# A Fast HRRP Synthesis Algorithm with Sensing Dictionary in GTD Model

R. Fan, Q. Wan, H. Chen, Y.L. Liu, and Y.P. Liu

Abstract—In the paper, a fast high-resolution range profile synthetic algorithm called orthogonal matching pursuit with sensing dictionary (OMP-SD) is proposed. It formulates the traditional HRRP synthetic to be a sparse approximation problem over redundant dictionary. As it employs a priori that the synthetic range profile (SRP) of targets are sparse, SRP can be accomplished even in presence of data lost. Besides, the computation complexity decreases from O(MNDK) flops for OMP to O(M(N+D)K) flops for OMP-SD by introducing sensing dictionary (SD). Simulation experiments illustrate its advantages both in additive white Gaussian noise (AWGN) and noiseless situation, respectively.

Keywords—GTD-based model, HRRP, orthogonal matching pursuit, sensing dictionary,.

### I. INTRODUCTION

HRRP is the phasor sum of the time returns from different scatterers on the target located within a resolution cell. As it is very simple and easy to realize, HRRP has been used to reflect target structure features in radar signal processing [1]-[6]. In GTD model, the radar target is no longer a point but composed of multiple scatterers along with radar line of sight (LoS). A process to identify radar target in GTD model is just the same as the process to estimate GTD model parameters (containing scattering mechanisms, intensity and scatter range cells).

In realistic environment, the returns are always inevitably interfered by passive or/and active jamming. For the case, the returns are corrupted or even invalid. We have to synthesize range profile from partial measurements because some samples are invalid. However, it's worth noting that the significant physical scatterers are sparse in actual targets, which implies that strong scattering cells are also sparse for the target's SRP. This is consistent with sparse signal representation of compressed sensing (CS) theory appeared in recent years [7]-[10]. The existed sparse signal recovery algorithms such as basis pursuit (BP) [7] and orthogonal match pursuit (OMP) [11] can be used to synthesize range profile from partial measurements. (For describing convenience, the measurement matrix in measurement system (see Eq. (3)) is called dictionary. Each column in dictionary is called an atom. Meanwhile, it calls that it is K-sparse if containing K nonzero entries in a vector.) However, in the GTD model with multiple scatterers, a few scattering mechanisms should be considered. With the increase of atom number in dictionary, the computational

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cost increases. Although a simplified scattering model can be used to approximate multiple scatterers model as discussed in section III, model mismatch can degrade the success recover probability, which deteriorates the cumulative distribute error (CDE) of SRP. Similar to [12]-[13], the SD is introduced to mitigate inter-atom interference (IAI) in this paper. Using SD, it can reduce computational complexity and mitigate the IAI so as to improve the recover probability of SRP. There are two main contributions in the paper. On the one hand, the SRP can be reconstructed from partial measurement data by introducing sparse property of HRRP. On the other hand, a faster reconstruct algorithm with SD (i.e., OMP-SD) than OMP algorithm is proposed.

In section II, it first presents the GTD scattered model in frequency domain and then, establishes measurement system in stepped frequency radar (SFR). After that, it briefly reviews existing algorithms to solve the model and presents approximate OMP algorithm (A-OMP) in section III. In section IV, it presents a strategy to construct SD. It mitigates model mismatch effectively. Besides, a fast algorithm (OMP-SD) to synthesize HRRP is proposed. Monte Carlo simulations illustrate the performance of the proposed algorithm both in AWGN and noiseless situation respectively in section V. Finally, some conclusions and further work are provided in Section VI.

Notation: It denotes vectors and matrices by boldface lowercase and uppercase letters, respectively. Uppercase Greek letters also represent matrix in this paper.  $(\cdot)^T$  denotes the transpose operation,  $(\cdot)^H$  denotes the conjugate transpose operation, Further,  $\|\cdot\|_2$  refers to the  $l_2$  norm for vectors.  $\|\cdot\|_\infty$  refers to the  $l_\infty$  norm for vectors. The  $vec(\cdot)$  operator vectorizes a matrix by stacking its columns.  $\mathbf{R} \in \mathbb{R}^{L \times M}$  and  $\mathbf{R} \in \mathbb{C}^{L \times M}$  denote a real-valued and complex-valued matrix and let  $\Re\{\cdot\}$  and  $\Im\{\cdot\}$  be real part and imaginary, respectively.  $(\cdot)^+$  denotes the M-P generalized inverse.

## II. PROBLEM FORMULATION

In this section, it briefly presents the GTD scatter model of SFR return signal. SF pulse trains are created by transmitting a train of M identical baseband pulse with different carrier frequencies. The carrier frequency of the m-th ( $m=0,1,\cdots,M-1$ ) pulse is  $f_m=f_0+\Delta f$ , where  $f_0$  is the initial frequency and  $\Delta f$  is the frequency step size. In the stretch processing [4], the range resolution is  $\Delta r=c/(2M\Delta f)$ , and the ambiguous range  $\Delta R=c/(2\Delta f)$  (c is the speed of light). For the convenience of signal modeling and derivation, it is assumed that the target is stationary and it falls in the range gate  $[L,L+L_0]$  in one CPI, where, the range from radar

antenna to target  $L=Q\Delta R$ , and  $L_0=N\Delta r$ ). (Q and N are nonnegative integers). Meanwhile, it assumes that the target can be present only the grid points and let us discretize the range space by  $\Delta r$  in  $L_0$ .

In one aspect angle, the parametric GTD scatter model of SFR at frequency  $f_m$  can be represented as follows [14]-[16],

$$y_m = \sum_{d=1}^{D} \sum_{l=1}^{N} G_d \left( j \frac{f_m}{f_0} \right)^{\alpha_d} \cdot \exp\left\{ -j \frac{4\pi}{c} f_m r_{dl} \right\} \cdot x_{dl} + u_m$$
(1)

where

$$x_{dl} = \begin{cases} 1, if \ scatter \ is \ present \ in \ r_{dl} \\ 0, otherwise \end{cases}$$
 (2a)

In (1),  $G_d$ ,  $\alpha_d$  are the complex amplitudes, scattering mechanism of d-th scatterer, respectively.  $r_{dl}$  denotes range space w.r.t. l-th range resolution cell in m-th pulse. D is the number of scatterers.  $u_m$  is the AWGN with mean zero and variance  $\sigma^2$ .

The  $y_m$  in (1) consists of N uniformly sampled time-domain data from the baseband echo signal of m-th pulse  $(N=2L_0/(c\Delta t)$  and  $\Delta t=1/(M\Delta f))$ . The model can be written into a matrix form as follows,

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{u} \tag{3}$$

where,  $\mathbf{y} \in \mathbb{C}^{M \times 1}$ ,  $\Phi \in \mathbb{C}^{M \times DN}$  and  $\mathbf{x} \in \mathbb{R}^{DN \times 1}$  are measurement vector, dictionary and HRRP index of the target, respectively.  $\Phi \triangleq [\Phi_1, \Phi_2, \cdots, \Phi_D]$  and

$$[\Phi_d]_{m,n} = G_d[j(1+m\Delta f/f_0)]^{\alpha_d} \cdot \exp(-j2\pi f_m(r_0+n/(M\Delta f)))$$
 OMP) algorithm is presented as follows.

 $\mathbf{x} = [\mathbf{x}_1^T \ \mathbf{x}_2^T \ \cdots \ \mathbf{x}_D^T]^T, \ \mathbf{x}_d \in \mathbb{R}^{N \times 1}, \ \alpha_d \in \Omega \ (\Omega \ \text{is a set composed of scattering mechanisms)}, \ \mathbf{u} \in \mathbb{C}^{M \times 1} \ \text{is the AWGN vector. For convenience of the later describing, it defines } \Phi \triangleq [\Phi_1, \Phi_2, \cdots, \Phi_D] \ \text{and} \ \Phi_d \triangleq [\phi_{d1}, \phi_{d2}, \cdots, \phi_{dN}].$   $\Phi_d$  denotes d-th block matrix of  $\Phi$ .  $\phi_{di}$  denotes the i-th atom of  $\Phi_d$ .  $d \in \Lambda \triangleq \{1, 2, \cdots, D\}$ . All atoms are normalized with  $l_2$ -norm throughout the paper.  $r_0$  is the radial distance from radar antenna to reference point on the target. In realistic settings, M << N < DN, hence (3) is an underdetermined system. It is to reconstruct of a high-dimension sparse vector  $\mathbf{x}$  from a small number of linear measurements  $\mathbf{y}$  and dictionary  $\Phi$ .

## III. THE A-OMP ALGORITHM

For the underdetermined system of linear equations in (3),  $l_1$ -norm minimization subject to constraints can be used to solve it as follows [7],

$$(P_1:) \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad subject \ to \ \|\mathbf{y} - \Phi \mathbf{x}\|_2 \le \varepsilon$$
 (5)

For the problem  $P_1$ , it can be solved by linear programming (LP). Nevertheless, general-purpose LP solvers require about  $O(D^3N^3)$  flops. Rather than minimizing an objective function in (5), many of the applications of  $P_1$  can be attacked heuristically by fitting sparse models, using greedy stepwise least squares. A widely used algorithm for sparse signal recovery is the OMP algorithm for the recovery of the support of the K-sparse signal in (3), which requires O(DNMK) flops [11]. OMP constructs a sparse solution to a given problem

by iteratively building up an approximation, the vector  $\mathbf{y}$  is approximated as a linear combination of a few atoms in dictionary  $\Phi$ , where the activeset of atoms to be used is built column by column, in a greedy fashion. At each iteration, a new atom that best correlates with the current residual is added to the activeset. The standard OMP algorithm can be found in [11]. For noiseless case, the exactly recovery condition (ERC) of OMP was derived by Troop [11]. T.cai et al. derived a new ERC both in the bounded noise and Gaussian noise [17].

In the paper, a few scatter mechanisms are considered. It increases the atom number in dictionary, and hence it increases computation. To decrease computation caused by multiple scatterers, it's a straight way to synthesize range profile of target that using a single scattering mechanism instead of multiple scattering mechanisms (i.e., to replace  $\Phi$  with  $\Phi_d$ ).

Just as the description in section II, the atoms in dictionary (i.e., columns of  $\Phi_d$ ) are normalized so that  $\|\phi_{di}\|_2=1$ , for  $i=1,2,\cdots,N$ . It denotes by  $c\subseteq S\triangleq\{1,2,\cdots,N\}$  the support of  $\mathbf{x}_d$ , which is defined as the set of indices corresponding to the nonzero components of  $\mathbf{x}_d$ . For matrix  $\Phi_d$ ,  $\Phi_d(c)$  denotes the matrix formed by picking the atoms of  $\Phi_d$  corresponding to indices in set c. Following the same convention as section 2,  $\phi_{di}$  represents the i-th atom of  $\Phi_d$ . It calls  $\phi_{di}$  a correct atom if the corresponding  $\mathbf{x}_{di}\neq 0$  and call  $\phi_{di}$  an incorrect atom otherwise. With slight abuse of notation, we use  $\Phi_d(c)$  to denote both the subset of atoms of  $\Phi_d$  with indices and the corresponding block matrix of  $\Phi_d$ . A detailed description of approximation orthogonal matching pursuit (A-QMP) algorithm is presented as follows.

## Algorithm 1 :A-OMP

# Input:

The measurement vector, y;

The dictionary,  $\Phi_d$ ,  $d \in \Lambda$ ;

the error threshold,  $\epsilon$ ;

#### **Main Procedures:**

- 1: Initialize the residual  $\mathbf{r}_0 = \mathbf{y}$  and initialize the subscript set of selected atom  $c_0$  is empty. Set i = 1.
- 2: Find the atom  $\phi_{t_i}$  that solves the maximization problem

$$t_i \triangleq \max_{t} |\phi_{dt}^H \mathbf{r}_{i-1}|, (t \in S, \phi_{dt} \text{ is the } tth \text{ atom in } \Phi_d)$$

and update  $c_i = c_{i-1} \cup \{t_i\}$ .

- 3: Let  $\mathbf{P}_i = \Phi_d(c_i)(\Phi_d(c_i)^H \Phi_d(c_i))^{-1} \Phi_d(c_i)^H$ . Denote the projection onto the linear space spanned by the elements of  $\Phi_d(c_i)$ . Update  $\mathbf{r}_i = (\mathbf{I} \mathbf{P}_i)\mathbf{y}$ .
- 4: If the stopping condition is achieved (i.e.,  $\|\mathbf{r}_i\|_2 \le \epsilon$ ), go to 5. Otherwise, set i = i + 1 and return to 2.
- 5: Pick out the range scattering cells w.r.t set  $c_i$ .
- 6: Calculate the scattering intensity in these range cells determined in the previous step with  $P_i^+y$ .
- 7: Reconstruct SRP using the scattering intensity and range scattering cells.
- 8: Return SRP.

Similar to OMP, the A-OMP is a stepwise forward selection algorithm and is easy to implement. A key component of A-OMP is the stopping rule which depends on the noise structure.

In the noiseless case the natural stopping rule is  $\mathbf{r}_i = 0$  ( $\mathbf{r}_i$  is the residual after i-th iteration, which is defined in Algorithm 1). That is, the algorithm stops whenever  $\mathbf{r}_i = 0$  is achieved. In this paper, both noiseless and the case of AWGN in which  $\mathbf{u}_i \sim N(0, \sigma^2)$  are considered. The stopping rule for each case and the properties of the resulting procedure are discussed in article [11].

As a special case of multi-scattering center, for a single scattering mechanism, the A-OMP algorithm procedure is the same as OMP, but it has a significant different physical meaning. Because the dictionary  $\Phi_d$  just as a sub-block of  $\Phi$ in (3). Thus, it is called approximate OMP in the paper. Once the subscribe set is determined with A-OMP or OMP, the SRP can be obtained with Least-Square (LS) solution.

# IV. OMP ALGORITHM VIA SENSING DICTIONARY (OMP-SD)

In (3), the sparse vector x can be obtained by OMP or A-OMP directly. However, two major problems cannot be avoided in this case. For OMP, it has to search all atoms in dictionary  $\Phi$  to find the best matched atom at each iteration (in the paper, the dictionary is M-by-DN dimension matrix); For A-OMP, it just needs to find the best matched atom in M-by-N dimension dictionary at each iteration, but it leads to model mismatch and increases CDE of SRP. Thus, an improved algorithm via sensing dictionary (i.e., OMP-SD) is developed to overcome drawbacks of both OMP and A-OMP. There are two advantages with OMP-SD to synthesize HRRP. On the one hand, as a result of the atoms in SD are independent on scattering mechanisms, it mitigates model mismatch. On the other hand, it reduces computation because of the searching dimensional of dictionary reduced from M-by-DN down to M-by-N.

# A. Dictionary Pre-processing

For the convenience of following analysis, the dictionary  $\Phi$  of in (3) are divided into D M-by-N dimensional block matrix firstly, which are denoted by  $\Phi_1, \Phi_2, \cdots, \Phi_D$  and each block matrix  $\Phi_d$ ,  $d \in \Lambda$ , corresponds to a different scattering mechanism. (3) can be rewritten as,

$$\mathbf{y} = [\Phi_1 | \Phi_2 | \cdots | \Phi_D] \mathbf{x} + \mathbf{u} \tag{6}$$

In (6), it considers  $\Phi_1$  (w.r.t the 1st scattering mechanism) as an example. In ideal condition, the Gram matrix  $\Phi_1^H \Phi_d = \mathbf{I}$ , but it is not the case because the dictionary is over complete, so it has to make  $\Phi_1^H\Phi_d \to \mathbf{I}$ ,  $(d \in \Lambda \text{ and } d \neq 1)$ , extremely, which needs to solve the problem  $\max_{d \in \Lambda} \|\mathbf{I} - \Phi_1^H \Phi_d\|_{\infty}$ . According to the idea, it should find an M-by-N SD  $\mathbf{W}$  ( being the same dimensional as block matrix  $\Phi_d$  ), which is independent on scattering mechanisms. The SD can be found by solving the problem  $P_2$  of the follows,

$$\left(\begin{array}{cc} \min & b_1 + \gamma b_2 \\ \end{array}\right) \tag{7a}$$

$$s.t. \|\mathbf{I} - diag(\mathbf{W}\Phi_d)\|_{\infty} \le b_1 \tag{7b}$$

$$(P_2:) \begin{cases} \min_{\mathbf{W}} b_1 + \gamma b_2 & (7a) \\ s.t. \|\mathbf{I} - diag(\mathbf{W}\Phi_d)\|_{\infty} \le b_1 & (7b) \\ s.t. \|\rho\|_{\infty} \le b_2 & (7c) \\ \rho = vec\left((\mathbf{W}^H \Phi_d)_{k,l}\right), k \ne l & (7d) \\ d \in \Lambda & (7e) \end{cases}$$

(7d)

In (7a),  $\gamma$  is the regular factor. It sets 0.5 in the paper. Both  $b_1$  and  $b_2$  are unknown but determined variables. They reflect the IAI level between **W** and dictionary  $\Phi_d$ . As the problem  $P_2$  is a convex problem, the sensing dictionary W can be obtained offline with efficient algorithms. And it can be solved by software pockets such as cvx (http://stanford.edu/ boyd/software.html.).

# B. The Proposed Algorithm

The proposed algorithm (OMP-SD) is also a greedy algorithm but different from OMP and A-OMP. For OMP-SD, at each iteration, it requires a two-step search to select an atom. First, it determines the offset index of atom in SD W, which is not sensitive to scatter mechanism. And then, it further to determine the specific scatter mechanism in dictionary  $\Phi$ . After the two-step procedures, an actual atom is picked out. The OMP-SD is described as follows.

### Algorithm 2 :OMP-SD

#### **Input:**

The measurement vector, y; The dictionary,  $\Phi_1, \Phi_2 \cdots, \Phi_D$ , **W** the err threshold,  $\epsilon$ ;

#### **Main Procedures:**

- 1: To initialize the residual  ${f r}_0 = {f y}$  and initialize the subscribe set  $c_0$  is empty. set i = 1.
- 2: To find the matrix  $\Gamma_{t_i}$  that solves the maximization problem

$$t_i \triangleq \max_{t} \left| \mathbf{w}_t^H \mathbf{r}_{i-1} \right|$$

where,

$$\Gamma = [\Phi_1(t_i) \ \Phi_2(t_i) \ \cdots \ \Phi_D(t_i)]$$

3: To solve the maximization problem

$$\xi = \max_{d} \left| \Gamma^{H} \mathbf{r}_{i-1} \right|, d \in \Lambda$$

and update  $c_i=c_{i-1}\cup\{\xi_i\}$ . Where  $\xi_i=(\xi-1)N+t_i$ . 4: Let  $\mathbf{P}_i=\Phi(c_i)(\Phi(c_i)^H\Phi(c_i))^{-1}\Phi(c_i)^H$  denote the projection onto the linear space spanned by the elements of  $\Phi(c_i)$ . Update  $\mathbf{r}_i = (\mathbf{I} - \mathbf{P}_i)\mathbf{y}$ .

- 5: If the stopping condition is achieved (i.e.,  $\|\mathbf{r}_i\|_2 \leq \epsilon$ ), go to 6. Otherwise, set i = i + 1 and go back to 2.
- 6: Pick out the range scattering cells w.r.t set  $c_i$ .
- 7: Calculate the scattering intensity in these range cells determined in the previous step with  $P_i^+ v$ .
- 8: Reconstruct SRP using the scattering intensity and range scattering cells.
- 9: Return SRP.

As far as computational complexity is concerned, it requires DN times correlation operators to select an atom in OMP whileas it just requires N + D times for the proposed algorithm. So it requires about O(M(N+D)K) flops. It is approximate to the simplified model in which requires Ntimes. Similarly to in section 3 discussed, once the subscribe set is determined with the proposed algorithm, the SRP can be recovered with LS solution, too.

TABLE I: Geometry parameters for example scattering geometries [14]

Value of $\alpha_d$	Scatter mechanisms
-1	corner diffraction
-0.5	edge diffraction
0	point diffraction; straight edge specular
0.5	singly curved surface reflection
1	late plate at broadside; dihedral

TABLE II: Time consuming simulation results

SRP Algorithm	time(s)
OMP	1044
A-OMP	109
OMP-SD	121

#### V. SIMULATION AND EXPERIMENTAL RESULTS

In this section, 10000 trails Monte Carlo simulation has been done to illustrate the previous discussions. Assume the SFR operates at the following condition. Five scattering mechanisms are considered (i.e.,  $\alpha_d \in \Omega \triangleq \{-1, -0.5, 0, 0.5, 1\}$ ). An example scattering geometrics and the corresponding scattering parameters are shown in Tab. II.

In (4), it assumes that target is stationary in one CPI, and the distance from radar antenna to target L is regarded as constant. In simulation, the radial distance from radar antenna to reference point on the target (i.e.,  $r_0$ ) is unvaried and it can be eliminated. For simplicity, we set  $r_0 = 0$  and hence, the m-th row and n-th column element in (4) is rewritten as

$$[\Phi_p]_{m,n} = G_p[j(1+m\Delta f/f_0)]^{\alpha_p} \cdot \exp(-j2\pi f_m n/(M\Delta f)))$$
(8)

The range of the measured frequency band is from L band to S band (i.e., from 1GHz to 4GH), where the start frequency is  $f_0=1$ GHz and frequency step size  $\Delta f=10$ MHz. The number of pulses M=300. And it assumes that the target is 5m length. Five scatterers are located on 0.3m, 0.85m, 2.0m, 3.25m and 4m to target front-end, respectively. All scatterers have same intensity. What's more, it assumes that the stationary scatterer centers are present on the grid points. In each measurement, only 30 returned pulses are measured in one CPI (i.e., 300 pulses). The measurement vector y is contaminated by AWGN with SNR=20dB, 15dB, 10dB, 5dB and noiseless situation, respectively. In order to explain the essence of model mismatch, mutual incoherence property (MIP) is introduced which is defined as the same as in article [10],

$$\mu(\Phi) \triangleq \max_{\substack{1 \le i,j \le n \\ i \ne j}} \frac{\left|\phi_i^H \phi_j\right|}{\|\phi_i^H\|_2 \cdot \|\phi_j\|_2} \tag{9}$$

Noting that each atom in dictionary is normalized, hence Eq. (9) can be rewritten as another form,

$$\mu(\Phi) \triangleq \max_{\substack{1 \leqslant i,j \leqslant n \\ i \neq j}} \left| \phi_i^H \phi_j \right| \tag{10}$$

We calculate the IAI of original dictionary and SD with Eq. (10), respectively. In the original dictionary, the IAI minimum is 0.0762 and the maximum of inner-atom cross-correlation (i.e., MIP) is 0.3872. However, both of them are 0.2248 (i.e.,

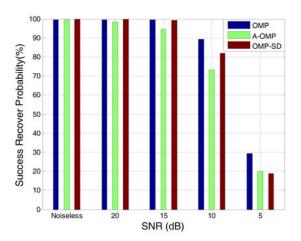


Fig. 1: Success recover probability w.r.t SNR

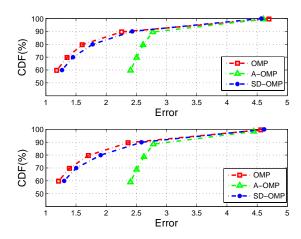


Fig. 2: Cumulative distribute error (upper plot, noiseless), (bottom plot,  $SNR=20\mathrm{dB}$ )

 $b_1=b_2=0.2248$  in (7a)) for the sensing dictionary. It mitigates the IAI.

Besides, simulation results about the three algorithms (i.e., OMP, A-OMP, OMP-SD) are shown in Fig. 1  $\sim$  Fig. 3. According to the simulation results, we can present three remarks in the following.

Remark 1: Fig. 1 shows that the success recovery probability is a monotonic decreasing relative to SNR for the three algorithms (i.e., OMP, A-OMP and OMP-SD). It is easy to understand that the OMP has the best recovery performance because it is match model and the A-OMP has worst recovery performance because of its model mismatch. However, the proposed method (OMP-SD) has an approximate performance compared to OMP and approximate computational complexity to A-OMP and it is confirmed in Tab. II. It should be noted that there is an exception for small SNR (< 10dB). When SNR is 5dB in Fig. 1, all of the three algorithms have a lower success recovery probability (less than 30 percent). Hence,

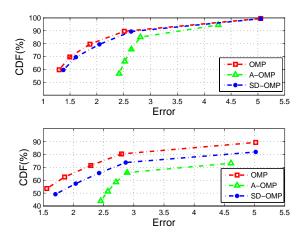


Fig. 3: Cumulative distribute error (upper plot,  $SNR=15 \mathrm{dB}$ ), (bottom plot,  $SNR=10 \mathrm{dB}$ )

objectively speaking, it is a drawback for these algorithm. But in moderately high SNR settings (i.e., greater than 15dB), the proposed algorithm has outstanding performance.

**Remark 2:** For the noiseless and three different SNR settings, Fig.  $2 \sim$  Fig. 3 show the cumulative distribute errors (i.e., CDE). It is widely used to evaluate recover performance in CS community such as [10]. From Fig.  $2 \sim$  Fig. 3 we can see the match model is best, while the mismatch model is worst although it requires least computation amount. However, OMP-SD shows that it has an approximate values of CDE compared to OMP. However, it has to point out that all of the three algorithms are not suitable for low SNR (< 10dB) settings.

**Remark 3:** The computational time results given in the Tab. II for the same computer platform. The computer used for experiments uses an Intel(R) Core(TM) i3 CPU M 330 4chip at 2.13 GHz, has random access memory of 4GB, and uses Windows 7 and Matlab R2012a (7.14.0.739) 64-bit(win64). There are sums of 10000 Monte Carlo trails time consuming results for OMP, A-OMP and OMP-SD, respectively. It confirms that OMP has the most computational cost but it is approximately computational cost between A-OMP and OMP-SD. Both of them have much lower computation cost compared with OMP.

#### VI. CONCLUSION AND FUTURE WORK

In this paper, a fast algorithm to synthesize range profile is proposed. For the SFR system in GTD model, the HRRP synthesis can be converted to solve a sparse approximation problem over redundant dictionaries. Different from A-OMP, the model mismatch is mitigated with SD. Better than OMP, the computational complexity is reduced. Finally, simulation results show the proposed algorithm is valid for both noiseless and noisy settings. In the future work, we plan to verify the performance of the proposed algorithm on real data.

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