A Fuzzy Multi-objective Model for a Machine Selection Problem in a Flexible Manufacturing System

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Abstract—This research presents a fuzzy multi-objective model for a machine selection problem in a flexible manufacturing system of a tire company. Two main objectives are minimization of an average machine error and minimization of the total setup time. Conventionally, the working team uses trial and error in selecting a pressing machine for each task due to the complexity and constraints of the problem. So, both objectives may not satisfy. Moreover, trial and error takes a lot of time to get the final decision. Therefore, in this research preemptive fuzzy goal programming model is developed for solving this multi-objective problem. The proposed model can obtain the appropriate results that the Decision Making (DM) is satisfied for both objectives. Besides, alternative choice can be easily generated by varying the satisfaction level. Additionally, decision time can be reduced by using the model, which includes all constraints of the system to generate the solutions. A numerical example is also illustrated to show the effectiveness of the proposed model.

Keywords—Machine Selection, Preemptive Fuzzy Goal Programming, Mixed Integer Programming, Application of Tire Industry.

I. INTRODUCTION

THE market conditions of a manufacturing industry are changing to more dynamic, more global and more customized driven. The manufacturing performance is no longer driven by the production cost. In contrast, it affects by quality, flexibility, delivery and customer service, which have become equally important [1]. Manufacturing companies need to distinguish themselves by increasing product quality, reducing manufacturing lead time and enhancing flexibility to the changing market.

Flexible manufacturing system (FMS) is the system that has the ability to produce a diverse range of parts efficiently and the capability to respond quickly to the part-mix changes [2]. The decisions related to FMS operations are of two types: pre-release and post-release decisions. Pre-release decisions include the FMS planning problem that deals with the pre-arrangements of jobs and tools before the processing begins, whereas post-release decisions deal with the scheduling problem of FMS [1]-[6]. Decision problems of post-release are part type selection, machine grouping, determination of production ratios, batching of the part type, allocation of tools among machines and operations, i.e. loading problem [1]. Machine loading problem in particular deals with the allocation of jobs to various machines under technological constraints. It can be divided a machine loading problem into five sub-problems: machine grouping, a part type selection, production rate determination, resource allocation and loading [5]. Formulation of all these problems in a single mathematical model may not be possible, it leads to a complex mathematical model, whose solution may be difficult to determine. Normally, integer programming, mixed-integer programming, dynamic programming, branch and bound models were developed for such kind of problems with different kind of objectives such as minimization of costs, minimization of setup time, minimization of the total system unbalance or balancing the workload per machine [3],[5]-[7]. Most of them consider a single objective function. However, in some case multiple objective functions are necessary. Heuristic methods were also presented due to the complexity of the problem in finding the optimal solution [6], [3]. They are largely based upon rules and rely on empirical experiences. Therefore, one of the limitations of a heuristic approaches is its difficulty to approximate results in a new or completely changed environment [7].

The case study company is one of the leading manufacturers of truck and bus tires. It is trying to increase its competitiveness by improving product quality and reducing production time. The Curing department is a crucial department that should be emphasized because it is the process that high product's quality can be produced. Moreover, it is also the bottleneck of the factory. Product quality is extremely important for the company because the product is related to customer safety and prestige of the company. Resources in the Curing department are high-priced machines including tire curing machines or pressing machines. These resources are limited resources. Therefore, the selection of pressing machines for ordering products is very important because it can increase both product quality and can reduce setup time or production lead time of the whole factory. So, two objective functions are considered. They are minimization of an average error of machines and minimization of the total setup time. To solve a Multiple Objective problem, there are several methods used in general such as fuzzy linear programming [8]-[15], compromise programming [16], [17], interactive approaches [13], [17], etc. However, the most popular one is Goal Programming (GP) [16]-[20]. In GP, a precise target is set for each objective as a goal. But, it is difficult for Decision Maker (DM) to clearly desire targets or goals. The Fuzzy Goal Programming (FGP) makes easiness by allowing vague aspirations of the DMs, which is suitable for the case study problem because target values of both objectives are unclear. Preemptive Fuzzy Goal Programming (P-FGP) has been applied to the problem. P-FGP is suitable for this problem since the first goal is extremely important than the second

goal. Additionally, setting the membership function for each goal makes easiness for DM in adjustment and decision.

The remainder of this paper is organized as follows. The problem description is discussed in Section II. Then, model formulation is illustrated in Section III. A case study is shown in Section IV. Finally, the conclusion of this research is provided in Section V.

II. PROBLEM DESCRIPTION

The case study company is one of the leading tire producers. It is trying to increase product quality and its productivity. The main process is the Curing Department, which is also the bottleneck of the factory. The process of this department is very crucial because it can improve product quality and can reduce production lead time. It is the assignment of the appropriate pressing machines and molds to all jobs in each week. There are two types of pressing machine; double-pressing machine (Type A) and single-pressing machine (Type B) and two types of mold; double-mold (Type X) and single mold (Type Y). A double-mold, a single mold and two single molds can be used in a double-pressing machine. But, only single-mold can be worked in a single-pressing machine. Each pressing machine and mold have different quality of each product. Therefore, the selection of a pressing machine and a mold for each task is very important. Availability of pressing machines and molds for production are also limited due to high-priced machines and molds. Moreover, mold changing time depends on the selected pressing machine and mold. Additionally, a rule of selecting a mold for a double-pressing machine should be followed, called *Cure law*. The rule mentions that the difference of cure time of molds which are selected for double-pressing machine should be less than 2 minutes/time. Otherwise, the high quality tire cannot be obtained. This rule is used for preventing uncooked rubber. Cured tires' quality depends on a uniformity of rubber in each pressing machine for each size and a geometry error of each pressing machine, which lead to the quality level of cured tires.

Presently, production planners need to consolidate all data and set a decision meeting among working team every time the plan is changed due to the limitations mentioned above. Information obtained from the Production planning department is customer demand, rough production plan and available molds and pressing machines of that period. The main objective of the factory is the quality of cured tires. Consequence of the changing molds plan may take time if the working team wants to reach an optimal quality. However, setup time is also important. It needs to compromise by a combination of DM's experience and the current situation, which may not be effective and time-consuming. The reduction of machine errors may not be satisfied and processing time is also unpredictable.

In order to reduce processing time, the selection of double-pressing machines should be firstly done. Afterward, the selection of single-press is processed. So, two models for the selection of couple-pressing machines and the selection of single-pressing machine are constructed.

III. MATHEMATICAL MODEL

In machine selection of the case study factory, two objectives are determined. Firstly, an average uniformity and geometry errors of all tasks should be minimized to ensure high product quality. Secondly, the total setup time should also be reduced to decrease the production lead time of the bottleneck process.

Notations of models can be represented as follows:

Index

i : Couple-pressing machine, $i = 1,\ldots,m_1$

j : Product of couple-pressing machine, $j = 1,\ldots,n_1$

k: Couple-mold or a single-mold that can match with another single-mold with acceptable quality, $k = 1,\ldots,o_1$

l : Single-pressing machine, $l = 1,\ldots,m_2$

g : Product of single-pressing machine, $g = 1,\ldots,n_2$

s: Single-mold, $s = 1,\ldots,o_2$

Decision Variables

$x_{ij} =$ 1 if a couple-pressing machine i is assigned for product j,

0 otherwise.

$y_{kj} =$ 1 if a couple-mold k is assign for product j,

0 otherwise.

$x_{lg} =$ 1 if a single-pressing machine l is assigned for product g,

0 otherwise.

$y_{sg} =$ 1 if a single-mold s is assign for product g,

0 otherwise.

Parameters

U_{ij} Uniformity error of a couple-pressing machine i for product j.

U_{lg} Uniformity error of a single-pressing machine l for product g.

G_i Average geometry error of a couple-pressing machine i.

G_l Average geometry error of a single-pressing machine l.

A The number of double-molds specified for changing.

B The number of single-molds specified for changing.

S_{ij} Size of a double-pressing machine i for product j.

S_{lg} Size of a single-pressing machine l for product g.

Z_{kj} Size of a double-mold k for product j.

Z_{sg} Size of a single-mold s for product g.

A. Multi-objective Model for Couple-Pressing Machine and Single- Pressing Machine

The selection of pressing machines and molds are based on types of pressing machine. Firstly, the selection of double-pressing machines is performed in order to reduce the production lead time because two molds can be assigned at the same time. Then, the selection of single-pressing machines is made for the remaining jobs. Some information about

availability of pressing machines and molds in each period and production plan in each week should be prepared before using the proposed models.

The integer programming model for the selection of couple-pressing machine can be mathematically represented by

$$\text{Minimize } z_1 = \frac{1}{n_1}\left[\sum_{i=1}^{m_1}\sum_{j=1}^{n_1} U_{ij}x_{ij} + \sum_{i=1}^{m_1}\sum_{j=1}^{n_1} G_i x_{ij}\right] \quad (1)$$

$$\text{Minimize } z_2 = \sum_{i=1}^{m_1}\sum_{j=1}^{n_1} T_{ij}x_{ij} \quad (2)$$

subject to

$$\sum_{j=1}^{n_1} x_{ij} = 1, \quad \forall i \quad (3)$$

$$\sum_{j=1}^{m_1} x_{ij} \geq 1, \quad \forall j \quad (4)$$

$$\sum_{i=1}^{m_1} x_{ij} = 2\sum_{k=1}^{o_1} y_{kj}, \quad \forall j \quad (5)$$

$$S_{ij}x_{ij} \geq Z_{kj}y_{kj}, \quad \forall j, \forall k \quad (6)$$

$$\sum_{k=1}^{o_1} y_{kj} = A \quad \forall j \quad (7)$$

The two objectives of the model are represented by Eq.(1)-(2). The first objective is to minimize an average error of pressing that comes from uniformity error and geometry error of selected double-pressing machines. The second objective is to minimize the total processing time. Each pressing machine can be assigned to only one job, but each job can assign to more than one pressing machine as shown in Eqs. (3)-(4). Two molds are used in a couple-pressing machine as shown in Eq.(5). The size of a pressing machine should be larger than molds assigned to the pressing machine for all jobs and for all molds as shown in Eq.(6). The number of couple-molds or couples of single-molds that can be matched with acceptable quality should be equal to the number of molds specified in the production plan as represented by Eq.(7).

By the same way, the integer programming model for selecting the single-pressing machine can be mathematically represented by

$$\text{Minimize } z_3 = \frac{1}{n_2}\left[\sum_{l=1}^{m_2}\sum_{g=1}^{n_2} U_{lg}x_{lg} + \sum_{l=1}^{m_2}\sum_{g=1}^{n_2} G_l x_{lg}\right] \quad (8)$$

$$\text{Minimize } z_4 = \sum_{l=1}^{m_2}\sum_{g=1}^{n_2} T_{lg}x_{lg} \quad (9)$$

subject to

$$\sum_{g=1}^{n_2} x_{lg} = 1, \quad \forall l \quad (10)$$

$$\sum_{l=1}^{m_2} x_{lg} \geq 1, \quad \forall g \quad (11)$$

$$\sum_{l=1}^{m_2} x_{lg} = \sum_{s=1}^{o_2} y_{sg}, \quad \forall s \quad (12)$$

$$S_{lg}x_{lg} \geq Z_{sg}y_{sg}, \quad \forall g, \forall s \quad (13)$$

$$\sum_{s=1}^{o_2} y_{sg} = B \quad \forall g \quad (14)$$

The first objective is to minimize average errors of pressing of single-pressing machines, represented by Eq.(8). The second objective is to minimize the total processing time as shown in Eq.(9). A pressing machine can be assigned to only one job but each job can be assigned to more than one pressing machine as shown in Eqs.(10)-(11). One mold is used in a single-pressing machine as shown in Eq.(12). Size of pressing machine should be larger than assigned molds for all jobs and for all molds, denoted by Eq.(13). The number of single-molds should be equal to the number of molds specified in the production plan as exhibited in Eq.(14).

B. Preemptive Fuzzy Goal Programming

In many Multiple Objective Decision Making (MODM) problems, some goals are extremely important than the others. So, the DM cannot simultaneously consider the attainments of all goals. Differentiating goals into different levels of importance, in which higher level goal must firstly be satisfied before the low level goals get consideration, is called preemptive or lexicographic ordering. The fuzzy goal programming with a priority structure for ordering goals is called "Preemptive Fuzzy Goal Programming (P-FGP)" [19], [20]. The P-FGP model can be shown as follows,

$$\text{lex max} = [p_1 f_1(\lambda), p_2 f_2(\lambda),...,p_t f_t(\lambda)], \quad (15)$$

subject to

$$\lambda_k + \delta_k^- - \delta_k^+ = \lambda_k^*, \quad \text{for all } k. \quad (16)$$

$$\delta_k^-, \delta_k^+ \geq 0, \quad \text{for all } k. \quad (17)$$

$$\delta_k^- \delta_k^+ = 0, \quad \text{for all } k. \quad (18)$$

$$\lambda_k \in [0,1] \quad \text{for all } k. \quad (19)$$

Where λ_k is the satisfactory level of goal k. λ_k^* is the acceptable satisfactory level of goal k. δ_k^+ and δ_k^- are the positive and negative deviations of the satisfactory level of goal k.

In the P-FGP, with assumed triangular membership function and that there exist T priority levels (each priority may include m_k goals for $k = 1,2,...,K$) that preemptive weights are $p_t >>> p_{t+1}$ whereas $f_t(\lambda)$ is the satisfactory function of priority t. The problem is then partitioned into T sub-problems or T fuzzy goal programming. For easiness, the goals are ranked in agreement with the following rule: if $r < s$, then the goal set $G_r(x)$ has higher priority than the goal set $G_s(x)$ [20].

In the case study both objective functions are imprecise depending on DM's preference. However, the first objective (to minimize the average errors of pressing) is extremely more

important than the second objective (to minimize the total processing time) because of customer safety, prestige of the company and high product cost so quality should be ensured. Then, P-FGP is applied in the proposed model.

C. Membership Functions

In this research, fuzzy set is applied to each goal of the objective function. Defining the membership function of each goal is based on the Positive-Ideal Solution (PIS) and the Negative-Ideal Solution (NIS) [18]-[19]. The PIS is the best possible solution when each objective function is optimized. The NIS is the feasible worst value of each objective function. So, the PIS is used to set the most preferred value and has the satisfactory degree of 1. By the same way, the satisfactory degree of 0 is assigned to the NIS. Acceptable deviation from the goal can be calculated from the difference between PIS and NIS or it can be evaluated by DM. Then, the triangular membership function of the kth goal based on the DM's preference can be shown as Fig.1. Mathematical representation of the membership function can be shown as follows Eq.(20).

$$\mu(z_k) = \begin{cases} 0 & \text{, if } z_k \leq \tau_k - \Delta_k \\ 1 - \left(\dfrac{\tau_k - z_k}{\Delta_k} \right) & \text{, if } \tau_k - \Delta_k \leq z_k \leq \tau_k \\ 1 - \left(\dfrac{z_k - \tau_k}{\Delta_k} \right) & \text{, if } \tau_k \leq z_k \leq \tau_k + \Delta_k \quad (20) \\ 0 & \text{, if } z_k \geq \tau_k + \Delta_k \end{cases}$$

where $\mu(z_k)$ is the membership function of the kth goal. τ_k is the specified target for the kth goal and assigned by the PIS. $\Delta_k = |\text{PIS-NIS}|$ is the acceptable deviation of kth goal.
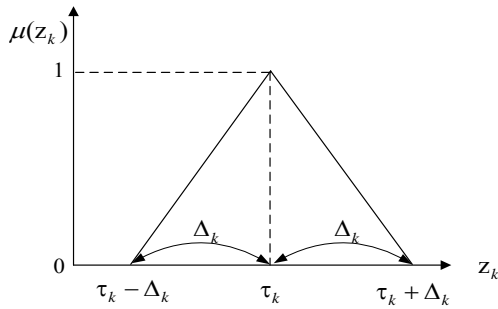

Fig. 1 The membership function of kth goal

D. Model Formulation

As mentioned previously, the proposed model has two goals to be considered. In the P-FGP, we need to satisfy the satisfactory level (λ_k) of each goal. These are the satisfactory level of both goals. Moreover, the first goal is defined more important than the second goal. So, two priority levels are constructed. Fuzzy goal equations can be derived as follows,

$$Z_1 + \Delta_1 \left(\delta_1^- - \delta_1^+ \right) = \tau_1 \quad (21)$$

$$Z_2 + \Delta_2 \left(\delta_2^- - \delta_2^+ \right) = \tau_2 \quad (22)$$

Then, the Fuzzy Multiple Objective Decision Making (FMODM) model can be shown as,

$$\text{lex max} = [\lambda_1, \lambda_2], \quad (23)$$

subject to

$$\lambda_k + \delta_k^- - \delta_k^+ = \lambda_k^*, \quad \text{for all } k. \quad (24)$$

$$\lambda_k \leq \mu(z_k), \quad \text{for all } k. \quad (25)$$

Then, FMODM model can be adapted to the multi objective problem of pressing machine and mold selection. Two models are constructed. They are solved consecutively. These models can be represented as follows:

FMODM for double-pressing machine and mold selection

$$\text{lex max} = [\lambda_1, \lambda_2],$$

subject to

$$\lambda_k + \delta_k^- - \delta_k^+ = \lambda_k^*, \quad k = 1 \quad (26)$$

$$\lambda_k \leq \mu(z_k), \quad k = 1,2. \quad (27)$$

$$\frac{1}{n_1} \left[\sum_{i=1}^{m_1} \sum_{j=1}^{n_1} U_{ij} x_{ij} + \sum_{i=1}^{m_1} \sum_{j=1}^{n_1} G_i x_{ij} \right] + \Delta_1 \left(\delta_1^- - \delta_1^+ \right) = \tau_1 \quad (28)$$

$$\sum_{i=1}^{m_1} \sum_{j=1}^{n_1} T_{ij} x_{ij} + \Delta_2 \left(\delta_2^- - \delta_2^+ \right) = \tau_2 \quad (29)$$

$$\delta_k^-, \delta_k^+ \geq 0, \quad \forall k \quad (30)$$

$$\delta_k^- \delta_k^+ = 0, \quad \forall k \quad (31)$$

$$\lambda_k \in [0,1], \quad \forall k \quad (32)$$

$$(3) - (7)$$

FMODM for Single-Pressing Machine and Mold Selection

$$\text{lex max} = [\lambda_3, \lambda_4],$$

subject to

$$\lambda_k + \delta_k^- - \delta_k^+ = \lambda_k^*, \quad k = 3 \quad (33)$$

$$\lambda_k \leq \mu(z_k), \quad k = 3,4. \quad (34)$$

$$\frac{1}{n_2} \left[\sum_{l=1}^{m_2} \sum_{g=1}^{n_2} U_{lg} x_{lg} + \sum_{l=1}^{m_2} \sum_{g=1}^{n_2} G_l x_{lg} \right] + \Delta_3 \left(\delta_3^- - \delta_3^+ \right) = \tau_3 \quad (35)$$

$$\sum_{l=1}^{m_2} \sum_{g=1}^{n_2} T_{lg} x_{lg} + \Delta_4 \left(\delta_4^- - \delta_4^+ \right) = \tau_4 \quad (36)$$

$$\delta_k^-, \delta_k^+ \geq 0, \quad \forall k \quad (37)$$

$$\delta_k^- \delta_k^+ = 0, \quad \forall k \quad (38)$$

$$\lambda_k \in [0,1], \quad \forall k \quad (39)$$

$$(10) - (14)$$

IV. A CASE STUDY

Pressing machine selection by the proposed model is shown in the following example. There are two plans from the production planning department; a new entry plan and a plan for existing molds. A new entry plan is a plan that is prepared for representing jobs and molds those need to be assigned to

available pressing machines in each week as shown in Table I.

TABLE I
NEW ENTRY PLAN

Job No.	Mold in	No. of mold plan (A,B)	No .of mold in storage		
			Type A	Type B	Total
1	Size 102	4 (B)	5	9	14
2	Size 141	2 (B)	0	2	2
3	Size 18	2 (B)	0	4	4
4	Size 138	1 (A)	1	8	9
	Total	9	6	23	29

A plan for existing molds is shown the using molds those will be removed and replaced by the molds plan for new entry jobs as shown in Table II.

Mold for each job of the new entry plan is assigned according to type of product. In Table I, nine molds are considered to enter. New entry molds should match with available pressing machines those have exiting molds in Table II. In this example, there are four couple-molds (eight molds) and one single-mold (total nine molds) to be assigned in this week. Two couple-molds for size 102, a couple-mold for size 141, a couple-mold for size 18 and a single-mold for size 138. These molds should be efficiently assigned to the available pressing machines in Table II according to technological constraints.

The available pressing machines are listed in Table II. The number of an entry mold is firstly verified, so there is not any problem about lacking of entry molds. There are twenty-two molds available, which consist of four single-molds and nine couple-molds (totally twenty-two molds). Four couple-molds and one single-mold are assigned to the new entry jobs for this week. Then, the remaining molds are 13 molds.

TABLE II
PLAN FOR EXITING MOLDS

Mold list in existing	Pressing machine name	No. of existing mold		No. of exit mold	No. of remaining mold
		Type A	Type B		
Size 25	D1-2	-	2	2	2
Size 25	D15-16	-	2		
Size 132	D9-10	-	2	4	2
Size 132	I7-8	-	2		
Size 132	K1-2	-	2		
Size 133	C3-4	-	2	2	2
Size 133	C7-8	-	2		
Size 137	I1-2	-	2	1	7
Size 137	J3-4	-	2		
Size 137	F02	1	-		
Size 137	F03	1	-		
Size 137	F06	1	-		
Size 137	G01	1	-		
	Total	4	18	9	13
	Grand Total	22		9	13

TABLE III
DIMENSION OF PRESSING MACHINES AND MOLDS

M/c Type	Pressing machine name	Pressing machine Dimension ($S_{ij} S_{lg.}$)	Dimension of mold for each size ($Z_{ij}, Z_{lg.}$)			
			Size 18	Size102	Size 141	Size 25
			A ≥301 B ≥55	A ≥ 301 B ≥63.5	A ≥301 B ≥55	
B	C3-4	55	/	x	/	/
	C7-8	55	/	x	/	/
	D1-2	65	/	/	/	/
	D9-10	63.5	/	/	/	/
	D15-16	63.5	/	/	/	/
	I1-2	63.5	/	/	/	/
	I7-8	63.5	/	/	/	/
	J3-4	63.5	/	/	/	/
	K1-2	63.5	/	/	/	/
A	F02	401	/	/	/	/
	F03	401	/	/	/	/
	F06	301	/	/	/	/
	G01	301	/	/	/	/

Note : (x)cannot assign (/) can assign

TABLE IV
MOLD CHANGING TIME AND % ERROR OF A COUPLE –MOLD
ON EACH PRESSING MACHINE

Pressing machine name	Time to change size of mold (T_{ij},)			%Error of each mold ($U_{ij}(x_{ij}) + G_i(x_{ij})$)		
	Size 18	Size 102	Size 141	Size 18	Size 102	Size 141
C3-4	460	560	520	0.04	0.09	0.08
C7-8	460	560	520	0.29	0.33	0.24
D1-2	440	520	460	0.07	0.115	0.02
D9-10	460	560	520	0.295	0.32	0.335
D15-16	440	520	460	0.26	0.365	0.29
I1-2	460	460	440	0.33	0.36	0.425
I7-8	460	560	520	0.22	0.28	0.295
J3-4	460	460	440	0.335	0.35	0.29
K1-2	460	560	520	0.05	0.03	0.085

TABLE V
MOLD CHANGING TIME AND %ERROR OF EACH SINGLE –MOLD
ON EACH PRESSING MACHINE

Pressing machine name A	Time to size change per each mold (T_{lg},)	%Error of each mold ($U_{lg}(x_{lg}) + G_i(x_{lg})$)
	Size 138	Size 138
F02	160	0.33
F03	160	0.27
F06	160	0.26
G01	160	0.16

In order to assign entry molds to the suitable pressing machines, it is necessary to consider technological information about the dimension of pressing machines and entry molds, machine errors and mold changing time as constraints. Table III is shown the dimension of entry molds and the dimension of available pressing machines. The dimension of the selected mold should be less than the pressing machine. For example the entry mold size 102 cannot assign to pressing machine C3-4 and C7-8 because the dimension of mold size 102 is bigger than pressing machine's dimension. Mold changing time and % error of a couple-mold on each pressing machine and a single-mold on each pressing machine are shown in Table IV

and V, respectively. These constraints are considered in the proposed model.

Firstly, PIS and NIS of both objective functions are obtained by the optimization of each objective. PIS and NIS of the average machine errors objective are 0.093% and 0.248%, respectively. PIS and NIS of the total setup time objective are 3,640 min/week and 4,120 min/week, respectively. These PIS and NIS are used to construct the membership function of each objective. The best answer of the first objective function of the problem is

Job#1: mold 102 is assigned to pressing machine I7-8 and K1-2

Job#2: mold 141 is assigned to pressing machine D1-2

Job#3: mold 18 is assigned to pressing machine C3-4

Job#4: mold 138 is assigned to pressing machine G01

On the other hand, if the total setup time (the second objective) is set, the optimal solution is

Job#1: mold 102 is assigned to pressing machine I1-2 and K1-2

Job#2: mold 18 is assigned to pressing machine D15-16

Job#3: mold 141 is assigned to pressing machine D1-2

Job#4: mold 138 is assigned to G01

These two objectives need to be compromised. Then, the proposed method is applied. Firstly, the selection of couple-pressing machines is deciding to reduce the overall processing time. After that the selection of single-pressing machines for assigned molds is performed. Then, the proposed model for selecting double-pressing machines can be shown as:

FMODM for Double-Pressing Machine and Mold Selection

$$\text{lex max} = [\lambda_1, \lambda_2],$$

subject to

$$\lambda_1 + \delta_1^- + \delta_1^+ = 0.88$$

$$\lambda_1 \leq 1 - \left(\frac{Z_1 - 0.093}{0.155} \right)$$

$$\lambda_2 \leq 1 - \left(\frac{Z_1 - 3{,}640}{480} \right)$$

$$\frac{1}{8} \left[\sum_{i=1}^{9} \sum_{j=1}^{4} U_{ij} x_{ij} + \sum_{i=1}^{9} \sum_{j=1}^{4} G_i x_{ij} \right] + 0.155 \left(\delta_1^- - \delta_1^+ \right) = 0.093$$

$$\sum_{i=1}^{9} \sum_{j=1}^{4} T_{ij} x_{ij} + 480 \left(\delta_2^- - \delta_2^+ \right) = 3{,}640$$

$$\sum_{j=1}^{4} x_{ij} = 1, \quad \forall i$$

$$\sum_{j=1}^{9} x_{ij} \geq 1, \quad \forall j$$

$$\sum_{i=1}^{9} x_{ij} = 2 \sum_{k=1}^{9} y_{kj}, \quad \forall j$$

$$S_{ij} x_{ij} \geq Z_{kj} y_{kj}, \quad \forall j, \forall k$$

$$\sum_{k=1}^{9} y_{kj} = 8 \quad \forall j$$

$$\delta_k^-, \delta_k^+ \geq 0, \quad \delta_k^- \delta_k^+ = 0, \lambda_k \in [0,1], \quad \forall k$$

By the same way, PIS and NIS of both objective functions are calculated for the selection of single-pressing machines. The model can be solved by the following model.

FMODM for Single-Pressing Machine and Mold Selection

Single objective optimization of both an average % error and the total setup time are the same. So, there is only one solution that the average % error is 0.16 and the total setup time is 160 min/week.

The proposed P-FGP models can obtain the compromised solution as shown in Table VI. The average % error of the single-pressing machines is 0.160 and the average % error of the double-pressing machines is 0.112. The total setup time for the single-pressing machines and double pressing machines are 160 min and 3,880 min, respectively. The satisfaction levels of the first and the second objectives are 1 and 0.88%. Then, pressing machines and molds selection plan are

Job#1: mold 102 is assigned to pressing machine I1-2 and K1-2

Job#2: mold 141 is assigned to pressing machine D1-2

Job#3: mold 18 is assigned to pressing machine C3-4

Job#4: mold 138 is assigned to pressing machine G01

TABLE VI
SOLUTION RESULTS OF BOTH SINGLE AND THE PROPOSED MODELS

Approach	Satisfactory level (λ_k)	Average %error	Satisfactory level (λ_k)	Total time loss (min/week)
Single objective model of an average % error	-	0.16 [a] 0.093 [b]	-	160 [a] 4,120 [b]
Single objective model of the total setup time)	-	0.16 [a] 0.248 [b]	-	160 [a] 3,640 [b]
P-FGP	1 0.92	0.16 [a] 0.106 [b]	1 0	160 [a] 4,120 [b]
	1 0.88*	0.16 [a] 0.112[b]	1 0.5*	160 [a] 3,880 [b]

Note: * Best compromise solution
[a] : Pressing machine /Mold A type [b] : Pressing machine /Mold B type

The average % error 0.112 is acceptable for high product quality. Setup time from the proposed model can be reduced from 4,120 to 3,880 min/week or 240 min/week, when comparing with an optimal solution of the first objective.

The proposed model is tested and compared with the single objective optimization of the first objective, the single objective optimization of the second objective and trial and error by working team with 4 week data sets. The results from Figs. 3-6 show that the suitable pressing machines can be selected by the proposed model. So, an average error and setup time for both types of pressing machines are low. Most of the proposed solutions are equal or better than the single objective optimization of the first objective, the single objective optimization of the second objective and trial and

error by working team. This model can be easily used to solve the machine selection. Meeting time for assignment can also be extremely reduced.

V. Conclusion

This research proposed the preemptive fuzzy goal programming model for machine selection of the tire manufacture. Two main objectives are determined; minimization of an average machine error and minimization of the total setup time. Conventionally, trial and error based on the discussion among the working team is used to select the pressing machine for each task due to the complexity and many technological constraints of the problem. So, the quality of products is unpredictable and the total setup time is also

high. Moreover, long meeting time is also needed to obtain the suitable plan. Therefore, in this research preemptive fuzzy goal programming model is developed for the problem to compromise these two objectives. The proposed model can obtain the appropriate results that DM satisfies for both objectives. Moreover, alternative choices can be easily generated by varying the levels of satisfaction. Besides, the model is also tested and compared with the current method and single objective optimization methods. The results of the comparison show that this model can find the best solution for the company.

Further research can be done by further determining limited equipment selection for each pressing machine.
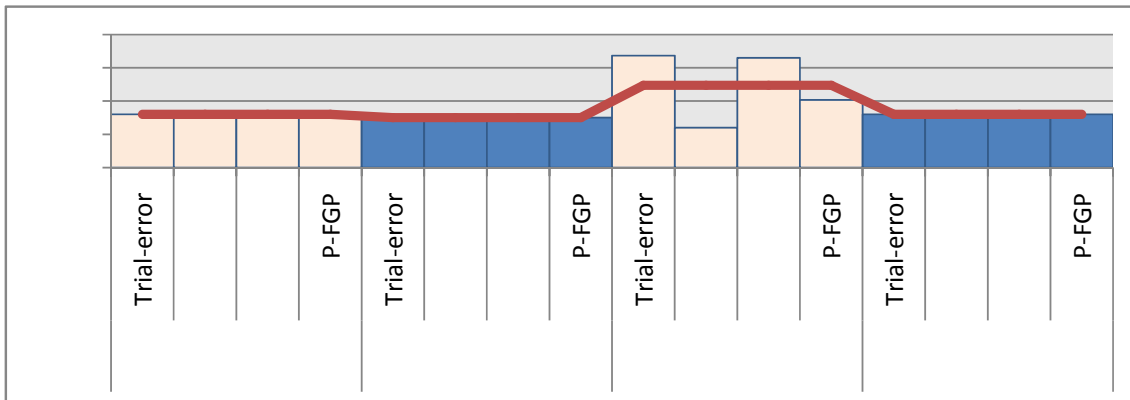

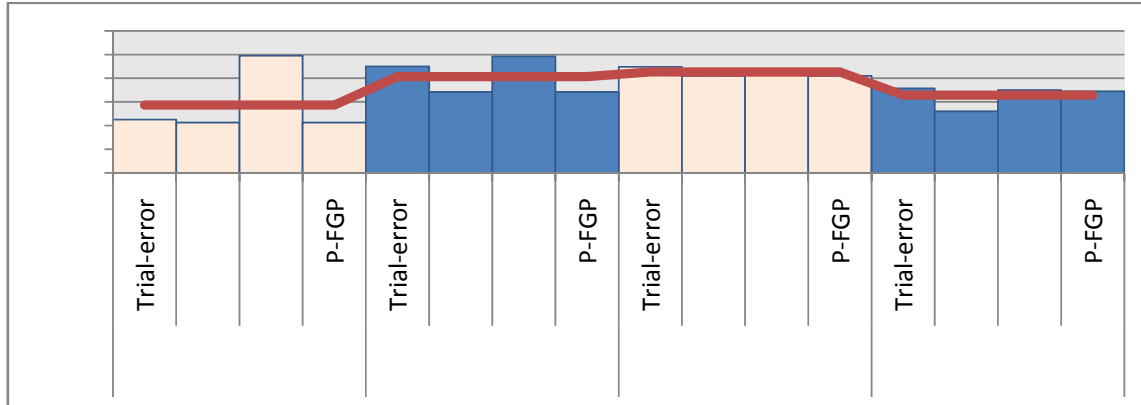Fig. 2 An average % error of single-pressing machines by each method

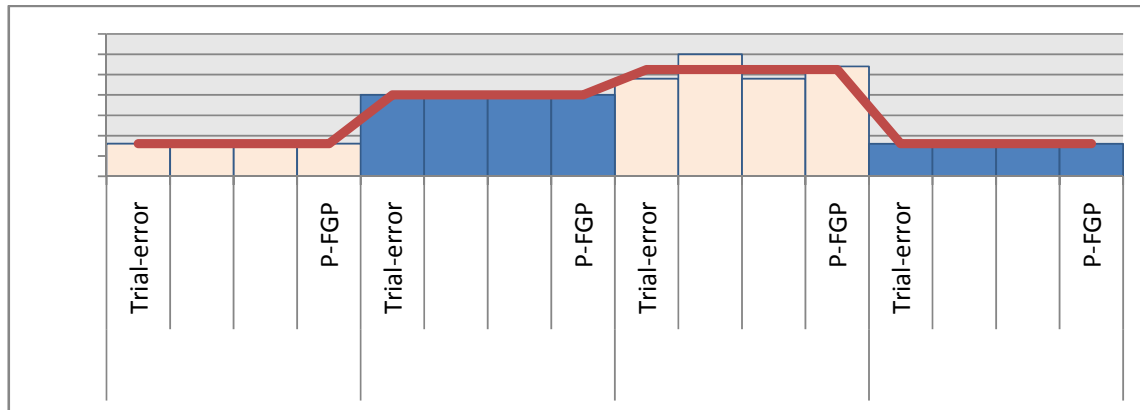Fig. 3 An average % error of double-pressing machines by each method

Fig. 4 The total setup time of single-pressing machines by each method (min/week)
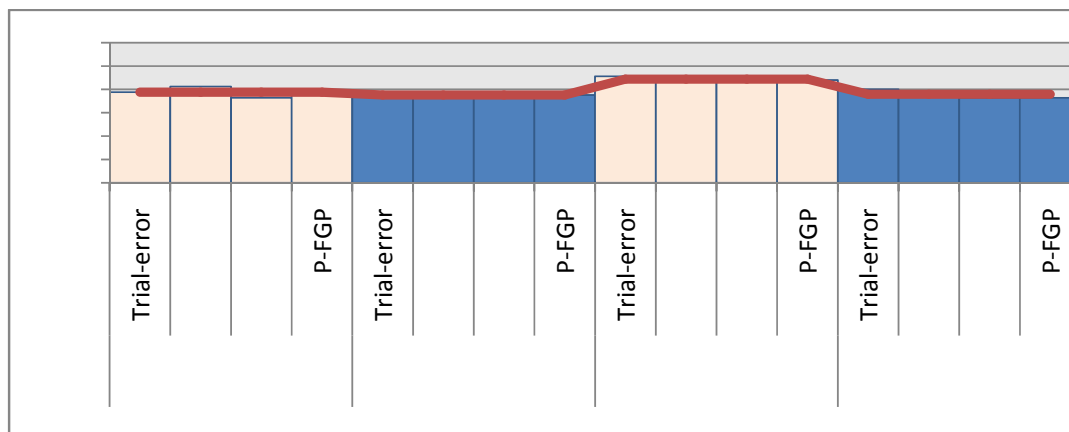

Fig. 5 The total setup time of double-pressing machines by each method (min/week)

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