A Location-Allocation-Routing Model for a Home Health Care Supply Chain Problem

Amir Mohammad Fathollahi Fard, Mostafa Hajiaghaei-Keshteli, Mohammad Mahdi Paydar

Abstract—With increasing life expectancy in developed countries, the role of home care services is highlighted by both academia and industrial contributors in Home Health Care Supply Chain (HHCSC) companies. The main decisions in such supply chain systems are the location of pharmacies, the allocation of patients to these pharmacies and also the routing and scheduling decisions of nurses to visit their patients. In this study, for the first time, an integrated model is proposed to consist of all preliminary and necessary decisions in these companies, namely, location-allocation-routing model. This model is a type of NP-hard one. Therefore, an Imperialist Competitive Algorithm (ICA) is utilized to solve the model, especially in large sizes. Results confirm the efficiency of the developed model for HHCSC companies as well as the performance of employed ICA.

Keywords—Home health care supply chain, location-allocation-routing problem, imperialist competitive algorithm, optimization.

I. INTRODUCTION AND LITERATURE REVIEW

THIS paper maintains HHCSC which concludes a set of pharmacies and nurses to supply the demand of patients. According to the official reports from recent studies [2], [10], old persons would like to take the nursing and care services at home instead of a formal setting in hospital. The main decisions and services include selecting the position of pharmacies and delivering the drugs or medical instruments from a pharmacy to the patients, and also the transportation cost for biological samples from patient's homes to pharmacies [6]-[10]. This issue has been motivated by many researchers especially in last decade. Basically, the models in the literature were a type of Vehicle Routing Problem (VRP) [10]. In the term of VRP, generally a company wants to supply the demand of customers by using some vehicles for optimizing the transportation cost and traveled distance between customers. The complexity of this problem directly increases by increasing the number of supplying centers and customers. For HHCSC companies, the biological samples are the goods which should be transformed from patients and pharmacies. In addition, VRP is classified as an NP-hard problem. In the proposed problem, the authors not only consider the routing of biological samples, but also the

Amir Mohammad Fathollahi Fard is a master student in Industrial Engineering at University of Science and Technology of Mazandaran, Behshar, Iran (e-mail: amirfard@mazust.ac.ir).

Mostafa Hajiaghaei-Keshteli is an assistant professor in Industrial Engineering Department at University of Science and Technology of Mazandaran, Behshar, Iran (corresponding author, e-mail: mostafahaji@mazust.ac.ir).

Mohammad Mahdi Paydar is an assistant professor of industrial engineering, University of Babol Noshirvani University of Technology, Babol, Iran (e-mail: paydar@nit.ac.ir).

location of pharmacies and the allocation of patients to pharmacies are added into the proposed HHCSC problem.

In recent studies, using heuristics and metaheuristics are attended by researchers to solve such NP-hard problems [7], [9]. For example, Heirmann et al. [8] proposed a multi-modal home health care scheduling and routing problem. They considered a real case study in Austria to generate their test problems. Another contribution of their study was to design a two-stage approach to address their model. Constraint programming approach was utilized in the first stage, while the second stage was addressed by various popular metaheuristics including Variable Neighborhood Search (VNS), Memetic Algorithm (MA), Scatter Search (SS) and a hyper heuristic simulated annealing algorithm. In another same work, Fikar and Hirsch [3] developed a two-stage metaheuristic to solve their proposed home health care routing and scheduling model. Their numerical example was provided by the Austrian Red Cross (ARC). In their model, they not only considered the walking to patients, interdependencies of services and time window, but also they added different types of transport systems working and time regulations for the first time into this research area.

Recently, a bi-objective model was explored by Braekers et al. [2] for the first time by considering the total cost of routing and scheduling of nurses and also client inconvenience, simultaneously. They aimed to find an interaction between two conflicting goals by using a metaheuristic structured by local searches. Furthermore, Shi et al. [10] considered a fuzzy logic for customers' demand and introduced a hybrid algorithm by combining Genetic Algorithm (GA) and simulation rules to solve it.

According to the literature review [4] and related recent studies [2]-[10], this paper proposes for the first time a location-allocation-routing model for HHCSC problems. To achieve the robust solutions for this presented NP-hard problem, an ICA is utilized. To summarize the rest of the paper, Section II describes the proposed real assumptions and the mathematical model. Section III illustrates the proposed algorithm to tackle the problem. Instances and experiments are investigated in Section IV. Finally, the future lines of our study and conclusions are stated in the last section.

II. PROBLEM FORMULATION AND MODEL DESCRIPTION

A. Problem Definition

In the proposed model, a Location-Allocation-Routing (LAR) strategy is proposed for the first time. The aim of the proposed problem is to select the locations of pharmacies, allocation of patients to the nearest pharmacies and the routing

and scheduling decisions to allocate the nurses for the patients.

To present the proposed HHCSC problem, this paper considers a city or a town by M patients distributed whole the city. A company wants to supply the home care services for these patients. In addition, there existed P pharmacies by opening cost of FC_p ordered as MAX_p number by this company. Each pharmacy has capacity CAP_p to supply the demand of A_i for required drugs for the patients. In addition, each pharmacy has N_p nurses. Also, consider a graph G=(V,A), where V is the set of vertices and A is the set of arcs. Let $X = \{X_i \mid i = 1,...,M\}$ be used for the set of M patients and let X_0 be the pharmacy. In our geographical space in two dimensions, the distance between each patient and pharmacy is D_{ip} and also between patients with each other is shown by D_{ij}. Nurses start from their pharmacy to visit the patients and back to it. Besides, different types of transport systems (e.g. car, public and so on) are considered, which have the capacity of CAP_k and also the transportation cost of TC_k. In addition, according to the time window, each patient has the earliest time (E_i) and the latest time (L_i) to visit by nurses. Also, to undertake the HHC services for patients such as nursing, cleaning and so on, a specified time for each patient (W_i) is considered. The nurses traveling from patient i to patient j, need a specified time (T_{ij}). Regarding the policy of the company, the summation of distance traveled for each nurse and his/her employed transport system would be lower than a predefined number ($MDIS_{pnk}$). If for this pharmacy this law is not observed, a penalty value should be considered for the extra traveled distance (PEN). To assign the patients to the nearest pharmacy, the allocation cost per unit distance is considered by CS. In the following, the real assumptions for the illustrated problem are addressed.

B. Assumptions

- The illustrated HHCSC considers many patients, a few pharmacies with their nurses and different types of vehicles.
- The potential locations for pharmacies are predefined and should be decided.
- The capacity of each pharmacy is limited according to each location.
- Between each pharmacy no flows exist.
- For each nurse, the start and the end points are specified according to his/her pharmacy.
- The working time for each patient is predefined and estimated by nurses and pharmacies.
- A time window exists for the patients to be available at a certain time.
- To transform the biological samples and nurses to visit the patients, several types of vehicles are considered.
- Each patient should assign to only one pharmacy to get the home care services.

- For overall distance traveled according to type of vehicle and nurses to visit patients, a maximum desired number is considered for all pharmacies of the company.

C. Notations

The proposed model is formulated by the following notations:

Indices:

i, j	Set of patients, $i, j \in \{1, 2, ..., M\}$
p	Set of potential pharmacy location, $p \in \{1, 2, ..., P\}$
n	Set of nurses, $n \in \{1, 2, ..., N_p\}$
k	Set of transport systems, $k \in \{1, 2, ..., K\}$

Parameters:

N_p	The maximum number of nurses for pharmacy p.
D_{ip}	Distance between patient i and pharmacy p.
D_{ij}	Distance between patient i and patient j.
TC_k	Transportation cost per unit of distance for transport system k.
CAP_k	The capacity of transport system k.
FC_p	Fixed opening cost of pharmacy p to be established.
W_i	The working time for the nurse to service the patient i.
E_i	The earliest service time for the patient i.
L_i	The latest service time for the patient i.
T_{ij}	The traveling time from patient i to patient j.
PEN	Amount of penalty for nurses and pharmacies for over distance traveling between patients according to adopted policy of company ($1 < PEN < 5$).
$BIG1$	A positive large number, this parameter is generated to construct the time window constraints.
$BIG2$	A positive large number, this parameter is generated to construct the allocation constraints.
CS	Unit allocation cost of demand per unit distance.
MAX_p	Maximum desired number of established sites for pharmacies.
$MDIS_{pnk}$	Maximum desired traveled distance for the nurse n for the pharmacy p by using transportation system k.
A_i	Demand of patient i (amount of drugs required for patient i).
CAP_p	Capacity of pharmacy p (amount of drugs).

Decision variables:

Z_{ip}	1 if patient i assigned to pharmacy p, otherwise 0.
X_{ijn}^{pk}	1 if for pharmacy p, the nurse n by using transport system k visits the patient i before patient j, otherwise 0.
S_{in}^p	Denotes the time at which nurse n for pharmacy p begins to service the patient i. it should be noted, if in an event the given nurse n does not service patient i, S_{in}^p does not mean anything.
Y_p	1 if pharmacy p is established, otherwise 0.
O_{pnk}	Overall distance traveled by nurse n from pharmacy p by using transport system k.

D. Formulation

A mixed integer linear programming is supposed to address the formulation as follows:

$$Min(\sum_{p=1}^{P} FC_p \times Y_p + \sum_{i=1}^{M} \sum_{p=1}^{P} D_{ip} \times CS \times Z_{ip} \qquad (1)$$

$$\sum_{p=1}^{P} \sum_{k=1}^{K} \sum_{n=1}^{N_p} \sum_{i=1}^{M} \sum_{j=1}^{M} D_{ij} \times TC_k \times X_{ijn}^{pk} + \sum_{p=1}^{P} \sum_{n=1}^{N_p} \sum_{k=1}^{K} O_{pnk} \times TC_k \times PEN)$$

$$s.t.\sum_{p=1}^{P} Y_p \leq MAX_p \qquad (2)$$

$$\sum_{i=1}^{M} Z_{ip} = 1, \forall p \in P \qquad (3)$$

$$\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{k=1}^{K}\sum_{j=1}^{M} X_{ijn}^{pk} = 1, \forall i \in M \qquad (4)$$

$$\sum_{i=1}^{M} A_i \times \sum_{j=1}^{M} X_{ijn}^{pk} \leq CAP_k, \forall k \in K, n \in N_p, p \in P \qquad (5)$$

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{j=1}^{M} X_{ojn}^{pk} = 1, \forall n \in N_p \qquad (6)$$

$$\sum_{i=1}^{M} X_{ihn}^{pk} - \sum_{j=1}^{M} X_{hjn}^{pk} = 0, \forall h \in M, p \in P, k \in K, n \in N_p \qquad (7)$$

$$\sum_{i=1}^{M} X_{i0n}^{pk} = 1, \forall p \in P, k \in K, n \in N_p \qquad (8)$$

$$S_{in}^{p} + T_{ij} + W_i - BIG1 \times (1 - X_{ijn}^{pk}) \leq S_{jn}^{p}, \forall i, j \in M, p \in P, k \in K, n \in N_p \qquad (9)$$

$$E_i \leq S_{in}^{p} \leq L_i, \forall i \in M, p \in P, n \in N_p \qquad (10)$$

$$O_{pnk} = \max(0, (\sum_{i=1}^{M}\sum_{j=1}^{M} D_{ij} \times X_{ink}^{pk}) - MDIS_{pnk}) \qquad (11)$$

$$Z_{ip} \leq Y_p \times BIG2, \forall i \in M, p \in P \qquad (12)$$

$$\sum_{n=1}^{N_p}\sum_{k=1}^{K} X_{ijn}^{pk} \leq Z_{ip} \times BIG2, \forall i, j \in M, p \in P \qquad (13)$$

$$S_{in}^{p}, O_{pnk} \in R \qquad (14)$$

$$Y_p, X_{ijn}^{pk}, Z_{ip} \in \{0,1\} \qquad (15)$$

The objective function minimizes the total cost of the LAR strategy of the model in the first equation, concluding four parts. The first one is decided as the location of pharmacies. The second part is considered as the total cost of allocation of patients to the nearest pharmacy. The last two terms are chosen as the routing decisions of the model. The third one is computed as the transportation cost and also the policy of overall transportation cost is considered in the last part of the first equation.

Equation (2) represents the total number of located pharmacies. Equation (3) shows that each patient should be assigned to only one pharmacy. Equation (4) indicates each patient is visited once only. Equation (5) ensures that the used transport system should have enough capacity to transform the patient's drugs. Equation (6) represents that each nurse starts from their pharmacy. Equation (7) means that the nurse visits the patient and then leaves the patient. Also, (8) ensures that each nurse should back to its pharmacy after all visiting the patients. Equation (9) explores that each nurse n from pharmacy p cannot arrive at patient j before, $S_{in}^{p} + T_{ij} + W_i$, since the nurse requires the working time W_i, as well as traveling time from patient i to patient j, here *BIG1* is a large scalar. In addition, (10) guarantees that the time window exists in this study. Equation (11) computes the over traveled distance for the nurses from a maximum desired distance by considering the pharmacy and the employed transport system. Equation (12) represents that patients can assign to a pharmacy, if it is open. Equation (13) also states the nurses of a pharmacy can visit their patients, if their patients are allocated to this pharmacy. In this regard, BIG2 is a large scalar to control the marks of both decision variables. Equations (14) and (15) enforce the non-negativity restrictions and binary variables.

III. IMPERIALIST COMPETITIVE ALGORITHM

To solve the proposed problem, ICA proposed by Atashpaz-Gargari and Lucas [1] is utilized. To implement a metaheuristic, an encoding plan is necessary to employ for any mathematical model [11]. In this study, we utilize a two-stage technique, namely, *Random-Key* which was utilized in several recent papers [5], [7]. The used procedures to encode the problem are similar to a recent paper by Sadeghi-Moghaddam et al. [9]. So, we refer the reader to see their paper.

ICA is a type of evolutionary algorithm which maintains a population of solutions to search by an interaction between exploration and exploitation phases, intelligently. The performance of this algorithm was confirmed in several articles. To see the details and explanation of ICA, this recent paper would be suggested: Fathollahi Fard et al. [5].

IV. EXPERIMENTAL RESULTS

Due to the novelty of the proposed model, there aren't any benchmarked instances in the literature to be available for this study [4], [8], [10]. In this regard, we design six random test problems in small and large sizes as addressed by Tables I and II.

TABLE I
THE INSTANCES FOR THE TEST PROBLEM

Instance	Number of pharmacies (P)	Number of nurses (N_p)	Number of transport systems (K)	Number of patients (M)
S1	4	2	2	10
S2	5	3	2	15
S3	5	3	2	20
L4	34	4	5	130
L5	34	3	6	140
L6	36	4	6	150

TABLE II
THE PARAMETERS AND THEIR SURFACES

Parameters	Surfaces
(x_i, y_i)	$1000 \times (U(0,1), U(0,1))$
(x_j, y_j)	$1000 \times (U(0,1), U(0,1))$
(x_p, y_p)	$1000 \times (U(0,1), U(0,1))$
D_{ij}	$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
D_{ip}	$\sqrt{(x_i - x_p)^2 + (y_i - y_p)^2}$
A_i	$Rand\{5, 10, 15, ..., 100\}$
PEN	For small sizes: 1.5, large sizes: 4.5
CAP_k	$Rand\{200, 250, ..., 500\}$
TC_k	$rand\{3, 4, ..., 8\}$
W_i	$rand\{5, 10, ..., 60\}$
E_i	$rand\{0, 1, ..., 10\}$
L_i	$rand\{500, 600, ..., 5000\}$
T_{ij}	$\dfrac{D_{ij}}{\sum_{i=1}^{M} \sum_{j=1}^{M} D_{ij}} \times 60$
$MDIS_{pnk}$	$rand\{25000, 30000, ..., 50000\}$
CS	2

To enhance the efficiency of the used ICA for the developed problem, the algorithm should be tuned [7]. In this study, ICA has been tuned by Taguchi experimental design method and the amounts of final parameters are chosen as follows:

Maximum number of iteration equals to 200. The number of population is equaled to 100. Also, the number of empires equals to 15. Colonies mean the cost coefficient is estimated by 0.1. Finally, the probability of success in revolution of colonies is 0.4.

It should be mentioned, the related codes were written in C++ language by using Microsoft Visual Studio 2014. In addition, the algorithm was run in a computer with processor type: Core 2 Duo-2.26 GHz and 2 GB of RAM.

ICA has been run for 10 run times. The best, worst and the average of the solutions are computed. Furthermore, the standard deviation of the algorithm among 10 run times is also considered with this study. In addition, for the small sizes, an exact solver using by *GAMS software* is used to validate the results of metaheuristic. In this regard, the Gap of solutions is computed by considering the outputs of exact method (EX). It should be noted, EX is not available for large sizes in a logical time. Thus, it is used only for small sizes. Table III shows the results of the proposed HHCSC problem. According to the results, the performance of ICA is confirmed to solve the model. In addition, the convergence behavior of algorithm in one of test problem is depicted by Fig. 1 to validate the convergence rate of the algorithm.

TABLE III
THE RESULTS OF ALGORITHM (CPU=THE PROCESS TIME OF RUNNING, OUT=THE AVERAGE OF SOLUTIONS, B=THE BEST SOLUTION, W=THE WORST SOLUTION, ST=STANDARD DEVIATION, GAP=GAP OF SOLUTION OF ICA FORM EX)

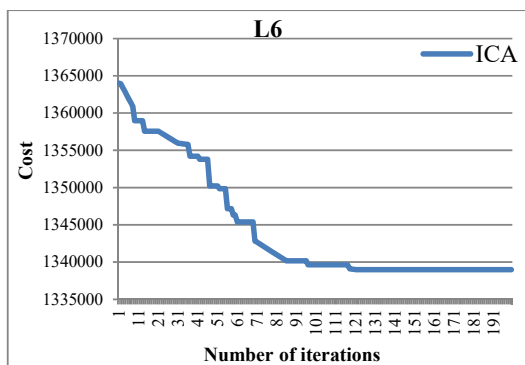| Algorithm		S1	S2	S3	L4	L5	L6
EX	OUT	1.99E+5	1.82E+05	1.59E+05	-	-	-
	CPU	9.37	14.74	30.76	-	-	-
ICA	B	1.99E+5	1.84E+5	159760.5	1078549	1315560	1335600
	W	1.99E+5	1.84E+5	161750.6	1089818	1329836	1348772
	OUT	1.99E+5	1.84E+05	160510.7	1082802.4	1321322.9	1338998.3
	CPU	4.87	4.65	5.13	41.77	46.46	50.59
	ST	5098.35	4676.98	8543.28	3804.2783	5123.047	4413.4439
	Gap	1.55E-03	7.89E-03	0.0076	-	-	-

Fig. 1 The convergence behavior of ICA in L6 test problem

V. CONCLUSION AND FUTURE WORKS

In this paper, for the first time in the literature of the home health care problem, a location-allocation-routing model was developed. The proposed model considered all the main decisions for home health care developers. The model was addressed by an ICA. In addition, an exact method using GAMS software was utilized to validate the results of the algorithm in small sizes. The efficiency and the performance of the algorithm were tested by different measurements in the experimental section.

For future works, more comprehensive analyses on the proposed model are required to be explored. Some other metaheuristics and heuristics can be ordered to solve the problem. Moreover, some real assumptions can be added into the present work. For instance, customer inconvenience and considering different types of services may be added into the presented model. In addition, some more scheduling constraints to consider the break regulations of the works of nurses may be suggested for future works.

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Amir Mohammad Fathollahi Fard was born in Sari in north of Iran. He earned his B.Sc of Industrial Engineering in University of Science and Technology of Mazandaran (USTM), Behshahr, Iran. Now, he is a master student in Industrial Engineering at USTM. His main interested researches are about Supply Chain, Health Care and heuristic optimization methods to solve the NP-hard problems.

Mostafa Hajiaghaei-Keshteli was born in Babol in north of Iran. He earned his B.Sc degree in Iran University of Science and Technology and his M.Sc degree in University of Science and Culture, Tehran, Iran and also his PHD degree from Amirkabir University of Technology (Poly technique Tehran). All of them are in Industrial Engineering. Now, he is an assistant professor in University of Science and Technology of Mazandaran, Behshahr, Iran. His main researchers are included of supply chain and transportation systems, health care and evolutionary computation and related algorithms to solve the NP-hard problems.

Mohammad Mahdi Paydar was born in Qaemshahr in north of Iran. He earned his both B.Sc and M.Sc degree in Industrial Engineering from Mazandaran University of Science and Technology, Babol, Iran. He also earned his PHD degree in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran. Now, he is an assistant professor in Babol Nooshirvani University of Technology, Babol, Iran. His main researches are about supply chain and logistic systems and cellular manufacturing.