$  the index that makes reference to the customers, where  $j \in J$ ; and  $k$  the index that identifies each candidate location for collection points, where  $k \in K$ .

Parameters and variables:

$C_{ij}$	Cost per unit distance of taking customer order $i$ to customer $j$ .
$C_{ik}$	Cost of transporting the customer's order and from the collection center $k$ .
$C_{Fk}$	Cost of opening the collection centre $k$ .
$D_{ij}$	Distance between client $i$ and client $j$ .
$D_{ik}$	Distance between client $i$ and collection center $k$ .
$N$	Number of collection centers
$W$	Vehicles capacity
$P_i$	Probabilistic demand customer $i$
$X_{ijk}$	1 if customer $i$ precedes customer $j$ in the order preparation in the collection center $k$ 0 in the opposite case
$Y_{ik}$	1 if customer $i$ precedes customer $j$ in the order preparation in the collection center $k$ . 0 in the opposite case
$Z_k$	1 if collection centre $k$ is open 0 in the opposite case

The objective function corresponds to:

$$\text{Min} Z = \sum_{k=1}^M C_{Fk} \cdot Z_k + \sum_{k=1}^M \sum_{(i,j) \in A} C_{ij} \cdot D_{ij} \cdot X_{ijk} + \sum_{k=1}^M \sum_{(i,j) \in A} C_{ik} \cdot D_{ik} \cdot Y_{ik} \quad (1)$$

Subject to:

$$\sum_{k \in K} \sum_{(i,j) \in A} X_{ijk} = 1 \quad \forall i \neq j \quad (2)$$

$$\sum_{k \in K} Z_k = N \quad (3)$$

$$\sum_{i \in I} \sum_{j \in J} X_{ijk} < M Z_k \quad \forall (k \in K) \quad (4)$$

$$\sum_{i \in I} P_i \sum_{j \in A} X_{ijk} \leq W \quad (5)$$

$$\sum_{k \in K} Y_{ik} = 1 \quad (6)$$

$$\sum_{i \in I} X_{ijk} - \sum_{i=1}^N X_{jik} = 0 \quad \forall (i \neq j); \forall (k \in K) \quad (7)$$

$$X_{ijk} \in (0,1) \quad (8)$$

$$Y_{ik} \in (0,1) \quad (9)$$

$$Z_k \in (0,1) \quad (10)$$

where (2) guarantees the existence of a single route to send the coal blends to the customers from the collection center  $k$ , guaranteeing the continuity of the route, (3) is the restriction that establishes the number of collection centers to set up; (4) guarantees that deliveries are dispatched only from the open collection centers, where “ $M$ ” is a parameter denominated a very large number. The restriction (5) establishes the vehicle capacity; (6) is the restriction that determines that each customer is assigned to a collection center; (7) establishes that each of the routes begins and ends in the same collection center. Finally, variables (8), (9), and (10) represent the binary variables utilized.

Because LRP is a NP-Hard type problem, a heuristic approach is more convenient to find a solution to the problem rapidly [19], [20]. In order to solve the problem, described in this research, it is proposed to use the particle swarm PSO algorithm. This meta-heuristic was proposed by [21] where each solution is denominated particle, and a set is the population or swarm and the search space is the area to explore. Each iteration seeks to find the optimal by means of the movement of particles, whose movement depends on the social information -g best and cognitive information p-best. A swarm particle is presented as follows:  $D$ : dimension of the particle,  $N$ : number of particles,  $X_i$ : position of the  $i$  th particle,  $V_i$ : velocity of the  $i$  th particle,  $P_i$ : best solution for  $i$  Th particle,  $P_{ig}$ : best global solution for the particles swarm.

The position update after each iteration is accomplished as:

$$x_{il}(t+1) = x_{il}(t) + v_{il}(t+1) \quad (11)$$

The velocity update of the  $i$ th particle is calculated through:

$$v_{il}(t+1) = w(t) * v_{il}(t) + (c_p * \text{ran}()) [p_{il}(t) - x_{il}(t)] + (c_l * \text{ran}()) [p_{li}(t) - x_{il}(t)] + (c_g * \text{ran}()) [p_{gi}(t) - x_{il}(t)] \quad (12)$$

The velocity of each particle is updated considering the following criteria: inertia, which leads the movement to the previous position, cognitive information, which leads the movement to the best position. Social information leads the

movement to the best position in the surrounding and to the best global position.

The CRLP solution is composed of the solution of two parts: customer allocation and vehicles routing. For this purpose, the method published by [22] is used; this method is transformed in position values through:

$$x_{il} = x_{min} + \frac{(x_{max} - x_{min})}{n * [s_{il} - 1 + \text{ran}()]} \quad (13)$$

$x_{il}$  = Dimension  $l$  of the particle in the swarm search-space.  $x_{min}$  = Minimum value of the particle position within the swarm search-space.  $x_{max}$  = Maximum value of the particle position within the swarm search-space.  $N$  = Number of candidate location and number of customers.  $S_{il}$  = Dimension  $l$  of the swarm solution  $I$ ,  $\text{Ran}$  = Random number.

Based on the aforementioned operations, the memetic algorithm proposed in this research is presented as:

Step 1: Define number of collection centers to install- $N$

Step 2: Identify number of swarm particles- $n$

Step 3: Locate  $N$  collection centers for  $i=1,2..n$

Step 4: Allocate customers to collection centers to installed collection centers, considering their operation characteristics.

Step 5: Build the a priori route for each solution  $S_i$

Step 6: If the criterion is met, continue to step 7; if not, return to step 5

Step 7: Calculate the fitness for solution  $S_i$ .

Step 8: If  $S_i$  fitness is  $<$  than  $P_g$ , update  $g\text{best}$   $P_g = P_i$

Step 9: If  $P_i$  fitness is  $<$  than  $P_g$ , update  $P_g = P_i$ .

Step 10: If criterion is met, continue to step 11, if not, return to step 7.

Step 11: Codify  $S_i$  solution.

Step 12: Calculate velocity  $V_i$  of the particle  $I$ .

Step 13: Update position  $X_i$  of the particle  $I$ .

Step 14: Decodify and correct solution  $S_i$

Step 15: Stop the algorithm if the stop criterion is met, if not, return to step 5.

The flow diagram of the algorithm described previously is presented in Fig. 1.

#### IV. APPLICATION OF THE MATHEMATICAL MODEL TO A LOGISTIC MINING CENTRE OF BOYACÁ-COLOMBIA

The number of particles that will compose the swarm in this research, is proposed as a variable depending on the number of combinations generated between candidate locations for collection centers and number of collection centers to establish.

For the CLRP model proposed in this research, it is assumed to be a distribution network with a probabilistic demand and vehicles with definite capacity, considering the particularity that, in this logistic system, transportation cost varies depending on the type of transport used: owned or rented.

In order to know the list of customers of the enterprise and analyse their behaviour, historical records of the objective enterprise are analysed. The distance between customers was

calculated using the tool Google maps. An example of the route designed using this app is shown in Figs. 1 and 2.

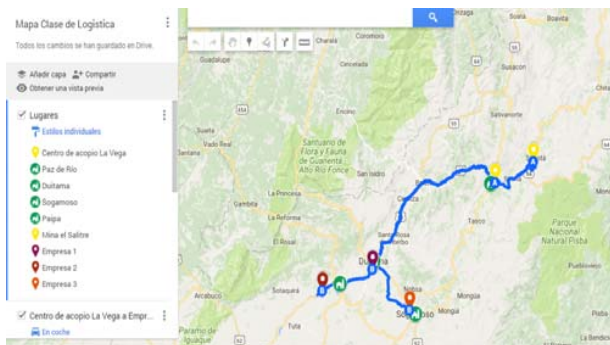


Fig. 1 Route design



Fig. 2 Characterization of a collection center

The definition of candidates lots for location of collection centers was the result of a preliminary study by the directors, which established that these locations are found in the northern region of the department of Boyacá. The fixed cost of installation was calculated according to the value of commercial lease for square meter.

For each iteration, it is necessary to convert the PSO search-space solution to the language used to represent the original problem solution, therefore, the CLRP problem contains information of the customer allocation problem, as well as, the vehicles routing problem.

## V.CONCLUSION

The present work pretends to illustrate a successful case of application of the current optimization technologies in a Colombian region eliminating the nullity paradigm in the replication of North American and European models in South America, which pretends to have a positive impact, not only in logistic problems, but also to standardize the process in the collection centers, which allows an adequate decision-making based on a historical record of situations that can alter the system performance negatively.

The goal of this research is: the formulation of a mathematical model, the design of a memetic algorithm and

the definition of input parameters for the model. Further research could establish the necessity to validate the algorithm developed in the software MATLAB, developing different scenarios for test or instances.

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