# A Linearization and Decomposition Based Approach to Minimize the Non-Productive Time in Transfer Lines 

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#### Abstract

We address the balancing problem of transfer lines in this paper to find the optimal line balancing that minimizes the nonproductive time. We focus on the tool change time and face orientation change time both of which influence the makespane. We consider machine capacity limitations and technological constraints associated with the manufacturing process of auto cylinder heads. The problem is represented by a mixed integer programming model that aims at distributing the design features to workstations and sequencing the machining processes at a minimum non-productive time. The proposed model is solved by an algorithm established using linearization schemes and Benders' decomposition approach. The experiments show the efficiency of the algorithm in reaching the exact solution of small and medium problem instances at reasonable time.


Keywords-Transfer line balancing, Benders' decomposition, Linearization.

## I. Introduction

THE transfer line balancing (TLB) problem concerns products that are manufactured in massive amounts. The line is composed of a number of workstations arranged in series. The product visits each workstation where a number of manufacturing operations are implemented. It is required to distribute these manufacturing operations on the existing workstations to achieve specific objectives, i.e minimizing cost, minimizing cycle time, minimizing non-productive time, and maximizing utilization. Certain capacity and technological constraints must be considered e.g., machine cycle time, precedence among the operations, inclusion constraints, and exclusion constraints. The problem is represented in the literature by different mathematical formulations, and solved through exact and metaheuristics algorithms.

Dolgui et al. [1] solve a TLB problem using parametric decomposition and graph optimization methods to minimize costs of machines, tools, labour, area, and maintenance. Dolgui et al. [2]partition the manufacturing operations into blocks where operations in the same block are implemented in parallel. Mixed integer programming models of this problem are proposed in [3] and [4]. Heuristic algorithms to solve this problem are developed in [5]-[8].Dolgui et al. [9] study a TLB problem comprising multiple spindle head machines. The problem is reduced to a shortest path problem. Integer programming models of this problem are proposed in [10] and

[^0][11]. The problem is reduced to a set covering problem in [12]. A heuristic method is devised in [13] to handle large scale instances of the multiple spindle machines. Dolgui et al. [14] solve this problem using a branch and bound algorithm. A more complex problem is studied in [15], where each station includes multi-spindle heads activated in a serial-parallel mode.

Gurevsky et al. [16] deal with a TLB problem where the precedence relationship between two operations allows implementing both operations simultaneously. The problem is solved using a genetic algorithm. Large instances of the TLB problem with non-strict precedence relationships is solved in [17] using a greedy randomized adaptive search and a genetic algorithm. A transfer line comprising machines with rotary tables is studied in [18]. The problem is represented as a constrained shortest path problem. Large scale industrial cases of the TLB problem is tackled using an ant colony optimization algorithm in [19]. Masood[20] studies a TLB problem to reduce the cycle time and increase machine utilization in critical workstations. Das et al. [21] investigate a TLB problem to minimize the non-productive time. The problem is solved in two stages using two mathematical models.

In this paper, we consider the same objective considered in [21], where a TLB problem is investigated to minimize the non-productive time. The problem is defined at an automotive company that executes machining operations on the engine cylinder head. The non-productive time involves the tool change time and the face orientation change time. Such nonproductive times significantly influence the makspane of the line and hence they required to be at minimum. We propose a new mathematical programming model to represent the problem. We consider capacity and technological constraints imposed on the manufacturing process of the part. The model aims at finding the optimal configuration of the transfer line that minimizes non-productive time. The model involves binary variables and nonlinear terms which bring difficulties in solving the model directly. To resolve these difficulties, the model is first decomposed using Benders' decomposition approach and then linearized through applying a linearization scheme. The TLB problem has not been tackled before this way, therefore the main contribution of this paper lies in introducing a new approach providing exact solutions of TLB problems.

The paper is organized as follows. The problem is described in section 2. Section 3 discusses the proposed mathematical model. Section 4 shows the linearization-decomposition based algorithms. Computational experiments are given in Section 5.

# International Journal of Mechanical, Industrial and Aerospace Sciences <br> ISSN: 2517-9950 <br> Vol:7, No:2, 2013 

Section 6 provides the summary and the conclusions of the paper, and also gives potential extensions for future research.

## II. Problem Description

The transfer line under study is established to manufacture automotive engine cylinder heads. A cylinder head is composed of a number of design features, in which each of them requires a number of machining operations to be performed. These design features are located at different faces of the head. The cylinder head is mounted on the CNC machine through a titling fixture. The fixture can be oriented in all directions to reach different faces of the head. An orientation change is required when the next operation is done on a different face. Machining operations are executed by different cutting tools. Hence a tool change takes place when the next operation requires a different tool. The orientation change time and the tool change time affect the makespan of the transfer line. So they are required to be at minimum.

The optimal line configuration that minimizes the nonproductive time is our focus in this study. The line configuration is specified by the number of sequential stations in the line, the number of machines in each station, the design features assigned to each station, and the sequence of operations in each station. The problem involves different kinds of constraints concerning capacity restrictions and technological requirements. Capacity restrictions represent the limits on the number of machines in each workstation, the available time of each machine (the given cycle time), where all the machines are identical and have the same cycle time. Technological constraints concern the precedence relationship of the machining operations of each design feature, and the inclusion and exclusion restrictions among design features.

## III. Mathematical Representation

The problem defined in the previous section is represented by the mathematical model shown in equations (1) - (16). The model determines the number of workstations from the available candidates, the design features and the number of parallel machines assigned to each formed workstation. The model also specifies the sequence of machining operations in each workstation. Indices, parameters and decision variables considered in the model are defined below.

## Indices

$g$ : Index set of stations, $g=1, \ldots ., G$
$r$ : Index set of design feature, $\mathrm{r}=1, \ldots, R$
$o$ : Index set of the operations to be performed on each feature $r, o=1, \ldots, O^{r}$
$s$ : Index set of positions on the processing sequence in each station, $s=1, \ldots, S_{g}$

## Parameters

$F_{r r^{\prime}}$ : Time for orientation change between feature $r$ and feature $r^{\prime}$
$T_{\text {ror'or' }}$ :Time for tool change between processing operation $o$ on
feature $r$ and operation $o^{\prime}$ on feature $r^{\prime}$
$D_{r}:$ Total number of design featurer to be processed (demand)
$T O_{r 0}:$ Time for processing opeartion $o$ on feature $r$ $B_{r o}$ :Refixturing time for processing opearation $o$ on feature $r$ $U_{g}$ :Maximum number of machines allowed in station $g$

## $E$ :Available time of a machine

$I_{r r^{\prime}}: 0-1$ matrix, 1 if design features $r$ and $r^{\prime}$ must be assigned to one station and feature $r$ must precede feature $r^{\prime}$. $N I_{r r^{\prime}}: 0-1$ matrix, 1 if design features $r$ and $r^{\prime}$ must not be assigned to one station

## Decision Variables

$Z_{r g}$ :Equals 1 if design feature $r$ is assigned to station $g$ and 0 otherwise
$X_{\text {rosg }}$ : Equals lif process $o$ required for design feature $r$ is assigned to position $\sin$ station $g$ and 0 otherwise

## $S_{g}:$ Number of design features assigned to station $g$

$N_{g}$ : Number of machines that should exist in station $g$

$$
\begin{align*}
& \text { Min NPT }=\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{r^{\prime}=1}^{R} \sum_{o=1}^{O^{r}} \sum_{o^{\prime}=1}^{O^{\prime}} \sum_{s=1}^{S_{g}-1} F_{r r^{\prime}} X_{\text {rosg }} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g} \\
& +\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{o=1}^{o^{r}-1} \sum_{s=1}^{S_{g}-1} T_{\text {ror }(0+1)} X_{\text {rosg }} X_{r(o+1)(s+1) g} Z_{r g}+  \tag{1}\\
& \sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{o=1}^{O^{r}} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o^{\prime}=1}^{O^{\prime}} \sum_{s=1}^{S_{g^{\prime}}-1} T_{\text {ror } 0^{\prime}} X_{\text {rosg }} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g}
\end{align*}
$$

## Subject to

$$
\begin{align*}
& \sum_{g=1}^{G} Z_{r g}=1  \tag{2}\\
& I_{r r^{\prime}}\left(Z_{r g}-Z_{r^{\prime} g}\right)=0 \quad g=1, \ldots, G,  \tag{3}\\
& r=1, \ldots, R, r^{\prime}=1, \ldots, R: r^{\prime}>r \\
& N I_{r r^{\prime}}\left(Z_{r g}+Z_{r^{\prime} g}\right) \leq 1 \quad g=1, \ldots, G, \\
& r=1, \ldots, R, r^{\prime}=1, \ldots, R: r^{\prime}>r  \tag{4}\\
& \sum_{r=1}^{R} O^{r} Z_{r g}=S_{g}  \tag{5}\\
& g=1, \ldots, G \\
& \sum_{s=1}^{S_{g}} X_{\text {rosg }}=Z_{r g} \quad r=1, \ldots, R, o=1, \ldots, O^{r}, g=1, \ldots, G  \tag{6}\\
& \sum_{r=1}^{R} \sum_{o=1}^{O^{r}} X_{\text {rosg }} Z_{r g}=1 \quad g=1, \ldots, G, s=1, \ldots, S_{g} \tag{7}
\end{align*}
$$

$$
\begin{align*}
& \sum_{r=1}^{R} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o=1}^{O^{r}} \sum_{o^{\prime}=1}^{O^{r^{\prime}}} \sum_{s=1}^{S_{g}-1} F_{r r^{\prime}} X_{r o s g} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g}+ \\
& \sum_{r=1}^{R} \sum_{o=1}^{O^{r}-1} \sum_{s=1}^{S_{g}-1} T_{r o r(o+1)} X_{r o s g} X_{r(o+1)(s+1) g} Z_{r g}  \tag{8}\\
& +\sum_{r=1}^{R} \sum_{o=1}^{O^{r}} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o^{\prime}=1}^{O^{r^{\prime}}} \sum_{s=1}^{S_{g}-1} T_{r o r^{\prime} o^{\prime}} X_{r o s g} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g}+ \\
& \sum_{r=1}^{R} \sum_{o=1}^{O^{r}} D_{r}\left[T O_{r o}+B_{r o}\right] Z_{r g} \leq E N_{g} \quad g=1, \ldots, G \\
& N_{g} \leq U_{g} \quad g=1, \ldots, G  \tag{9}\\
& \sum_{s^{\prime}=1}^{S_{g}} X_{r o s^{\prime} g} \geq \sum_{s^{\prime}=1}^{S_{g}} X_{r(o+1) s^{\prime} g} \quad r=1, \ldots, R, \quad o=1, \ldots, O^{r},  \tag{10}\\
& g=1, \ldots, G, s=1, \ldots, S_{g} \\
& Z_{r g} \quad \text { binary }  \tag{11}\\
& r=1, \ldots, R, g=1, \ldots, G \\
& X_{\text {rosg }} \text { binary } \quad r=1, \ldots, R, o=1, \ldots, O^{r} \text {, } \\
& s=1, \ldots, S_{g} \quad g=1, \ldots, G  \tag{12}\\
& N_{g} \quad \text { integer }  \tag{13}\\
& g=1, \ldots, G \\
& g=1, \ldots, G \tag{14}
\end{align*}
$$

Equation (1) shows the objective function representing the non-productive time $(N P T)$. The first term of the equation depicts the orientation change time. The second and third terms represent the tool change time. A tool change should take place only in the following two cases: (i) between two consecutive operations of a design feature and (ii) between two operations belonging to two different design features.

Equation (2) is the assignment constraint that assigns each design feature to only one station. Inclusion and exclusion constraints are represented by equations (3) and (4) respectively. The number of sequence positions in each station is found by equation (5). Equations (6) and (7) are the assignment constraints that ensure assigning operation $o$ required for feature $r$ to only one position and to ensure that each position is assigned to only one operation, respectively. Equation (8) represents the time capacity constraint where the processing time, the refixuring time, and the non-productive time of each workstation should not exceed the available time of all machines. The processing time and the refixturing time of each operation are depicted in the fourth term of the left hand side of the constraint. Equation (9) shows the upper limit imposed on the number of machines allowed in each
workstation. Precedence relationship between operations of a given design feature is represented by equation (10). The remaining constraints represent the binary and integrality restrictions imposed on the decision variables.

The model involves two complexities. The first complexity is the multiplication of the binary variables, as in the terms involving $X_{r o s g} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g}$ and $X_{r o s g} X_{r(o+1)(s+1) g} Z_{r g}$. The second complexity belongs to the number of sequencing positions $S_{g}$ in a given workstation $g$. The model considers this number as a decision variable while it is required to specify the index $s$ representing the set of sequencing positions in workstation $g$. The following section shows how these two complexities are resolved using the proposed solution algorithm.

## IV. SOLUTION Approach

The problem under study is solved through decomposing the proposed model into two problems using the generalized Benders' decomposition approach introduced by Geoffrion [22]. The first problem, Benders' master problem, is solved to assign design features to workstations and to decide on the formed workstations from the available candidates. The second problem, Benders sub-problem, is solved to determine the sequence of operations in each workstation formed by Benders' master problem. The master problem sends the values of the decision variables $Z_{r g}$ and $S_{g}$ to the sub-problem, then the sub-problem finds the optimal solutions of the other variables given these values of $Z_{r g}$ and $S_{g}$. If the sub-problem is feasible, it sends a Benders' optimality cut to the Benders' master problem. If the sub-problem has no feasible solution, a combinatorial Benders' cut is added to the Benders' master problem in order to generate different values of the decision variable $Z_{r g}$. This decomposition scheme resolves the difficulty implied in the decision variable $S_{g}$ and reduces the multiplication of binary variables $X_{r o s g} X_{r^{\prime} o^{\prime}(s+1) g} Z_{r g} Z_{r^{\prime} g}$ and $X_{r o s g} X_{r(o+1)(s+1) g} Z_{r g}$ to only two variables multiplied together $Z_{r g} Z_{r^{\prime} g}$ in the master problem, and $X_{r o s g} X_{r^{\prime} o^{\prime}(s+1) g}$ and $X_{r o s g} X_{r(o+1)(s+1) g}$ in the sub-problem. These binary multiplications call for applying a linearization scheme to resolve the nonlinearity of these terms.

The objective function of the Benders' master problem considers only the term $\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{r^{\prime}=1}^{R} F_{r r^{\prime}} Z_{r g} Z_{r^{\prime} g}$ from the original objective function. As this term involves the multiplication of two binary variables, it needs applying a linearization scheme to replace it with its equivalent linear term. Therefore, the linearization scheme proposed in [23] is used to linearize this nonlinear term. The scheme replaces the multiplication of the two binary variables $Z_{r g} Z_{r^{\prime} g}$ by a new binary variable $Y_{r g r^{\prime} g^{\prime}}$, and adds the two auxiliary constraints shown in equations (18), (19). The objective function of the

Benders' master problem also involves a function $\alpha$ which provides a lower bound on objective function of the Benders' sup-problem. The Benders' optimality cut is shown in equation (16). The term $\lambda_{r g}^{y}$, appearing in this cut, represents the multiplier representing the effect of the binary variable $Z_{r g}$ on the objective function of the Benders' sub-problem. At each feasible iteration $y$, the Benders' sub-problem is solved for each $Z_{r g}$ to find the multiplier $\lambda_{r g}^{y}$. The combinatorial Benders' cut is shown in equation (17). In addition to these new constraints, the Benders' master problem considers the constraints related to assigning design features to stations, equation (2), and inclusion and exclusion constraints given in equations (3) and (4). The formulation of the Benders' master problem is depicted below.

$$
\begin{equation*}
\text { M in } M P=\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{r^{\prime}=1}^{R} F_{r r^{\prime}} Z_{r g} Z_{r^{\prime} g}+\alpha \tag{15}
\end{equation*}
$$

Subject to
Equations (2), (3), (4), (11)

$$
\begin{gather*}
\alpha \geq B S P^{y}+\sum_{r=1}^{R} \sum_{g=1}^{G} \lambda_{r g}^{y}\left(Z_{r g}-Z_{r g}^{y}\right) \quad y=1, \ldots, Y  \tag{16}\\
\sum_{r=1}^{R} \sum_{g=1: 1 Z_{r g}^{k}=0}^{G} Z_{r g}+\sum_{r=1}^{R} \sum_{g=1: 1 Z_{r g}^{k}=1}^{G} 1-Z_{r g} \geq 1 \quad k=1, \ldots, K  \tag{17}\\
Z_{r g}=\sum_{g^{\prime}=1}^{G} Y_{r g}^{r^{\prime} g^{\prime}} \quad g=1,2, \ldots . G, \quad r=1,2, \ldots, R,  \tag{18}\\
r^{\prime}=1,2, \ldots, R \\
Y_{r g}^{r^{\prime} g^{\prime}}=Y_{r^{\prime} g^{\prime}}^{r g}  \tag{19}\\
g=1,2, \ldots . G, g^{\prime}=1,2, \ldots . G, \quad r<r^{\prime} \\
r=1,2, \ldots . R, r^{\prime}=1,2, \ldots ., R,  \tag{20}\\
Y_{r g}^{r^{\prime} g^{\prime}} \text { is binary } \begin{array}{r}
r=1,2, \ldots . R, r^{\prime}=1,2, \ldots ., R, \\
g=1,2, \ldots . G, g^{\prime}=1,2, \ldots . G
\end{array}
\end{gather*}
$$

The Benders' sub-problem considers the constraints given by equations (5)-(10), (12)-(15). The objective function of this problem is the same objective function (1) except that the variable $Z_{r g}$ is considered as an input parameter. The multiplication of each of the two binary variables $X_{\text {rosg }} X_{r^{\prime} o^{\prime}(s+1) g}$ and $X_{\text {rosg }} X_{r(o+1)(s+1) g}$ should be replaced by one binary variable. Similarly as done before in the Benders' master problem, the linearization scheme adds the new binary variable $W_{\text {gros }}^{r^{\prime} o^{\prime}(s+1)}$ and the new constraints shown in equations (24) and (25). The linearized objective function and
constraints considered in the Benders sub-problem are shown below.

$$
\begin{align*}
& \text { Min } S P=\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o=1}^{O^{r}} \sum_{o^{\prime}=1}^{o^{\prime}} \sum_{s=1}^{S_{g}^{*}-1} F_{r r^{\prime}} W_{g r o s}^{r^{\prime}(s+1)} Z_{r g} Z_{r^{\prime} g}+ \\
& \sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{o=1}^{O^{r}} \sum_{s=1}^{s_{g}^{*}} T_{r o r(o+1)} W_{g r o s}^{r(o+1)(s+1)} Z_{r g}  \tag{21}\\
& +\sum_{g=1}^{G} \sum_{r=1}^{R} \sum_{o=1}^{O^{r}} \sum_{\substack{R}}^{r^{\prime}=1} \sum_{r^{\prime} \neq r}^{o^{\prime}=1} \sum_{s=1}^{s_{g}^{*}-1} T_{r o o^{\prime} o^{\prime}} W_{g r o s}^{r^{\prime} o^{\prime}(s+1)} Z_{r g} Z_{r^{\prime} g}
\end{align*}
$$

Subject to
Equations (5)-(10), (12)-(15)

$$
\begin{align*}
& \sum_{r=1}^{R} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o=1}^{O^{r}} \sum_{o^{\prime}=1}^{O^{r}} \sum_{s=1}^{s_{s}^{*}-1} F_{r r^{\prime}} W_{g r o s}^{r^{\prime} r^{\prime}(s+1)} Z_{r g} Z_{r^{\prime} g}+ \\
& \sum_{r=1}^{R} \sum_{o=1}^{o^{r}-1} \sum_{s=1}^{s_{g}^{*}} T_{\text {ror }(o+1)} W_{\text {gros }}^{r(o+1)(s+1)} Z_{r g}  \tag{22}\\
& +\sum_{r=1}^{R} \sum_{o=1}^{O^{r}} \sum_{\substack{r^{\prime}=1 \\
r^{\prime} \neq r}}^{R} \sum_{o^{\prime}=1}^{o^{\prime}} \sum_{s=1}^{o_{s}^{*}-1} T_{\text {ror } o^{\prime}} W_{g r o s}^{r^{\prime} o^{\prime}(s+1)} Z_{r g} Z_{r^{\prime} g} \\
& +\sum_{r=1}^{R} \sum_{o=1}^{O^{r}} D_{r}\left[T O_{r o}+B_{r o}\right] Z_{r g} \leq E N_{g} \quad g=1, \ldots, G \\
& Z_{r g}=Z_{r g}^{*} \quad: \lambda_{r g} \quad g=1, \ldots G, r=1, \ldots R  \tag{23}\\
& X_{r^{\prime} O^{\prime} g s^{\prime}}=\sum_{r=1}^{R} \sum_{o=1}^{O^{r}} W_{\text {gros }}^{r^{\prime} \sigma^{\prime} s^{\prime}} \quad g=1,2, \ldots . G, s=1,2, \ldots . S_{g},  \tag{24}\\
& r^{\prime}=1,2, \ldots, R, o^{\prime}=1,2, \ldots . . O^{r^{\prime}}, s^{\prime}=1,2, \ldots . S_{g}: s \neq s^{\prime} \\
& W_{\text {gros }}^{r^{\prime} s^{\prime} s^{\prime}}=W_{\text {gr'o's's }}^{\text {ros }} \quad g=1,2, \ldots . G, s=1,2, \ldots . S_{g}, \\
& r^{\prime}=1,2, \ldots, R, o^{\prime}=1,2, \ldots . O^{r^{\prime}}, s^{\prime}=1,2, \ldots . S_{g}: s \neq s^{\prime}  \tag{25}\\
& W_{\text {gros }}^{r^{\prime} \text { 's' }^{\prime}} \text { binary } \quad g=1,2, \ldots . G, s=1,2, \ldots . S_{g} \text {, } \\
& r^{\prime}=1,2, \ldots, R, \quad o^{\prime}=1,2, \ldots . O^{r^{\prime}}, s^{\prime}=1,2, \ldots . S_{g}: s \neq s^{\prime} \tag{26}
\end{align*}
$$

The Benders' sub-problem is separable for each workstation $g$ which leads to decomposing the Benders' sub-problem into $G$ sub-problems and then solving the sequencing problem of each workstation $g$ individually. The iterations between the master problem and the sub-problem stop when the value of function $\alpha$, representing the lower bound of the objective function (1), is greater than or equal to the value of the
objective function of the Benders' sub-problem which represents the upper bound of objective function (1).

## V. Computational Study

The computational experiments were implemented on Intel i7 CPU 870@2.93 GHz and 8 MP RAM. The algorithm is coded using AMPL. The commercial solver used to solve the mathematical models is CEPLX. Table 1 shows the uniform distributions used to randomly generate values of the parameters of the proposed model.

TABLE 1
Parameter Settings in the Tested Problems

| Parameter | Data range | Parameter | Data range |
| :--- | :--- | :--- | :--- |
| $F_{r r^{\prime}}$ | Uniform(1,5) sec | $T O_{r o}$ | Uniform(1,5) sec |
| $T_{r o r^{\prime} o^{\prime}}$ | Uniform(2,7) sec | $B_{r o}$ | Uniform(1,2) sec |
| $D_{r}$ | Uniform(5,12) unit | $U_{g}$ | Uniform(3,5) <br> machine |
| $E$ | Uniform(330,550) sec | $V_{g}$ | Uniform(5,10) sec |
| $U_{g}$ | Uniform(2,5) machine | $O^{r}$ | Uniform(2,4) <br> operations |

Table II illustrates the configuration of the six problem instances tested in our study. The second column depicts the number of design features, the number of operations, and the number of candidate workstations, respectively given by $R / O$ $/ G$. The third and fourth columns show the inclusion and exclusion restrictions imposed on the design features. For example, in the first problem, design features 1 and 2 must be processed in one workstation, while design feature 3 should be processed in a workstation different from the one processing design feature 1. The last column of the table gives the solution time of the algorithm. All the six problems are solved to optimality, since the upper bound given by the objective function of the Benders' sub-problem equals the lower bound given by function $\alpha$ represented in the Benders' master problem. As shown by the solution time, small instances are solved in seconds but the time increases to hours in larger instances. The algorithm failed to solve problems comprising more than 40 processes even when we allow 6 hours. This calls for developing an efficient metaheuristic algorithm to solve larger problem instances containing 25 to 30 design features with 80 to 100 operations. In this case, our algorithm can be used to evaluate the efficiency of the metaheuristic algorithms in reaching optimal solutions.

TABLE II
Configuration and Solution Time of the Tested Problems

| Problem <br> number | $R / O / G$ | Inclusion <br> relationships | Exclusion <br> relationships | Solution tim |
| :---: | :---: | :--- | :--- | :--- |
| 1 | $4 / 10 / 2$ | $[1,2]$ | $[1,3]$ | 5.2 sec |
| 2 | $6 / 17 / 2$ | $[1,4],[2,5]$ | $[3,5],[4,6]$ | 50.4 sec. |
| 3 | $8 / 25 / 2$ | $[3,4],[5,7]$ | $[2,7],[6,8]$ | 20.1 min. |
| 4 | $10 / 29 / 3$ | $[2,4],[3,5]$ | $[6,10]$ | 2.8 hr. |
| 5 | $12 / 35 / 3$ | $[1,4],[2,6]$, <br> $[4,5],[3,9]$ | $[6,9],[6,10]$, <br> $[9,10]$ | 3.4 hr. |
| 6 | $14 / 40 / 3$ | $[5,6],[5,9]$, <br> $[10,13]$ | $[1,8],[6,13]$, <br> $[8,10]$ | 3.7 hr. |

## VI. Summary, Conclusions, and Future Extensions

A transfer line balancing problem is investigated in this paper. This problem is very common in the manufacturing lines of automotive parts such as cylinder heads and cylinder block. We focus on minimizing the non-productive time associated with tool change and face orientation change. A new mathematical model is proposed to specify the optimal line configuration achieving the minimum non-productive time. We develop an algorithm using linearization schemes and Benders' decomposition approach to solve the model to optimality. Results given by a computational study show the efficiency of the proposed algorithm in solving small and medium problem sizes in relatively short time.

The problem can be extended in many directions. The transportation time of moving parts from one workstation to another can be considered in a future work. This nonproductive time, unlike the tool change time and the face orientation change time, favors processing the design features in a minimum number of workstations. In some transfer lines, there exists a precedence relationship among the design features, rather than the one considered in this paper which states precedence relations between the manufacturing processes. In addition, tool life limitation and tool magazine capacity may restrict the assignment of design features to workstations. All of these constraints could be added to our proposed model in a future work. Lastly, the experiments in this paper depicted the need for developing a metaheurisitc algorithm, i.e. genetic algorithm or ant colony optimization algorithm, to solve the large sizes of the defined problem in short time.

## AcknowLedgment

This research is supported by Tri-Success Grant Program (TSGP) 2011, University of Windsor.

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