International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:9, 2012

A DEA Model for Performance Evaluation in The Presence of Time Lag Effect

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***Abstract*—**Data Envelopment Analysis (DEA) is a methodology that computes efficiency values for decision making units (DMU) in a given period by comparing the outputs with the inputs. In many cases, there are some time lag between the consumption of inputs and the production of outputs. For a long-term research project, it is hard to avoid the production lead time phenomenon. This time lag effect should be considered in evaluating the performance of organizations. This paper suggests a model to calculate efficiency values for the performance evaluation problem with time lag. In the experimental part, the proposed methods are compared with the CCR and an existing time lag model using the data set of the 21st century frontier R&D program which is a long-term national R&D program of Korea.

***Keywords*—**DEA, Efficiency, Time Lag

I. Introduction

DEA evaluates the relative performances of comparable units having the authority to make decisions. DEA models use multiple input and output factors of DMUs in efficiency assignments. The efficiency of a DMU is defined as the ratio of weighted outputs to weighted inputs. Since the first Data Envelopment Analysis (DEA) model was developed by Charnes, Cooper and Rhodes (CCR)[1], there are various extentions of the original CCR model[2]-[6]. In special, Banker, Charnes, and Cooper(BCC) extended the CCR model to accommodate technologies that exhibit variable returns to scale[2]. There are many extended models of the basic DEA modelA Generalized DEA model (GDEA) was developed by Yu, Wei and Brockett[7]. Post and Spronk proposed an interactive DEA procedure (IDEA) and introduced a combined DEA model that solves a single linear program to compute the efficiency values of all DMUs[8].

DEA models generally assume that inputs are converted to outputs in the same period. However, this assumption may not be appropriate to some situations such as the performance evaluation of R&D activity and educational activity. Consider academic papers published or patents applied or registered as an output of a research activity. A published journal paper may result from research efforts over several years because the review process of an article for a journal could take some time. That is, production lead time is required to produce the output, such as journal papers or patents, from starting a research project. Furthermore, this production lead time varies according to research areas. In other words, there is a certain length of time lag between the consumption of inputs and the production of outputs. The length of time lag must be dependent on research areas on which is considered to obtain efficiency of research activity.

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In this case, general DEA models cannot be applied to obtain efficiency value of DMUs because inputs in a given period may contribute to outputs of several subsequent periods. Özgür Özpeynirci and Murat KöksLan [9] developed the Multi-period Input(MpI) model to capture the time lag between the inputs and outputs in DEA. This model was developed under assumption that the output of a period is produced by consuming the inputs of one or more previous periods from the output period. They added a lag period index t to Post and Spronk's model in order to take account for time lag effect. This model is to find the efficiency values of all DMUs for all periods by a single model.

We can interpret the time lag effect in production activity in other way. The input of a certain period can contribute to the outputs of one or more subsequent periods from the period. That is, the outputs of one or more periods can be thought to be produced by consuming a previous period. More exactly, the inputs of multiple periods can contribute to the outputs of multiple periods. However, it is not possible to explain the relation with the consumption periods of inputs and the production periods of outputs. Thus, this paper suggests a multi-period output model(MPO) model to capture another interpretation of time lag effect. Section II gives the suggested MPO model. A case study is described in section III.

II. Multi-period output model

For efficiency evaluation of long-term projects, the effect of time lag due to production lead time should be captured to obtain more exact efficiency of research activities. Most of DEA models didn't take consideration of the time lag effect in defining efficiency. At first, Özpeynirci and KöksLan suggested the Multi-period Input DEA(MPI) model to capture time lag effect that there might be between the consumption of inputs and the production of outputs.

In this section, we proposed another approach to deal with the time lag effect between the outputs and the inputs of a research activity in DEA context. The proposed model assumes that the input of a period could contribute to not only the outputs of the same period but also the outputs of future periods from the period.

A. Time spans for time lag models

The difference in the definition of efficiency should be clarified for some DEA models including the suggested model in this paper. To do this, it is required to compare the time span for efficiency measure used by models. Table I compares the time span to be used to calculate efficiency of a DMU by CCR, MPI and MPO model. Assume that the longest time lag for producing research output is 2 periods, i.e., PM = 2.

In the general DEA, i.e. CCR model, the inputs of a given period were assumed to be converted to the outputs in the same period. Thus, the efficiency of a DMU for the 1st period is defined by the ratio of the weighted output of the 1st period to the weighted input of the 1st period.

This efficiency measurement can't deal with time lag effect between the consumption of input and the production of outputs.

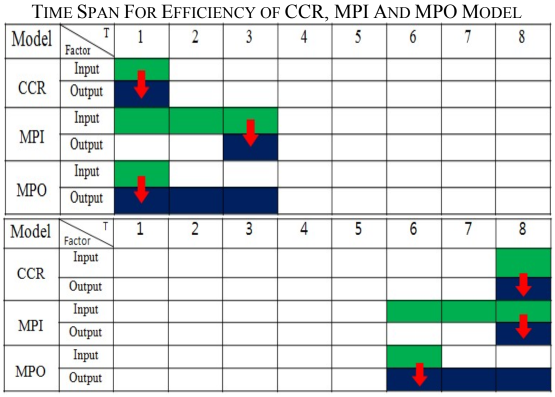
In the MPI model suggested by Özpeynirci and Kökslan[9], they thought that the outputs of a period are produced by the consumption of the inputs of one or more previous periods of the period.

The idea of MPI model is to measure the efficiency value of a DMU using the ratio of its weighted output for the third period to the weighted input for 1-3 periods. The efficiency of a DMU for the 8th period is calculated by the ratio of its weighted output for the 8th period to its weighted input for periods 6-8. That is, the underlying assumption of the efficiency definition is that the inputs of a DMU for some multiple periods contribute to the outputs of a single period, which is the last period of the multiple periods.

However, there can be another viewpoint for dealing with the time lag effect in productive activity such as R&D activity. That is, for a DMU, the inputs of a period can be interpreted to contribute to the outputs not only of the same period but also one or more subsequent periods of the period.

In the publication process of research papers, we often find that the consumption of inputs of a year could produce the outputs in the same year or in one or more years later. Thus, we suggest a multi-periods output(MPO) model based on this interpretation of time lag effect.

TABLE I
TIME SPAN FOR EFFICIENCY OF CCR, MPI AND MPO MODEL



For the first period, the efficiency value of a DMU is defined by the ratio of its weighted output for periods 1-3 to its weighted inputs in the first period in the case of MPO model. The efficiency value of the 6th period can also be calculated by the ratio of its weighted output for periods 6-8 to its weighted inputs in the 8th period.

As we can see in the Table I the definition of efficiency measure in MPO model is different from that of MPI model. Because of this difference, the periods for which we can get the efficiency values of DMUs depend on the model to be applied for considering time lag.

For the same data set, we can obtain efficiency values for periods 1-6 using MPO model while MPI model can give efficiency values for periods 3-8.

B. Multi-period Output (MPO) model

As explained in the last section, the MPO model assumes that the inputs of a period contribute to the outputs not only of the same period but also subsequent PM periods of the period. Let h_{jt} be the efficiency of the input used in the tth period by DMU j. Let x_{ijt} denote the amount of the ith input consumed by DMU j in the period t. Let v_{ijt} be the weight of the ith input consumed by DMU j in the period t. Let $Y_{rj(t+p)}$ represent the amount of the rth output produced by DMU j in the period t + p. Let u^{p}_{rjt} denote the weight of the rth output produced by DMU j p periods after from the period t. Since there is a time lag of PM periods, the outputs corresponding to the input of the 1st period are from the first period to the (PM+1)th period. The following MPO model finds the efficiency scores for periods [1, T-PM]. When PM = 0, (MPO) is equivalent to the combined DEA model.

(MPO)

$$\text{Max} \sum_{t=1}^{T-PM}\sum_{j=1}^{n} h_{jt} = \sum_{p=0}^{PM}\sum_{t=1}^{T-PM}\sum_{j=1}^{n}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rj(t+p)}$$

Subject to

$$\sum_{i=1}^{m} v_{ijt} X_{ijt} = 1 \qquad j = 1,\dots,n;\ t = 1,\dots,PM;\ i = 1,\dots,m;$$

$$\sum_{p=0}^{PM}\sum_{r=1}^{s} u^{p}_{rjt} Y_{rk(t+p)} - \sum_{i=1}^{m} v_{ijt} X_{ikt} \le 0 \qquad k = 1,\dots,n;\ j = 1,\dots,n;\ t = 1,\dots,T-PM;\ p = 0,\dots,PM;\ r = 1,\dots,s;$$

$$u^{p}_{rjt}, v_{ijt} \ge \varepsilon \qquad r = 1,\dots,s;\ i = 1,\dots,m;$$

III. CASE STUDY

The 21st century frontier R&D program is a long-term national R&D program of Korea, for selective and intensive development of strategic technologies to enhance national scientific competitiveness to the level of advanced countries by 2010.

The objective of the program is to develop the national economy through improving national competitiveness, public welfare, quality of life to the level of advanced countries and creating new industries through the development of future technologies. Projects were selected by two main selection criteria to achieve the objective of the program. One is the technology to develop high-tech products within ten years that can contribute to the improvement of national competitiveness. The other is a technology that has a principle researcher who can fully utilize the R&D experience and capabilities and can manage a large-scale, long-term project.

TABLE II
THE NUMBER OF THE LONG TERM PROJECTS IN EACH RESEARCH CENTER AND SUPPORTED YEARS

Research center	# of projects	Supported yrs
SG01	5	8
SG02	11	8
SG03	19	8
SG04	9	6
SG05	12	8
SG06	15	9
SG07	9	8
SG08	15	9
SG09	18	10
SG10	11	10
SG11	24	9
SG12	10	10
SG13	7	8
SG14	12	7
SG15	16	7

Currently, 24 research centers are being supported by this program since two project centers were launched in 1999. The committee of the program has launched three centers in 2000, five centers in 2001, nine centers in 2002, four centers in 2003 and finally 1 center in 2004. These 24 project centers have been launched by funding of three ministries in Korean government, that is, 18 centers by The Ministry of Science and Technology (MOST), 5 centers by The Ministry of Commerce, Industry & Energy (MOCIE) and 1 center by The Ministry of Information & Communication (MIC). Each center is granted about 9 million US dollar per year by the government and its running period is no more than ten years from its start. Each center is operated and managed under the responsibility of a director who is nominated by the government. That is, the director of each center organizes centers, selects subprojects, forms research teams, and allocate fund to subprojects by its own evaluation system under the responsibility of the director. Thus, the director and each center itself are responsible for the performance of each center and subprojects.

TABLE III
DATA OF A PROJECT IN RESEARCH CENTER SG09

yr	Fund (m \)	researcher			papers		Patent	
		Ph.D	MS	BS	SCI	Non SCI	appl	Reg.
1	300	0	10	13	0	0	2	0
2	300	0	9	12	0	1	3	2
3	300	4	12	6	3	3	4	0
4	150	5	5	2	3	0	2	0
5	180	5	5	2	2	2	5	3
6	180	5	5	2	8	7	8	0
7	180	3	0	6	1	0	6	4
8	180	3	0	3	5	0	3	6
9	180	0	3	5	2	0	9	2
10	180	0	2	4	9	0	5	2

The number of subprojects supported by these 24 centers is totally 2,670 subprojects. However, the long term projects are required to apply the MPO model considering time lag effect. Thus, the data about input factors and output factors of 15 research centers among these 24 centers will be used to analyze the efficiency of research activity. Table II summarizes the number of the long term projects in each center and their supported years. For instance, Table III shows a sample data set for a project supported by the research center SG09. There are four input variables, research expenditure(million Korean Won), the number of researchers(Ph.D, MS, BS). The number of published papers(SCI journal and non SCI journal) and the number of applied and registered patents are considered as outputs of research activity. We used CCR, MPI and MPO three models to evaluate the performance of the 21st century frontier R&D program, and compare these result to know the difference when we add the time lag to the evaluation. We used LINGO to run these models. Table IV shows the results of MPO model using the data set of 18 projects supported for 10 years by research center SG09.

TABLE IV
EFFICIENCY VALUES OF 18 PROJECTS IN CENTER SG09 FOR EACH PERIOD

project	1	2	3	4	5	6	7	8
A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B	0.0000	0.0000	0.3205	1.0000	1.0000	1.0000	1.0000	1.0000
C	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6723
D	1.0000	1.0000	1.0000	0.8887	1.0000	1.0000	1.0000	1.0000
E	1.0000	1.0000	1.0000	0.5515	0.6380	1.0000	1.0000	0.8062
F	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
G	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7997
I	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
J	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
K	0.5125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9213	0.8181
M	1.0000	1.0000	1.0000	0.7368	0.7368	0.8363	0.9458	0.9458
N	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7545	1.0000
O	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
P	1.0000	1.0000	0.9441	0.7221	0.6007	1.0000	0.5280	0.2877
Q	0.8338	1.0000	1.0000	1.0000	1.0000	0.5576	0.7336	0.7029
R	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

ACKNOWLEDGMENT

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011-0005239).

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