A Condition-Based Maintenance Policy for Multi-Unit Systems Subject to Deterioration

Nooshin Salari, Viliam Makis

Abstract—In this paper, we propose a condition-based maintenance policy for multi-unit systems considering the existence of economic dependency among units. We consider a system composed of N identical units, where each unit deteriorates independently. Deterioration process of each unit is modeled as a three-state continuous time homogeneous Markov chain with two working states and a failure state. The average production rate of units varies in different working states and demand rate of the system is constant. Units are inspected at equidistant time epochs, and decision regarding performing maintenance is determined by the number of units in the failure state. If the total number of units in the failure state exceeds a critical level, maintenance is initiated, where units in failed state are replaced correctively and deteriorated state units are maintained preventively. Our objective is to determine the optimal number of failed units to initiate maintenance minimizing the long run expected average cost per unit time. The problem is formulated and solved in the semi-Markov decision process (SMDP) framework. A numerical example is developed to demonstrate the proposed policy and the comparison with the corrective maintenance policy is presented.

Keywords—Reliability, production, maintenance optimization, Semi-Markov Decision Process.

I. INTRODUCTION

MOST modern systems are complex and contain many components which are subject to deterioration over time. For many operating systems such as aircrafts, medical equipment, and power generation systems, cost of downtime due to unexpected failure is high. Therefore, maintenance programs should be developed to increase availability and reduce the operating costs.

Maintenance models aim at determining a maintenance policy to optimize system performance considering certain criteria (i.e. cost, downtime, reliability, etc.). Maintenance models can be classified as corrective and preventive maintenance models. Corrective maintenance (CM) is performed after failure of the system to restore the system to operational condition, and preventive maintenance (PM) is performed before the system fails [1]. PM is performed to reduce the maintenance cost of the system, improve system reliability, and increase system availability. PM strategies are defined as time-based and condition-based preventive maintenance [2]. Condition-based maintenance (CBM) recommends maintenance decisions based on the information obtained from inspection data [3].

N. Salari is with the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON M5S 3G8, Canada (e-mail: nsalari@mie.utoronto.ca).
V. Makis is with the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON M5S 3G8, Canada (e-mail: makis@mie.utoronto.ca).

CBM uses data collected through system inspection and recommends the maintenance actions. Early studies in the area of CBM have focused mainly on single unit systems [4], [5], however in reality most of the systems consist of more than one unit [6].

Tian et al. [7] proposed a multi-component CBM policy based on the proportional hazards model, where there is economic dependency among components. Two levels of risk threshold are considered. Level 1 is used for preventive replacement while at level 2, opportunity to maintain other components arises. The replacement decision is made based on the age of the component and its hazard value. In another study [8], authors introduced a CBM policy for a multi-component system with economic dependency to minimize the total operating and maintenance cost by using artificial neural network. The maintenance policy is defined by two failure probability thresholds. Liu et al. [9] proposed a PM policy for a system with continuously degrading components. Maintenance action is triggered whenever the system reliability drops below a certain threshold. Zhu et al. [10] proposed a condition-based maintenance model for a multi-component system subject to continuous stochastic deterioration. They determined the optimal preventive maintenance limits for components and optimal joint maintenance interval by minimizing the long-run expected average cost rate.

An optimal age-based group maintenance policy for a deteriorating multi-unit system is studied by Shafiee et al. [11]. The system under study is subject to multiple types of independent degradation processes. When the degradation level of one of the components reaches its critical size, the system undergoes an unplanned maintenance action, otherwise a planned group maintenance task is conducted for the whole system at the operational age T. An optimization model is formulated to determine the optimal group maintenance time T minimizing the long-run maintenance cost per unit time.

These research papers focused on a particular system, and fail to comprehensively study a multi-unit production system. Thus, it is important to develop an effective and comprehensive CBM model taking into account production and demand. To develop an optimal maintenance model that can be applied to production system, we need to take into account production and demand rates for the system. A failure in a production system results in disruption of production, increased costs of downtime, and lost production [12].

There are recent studies on the maintenance policies for production systems. Salari and Makis [13] proposed two new CBM policies for a multi-unit production system taking into account production and demand rates. Units are subject to gradual deterioration, and maintenance is determined by the

total production rate of the system at the time of inspection. At each inspection time, units are inspected, and if the production rate of the system is less than or equal to a critical level, maintenance is performed. Authors determined the optimal levels of the total production rate of the system at an inspection time to perform maintenance for both maintenance policies minimizing the long run expected average cost per unit time.

The purpose of this paper is to develop a CBM model taking into account production rates of the units and the system's demand rate. This model describes well the real system considered in [13], but the objective in that paper was to determine the optimal production level to initiate maintenance, whereas our focus is to obtain the optimal level to initiate maintenance based on the total number of failed units in the system. We also consider operating cost for operational units in the system.

II. MODEL DESCRIPTION

Consider a system consisting of N identical units which are subject to gradually deterioration. Let $\{X_t\}_{t \geq 0}$ be a three-state continuous time homogeneous Markov chain with two working states $O = \{0, 1\}$ and a failure state F, $\Omega = \{0, 1, F\}$. State 0 represents healthy state, state 1 represents warning state and state F represents failure state. Average production rate for units in state 0 (p_0) is higher than the average production rate in state 1 (p_1). Inspection of the system is performed at discrete equidistant time epochs $k\Delta$, $k = 1, 2, ...$, and at each inspection time, the numbers of units in each state are updated. We assume that economic dependency exists between the units which includes a high setup cost for sending maintenance personnel and equipment to a remote site.

Maintenance policy is determined by the total number of failed units at each inspection time. If the total number of failed units at an inspection time drops below a critical level, maintenance is initiated.

If the decision is to perform maintenance, maintenance crew is sent to the field and maintenance is initiated. We assume that the total maintenance time is a random variable with a known distribution function $F(\tau)$. During replacement time, units in states 0 and 1 continue working and their states can change. Failed units are replaced correctively and units in the warning state are maintained preventively. After maintenance is performed, all the units are in state 0.

Our objective is to find the optimal number of units at an inspection time to initiate maintenance, minimizing the long run expected average cost per unit time for the system.

Let $S = \{(x, m) | x + m \leq N, x \geq 0, m \geq 0\}$ be the state space of the whole system at an inspection time where x is the number of failed units, and m represents the number of units in state 1. By knowing the number of units in states 1 and F, number of units in state zero is fully determined.

It is assumed that the sojourn times in states 0 and 1 are exponentially distributed with parameters $\nu_0 = q_{01} + q_{02}$ and $\nu_1 = q_{12}$. To model monotonic system deterioration, we assume that the state process of each unit is non-decreasing with probability 1. The instantaneous transition rates q_{ij},

$i, j \in \Omega$, are defined by:

$$q_{ij} = lim_{u \to 0} \frac{P(X_{t+u} = j | X_t = i)}{u} < +\infty, \ i \neq j$$

$$and \quad q_{ii} = -\sum_{j \neq i} q_{ij} \tag{1}$$

The transition probability matrix, $P_{ij}(t) = P(X_{s+t} = j | X_s = i)$ is obtained by solving the Kolmogorov backward differential equations [14].

Next, we will describe the system states for this particular system and derive the transition probabilities, the expected cost and the sojourn time for each state of the system.

III. COMPUTING THE TRANSITION PROBABILITIES

In this section, we formulate and solve the maintenance optimization problem in the SMDP framework.

A. State Definition

1) State $(0, 0)$: all the units are in healthy state.
2) State (x, m): there are x failed units in the system and m units in the warning state.

For the cost minimization problem, the SMDP is determined by the following quantities [14]:

- $P_{i,j}$ = the probability that the system will be in state $j \in S$ at the next decision epoch given the current state is $i \in S$.
- τ_i = the expected sojourn time until the next decision epoch given the current state is $i \in S$.
- C_i = the expected cost incurred until the next decision epoch given the current state is $i \in S$.

We note that each of these components depends also on the action taken in the current state i. Once transition probabilities, costs, and the sojourn times for each state are defined, the long-run expected average cost can be obtained by solving the following system of linear equations [14]:

$$V_r = C_r - g(L) \cdot \tau_r + \sum_{k \in S} P_{r,k} \cdot V_k \tag{2}$$

$$V_j = 0 \ for \ an \ arbitrarily \ selected \ single \ state \ j \in S$$

The optimal number of failed units to initiate maintenance (L^*) and the corresponding minimum long-run expected average cost per unit time can be obtained by iteratively solving the system of linear equations in (2).

B. Computing the Transition Probabilities

At each inspection time, units of the system are inspected and if the number of failed units is smaller than the critical level (L), system continues operating and the system state is updated at the next inspection time. However if the number of failed units at an inspection time exceeds a critical level, decision is made to perform maintenance.

Assume that the system state is (x, m) at an inspection time, and the decision is not to perform maintenance ($x < L$). Transitions can occur from state (x, m) to states (x', m'),

where $x' \geq x$. This transition means that $x' - x$ units failed by the next inspection epoch and there are m' units in the warning state at the next inspection time. Define i as the number of the units which failed by the next inspection time. When there is no maintenance at the current inspection time, transition can occur from state (x, m) to state $(x + i, m')$, where $i \geq 0$, which is given by:

$$P_{(x,m)(x+i,m')}(\Delta) = \qquad (3)$$
$$\begin{cases} \sum_{j=0}^{min\{i,m\}} \binom{n_0}{i-j}\binom{m}{j}\binom{n_0-i+j}{m'-m+j} & m' \geq m \\ \quad \times P_{00}(\Delta)^{n_0'} P_{01}(\Delta)^{m'-m+j} P_{02}(\Delta)^{i-j} \\ \quad \times P_{11}(\Delta)^{m-j} P_{12}(\Delta)^j \\ \\ \sum_{j=m-m'}^{min\{i,m\}} \binom{n_0}{i-j}\binom{m}{j}\binom{n_0-i+j}{m'-m+j} & m' < m \\ \quad \times P_{00}(\Delta)^{n_0'} P_{01}(\Delta)^{m'-m+j} P_{02}(\Delta)^{i-j} \\ \quad \times P_{11}(\Delta)^{m-j} P_{12}(\Delta)^j \end{cases}$$

where $n_0 = N - m - x$ and $n_0' = N - m' - x - i$.

Assume that system state is (x, m) at an inspection time, where $x \geq L$, the decision is made to perform maintenance, and crew is sent to the field to perform maintenance. There are two maintenance policies considered in this paper. First maintenance policy suggests corrective replacement of failed units, and transition can occur from state (x, m) to state $(0, m')$ where $0 \leq m' \leq N - x$. This transition probability can be written as follows:

$$P_{(x,m)(0,m')}(\tau) = \qquad (4)$$
$$\begin{cases} \sum_{i=0}^{N-x-m'} \sum_{j=0}^{min\{i,m\}} \binom{n_0}{i-j}\binom{m}{j}\binom{n_0-i+j}{m'-m+j} & m' \geq m \\ \quad \times \int_0^\infty P_{00}(u)^{n_0'} P_{01}(u)^{m'-m+j} \\ \quad \times P_{02}(u)^{i-j} P_{11}(u)^{m-j} P_{12}(u)^j \times f_\tau(u) \ du \\ \\ \sum_{i=m-m''}^{N-x-m'} \sum_{j=m-m'}^{min\{i,m\}} \binom{n_0}{i-j}\binom{m}{j}\binom{n_0-i+j}{m'-m+j} & m' < m \\ \quad \times \int_0^\infty P_{00}(u)^{n_0'} P_{01}(u)^{m'-m+j} \\ \quad \times P_{02}(u)^{i-j} P_{11}(u)^{m-j} P_{12}(u)^j \times f_\tau(u) \ du \end{cases}$$

Second maintenance policy suggests corrective replacement of failed units and preventive maintenance of units in the warning state. For this policy, transition occurs from state (x, m) to state $(0, 0)$. This transition probability can be written as follows:

$$P_{(x,m),(0,0)}(\tau) = 1 \qquad (5)$$

C. Expected Cost and Sojourn Time

The following cost components are considered in the model:
- C_I: Inspection cost per unit at each inspection time
- C_F: Failure replacement cost per unit
- C_P: Preventive maintenance cost per unit
- C_D: Cost rate of lost production, when the total production rate of the system is below the demand rate
- C_E: Profit rate from excess production
- C_{r0}: Operating cost rate of units in state 0
- C_{r1}: operating cost rate of units in state 1
- C_K: Set-up cost for performing maintenance

We consider different production rates for the units working in states 0 and 1. The total production rate of the system is determined based on the total number of units in states 0 and 1. Total production rate of the system at state (x, m) can be written as $PR = p_0 \times n_0 + p_1 \times m$, where $n_0 = N - x - m$. If at an inspection interval total production rate of the system (PR) drops below the demand rate (D), there is lost production in that interval. If the total production rate is above the demand rate, there is a profit from excess production in that interval.

Expected cost of lost demand or an excess production profit in state (x, m) in the time interval Δ is given by the number of units in each working state at the current inspection time, that is given by :

$$Cpe_{(x,m)}(\Delta) = E(\text{profit or cost of production}|(x,m)) = \quad (6)$$
$$C_D \times Max\{0, D \cdot \Delta - n_0 \cdot \int_0^\Delta (p_0 \cdot P_{00}(u) + p_1 \cdot P_{01}(u))du$$
$$-m \times \int_0^\Delta p_1 \cdot P_{11}(u)du\}$$
$$-C_E \times Max\{0, (n_0 \cdot \int_0^\Delta (p_0 \cdot P_{00}(u) + p_1 \cdot P_{01}(u))du$$
$$+m \cdot \int_0^\Delta p_1 \cdot P_{11}(u)du) - D \cdot \Delta\}$$

Expected operating cost of the system in state (x, m) in the time interval Δ is given by:

$$Cr_{(x,m)}(\Delta) = E(\text{operating cost}|(x,m)) = \qquad (7)$$
$$C_{r0} \times n_0 \times \int_0^\Delta (P_{00}(u) + P_{01}(u))du$$
$$+C_{r1} \times m \times \int_0^\Delta P_{11}(u)du$$

At each inspection time, units of the system are inspected and the states of the units are determined. By knowing the system state, the total production rate of the system at the current state is calculated. If the total number of failed units is below the critical level L, the action is to do nothing, otherwise maintenance is initiated.

If the decision is to do nothing in state (x, m), where $x < L$, the expected cost incurred is equal to:

$$C_{(x,m)}(\tau) = C_I \cdot (N - x) + C_K + Cpe_{(x,m)} + Cr_{(x,m)} \quad (8)$$
$$= C_I \cdot (N - n_F) + C_K + C_D \times Max\{0, D \cdot \Delta$$
$$-n_0 \cdot \int_0^\Delta (p_0 \cdot P_{00}(u) + p_1 \cdot P_{01}(u))du$$
$$-m \cdot \int_0^\Delta p_1 \cdot P_{11}(u)du\}$$
$$-C_E \times Max\{0, (n_0 \cdot \int_0^\Delta (p_0 \cdot P_{00}(u) + p_1 \cdot P_{01}(u))du$$
$$+m \cdot \int_0^\Delta p_1 \cdot P_{11}(u)du) - D \cdot \Delta\}$$
$$+C_{r0} \times n_0 \times \int_0^\Delta (P_{00}(u) + P_{01}(u))du$$
$$+C_{r1} \times m \times \int_0^\Delta P_{11}(u)du$$

If the decision is to initiate maintenance, crew is sent to the field and maintenance is performed following the particular maintenance policy.

First maintenance policy suggests corrective replacement of failed units. After replacement is performed, there are no failed

units in the system and all the units are working in state 0 or 1. For this policy, the expected cost incurred in state (x, m), when maintenance is required is equal to :

$$C_{(x,m)}(\tau) = C_I \cdot (N - x) + C_K + Cpe_{(x,m)} + Cr_{(x,m)} \quad (9)$$

$$+ C_F(x + \int_0^\infty E(\text{\# of units failing in } \tau | \tau = u) \cdot f_\tau(u) du))$$

$$= C_I \cdot (N - n_F) + C_K + C_D \times Max\{0, D \cdot E(\tau)$$

$$- n_0 \int_0^\infty \int_0^u (p_0 \ P_{00}(t) + p_1 \ P_{01}(t)) \ f_\tau(u) \ dt \ du$$

$$- m \int_0^\infty \int_0^u p_1 \ P_{11}(t) f_\tau(u) \ dt \ du\}$$

$$- C_E \times Max\{0, n_0 \int_0^\infty \int_0^u (p_0 \ P_{00}(t) + p_1 \ P_{01}(t)) \ f_\tau(u) \ dt \ du$$

$$+ m \cdot \int_0^\Delta p_1 \cdot P_{11}(u) du) - D \cdot E(\tau)\}$$

$$+ C_{r0} \cdot n_0 \cdot \int_0^\infty \int_0^u (P_{00}(u) + P_{01}(u)) f_\tau(u) \ dt \ du$$

$$+ C_{r1} \cdot m \cdot \int_0^\infty \int_0^u P_{11}(u) f_\tau(t) \ dt \ du$$

$$+ C_F(n_F + n_0 \int_0^\infty P_{02}(u) f_\tau(u) \ du + m \int_0^\infty P_{12}(u) f_\tau(u) \ du)$$

where $E(\tau) = \int_0^\infty u \cdot f_\tau(u) \ du$

Second maintenance policy prescribes corrective replacement of failed units and preventive maintenance of units in state 1, when the total number of failed units reaches or exceeds a critical level L. For this policy, the expected cost incurred in state (x, m), when maintenance is needed is equal to :

$$C_{(x,m)}(\tau) = C_I \cdot (N - x) + C_K + Cpe_{(x,m)} + Cr_{(x,m)} \quad (10)$$

$$+ C_F(x + \int_0^\infty E(\text{\# of units failing in } \tau | \tau = u) \cdot f_\tau(u) du))$$

$$+ C_P(\int_0^\infty E \ (\text{\# of units in state 1 at } \tau | \tau = u) \cdot f_\tau(u) du))$$

$$= C_I \cdot (N - n_F) + C_K + C_D \times Max\{0, D \cdot E(\tau)$$

$$- n_0 \int_0^\infty \int_0^u (p_0 \ P_{00}(t) + p_1 \ P_{01}(t)) \ f_\tau(u) \ dt \ du$$

$$- m \int_0^\infty \int_0^u p_1 \ P_{11}(t) f_\tau(u) \ dt \ du\}$$

$$- C_E \times Max\{0, n_0 \int_0^\infty \int_0^u (p_0 \ P_{00}(t) + p_1 \ P_{01}(t)) \ f_\tau(u) \ dt \ du$$

$$+ m \int_0^\infty \int_0^u p_1 \ P_{11}(t) f_{T_{R_2}}(u) \ dt \ du - D \times E(\tau)\}$$

$$+ C_{r0} \cdot n_0 \cdot \int_0^\infty \int_0^u (P_{00}(u) + P_{01}(u)) f_\tau(u) \ dt \ du$$

$$+ C_{r1} \cdot m \cdot \int_0^\infty \int_0^u P_{11}(u) f_\tau(t) \ dt \ du$$

$$+ C_F(n_F + n_0 \int_0^\infty P_{02}(u) f_\tau(u) \ du + m \int_0^\infty P_{12}(u) f_\tau(u) \ du)$$

$$+ C_P(n_0 \int_0^\infty P_{01}(u) f_\tau(u) \ du + m \int_0^\infty P_{11}(u) f_\tau(u) \ du)$$

After calculating the expected cost for each state, we need to calculate the mean sojourn time in each state of the system.

The expected sojourn time in state (x, m) when the action is to do nothing is equal to Δ because the next inspection occurs after Δ time units.

$$\tau_{((x,m),a=0)} = \Delta \quad (11)$$

The expected sojourn time in state (x, m) when the action is to do maintenance for both policies is equal to the total expected maintenance time.

$$\tau_{((x,m),a=1)} = E(\tau) \quad (12)$$

D. Numerical Example

In this section, we illustrate the proposed model and the maintenance policies with a numerical example.

We consider a multi-unit system consisting of 6 units subject to condition monitoring. We assume that the deterioration process of each unit of the system follows a continuous time homogeneous Markov chain with two operating states $\{0, 1\}$, and a failure state $\{F\}$ which is absorbing, $\Omega = \{0, 1, F\}$. The sojourn time in state 0 has an exponential distribution with parameter $\nu_0 = q_{01} + q_{02}$, and the sojourn time in state 1 has an exponential distribution with parameter $\nu_1 = q_{12}$.

$$q_{01} = 0.026 \times 10^{-2}, \ q_{02} = 0.014 \times 10^{-2}, \ \nu_0 = 0.04 \times 10^{-2},$$

$$\nu_1 = q_{12} = 0.3 \times 10^{-2}$$

The average production rate of a unit in state 0 is $p_0 = 40$, and in state 1, it is $p_1 = 28$. We assume a constant demand rate for the system $D = 150$.

The following costs will be considered in the experiment:

$$C_I = 100, \ C_F = 35000, \ C_P = 6500, \ C_D = 0.4, \ C_E = 0.2,$$

$$C_{r0} = 2, \ C_{r1} = 4, \ C_K = 4000$$

System is inspected every $\Delta = 300$ time units and the number of units in each state is determined. If the total number of failed units at an inspection time is greater than or equal to the critical level L, maintenance is initiated. Maintenance time follows a Gamma distribution with parameters $\alpha = 6$ and $\beta = 0.3$ for policy 1, and with parameters $\alpha = 6$ and $\beta = 0.2$ for policy 2. Gamma density function with parameters α and β is given by:

$$f(t) = \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\beta t} \quad (13)$$

Table I presents the optimal levels to initiate maintenance and the corresponding expected average costs per unit time for both policies.

TABLE I
OPTIMAL LEVELS TO INITIATE MAINTENANCE AND THE
CORRESPONDING EXPECTED AVERAGE COSTS FOR BOTH POLICIES

Policy	Optimal level (L^*)	Expected average cost rate
Corrective maintenance	3	51.26
Corrective and preventive maintenance	2	44.94

Results in Table I indicate that optimal level to initiate maintenance for policy 1 is $L^* = 3$. This optimal value indicates that at an inspection time a decision is made to initiate maintenance, if there are at least 3 failed units in the system. The corresponding expected average cost for this policy is 51.26. The optimal level to perform maintenance for policy 2 is equal to $L^* = 2$, with the expected average cost

rate of 44.94. The optimal level to initiate maintenance for the second policy is lower than the first policy, as we considered PM in the second policy. By applying Policy 2, more units are operating in the healthy state after maintenance compared to Policy 1. Units in state 0 have higher production rate, and the total production rate of the system is higher after maintenance for Policy 2. As a result, there is a lower expected cost of lost production and a higher expected profit from selling extra production for Policy 2 compared to Policy1.

We observe that Policy 2 gives lower expected average cost rate compared to Policy 1. Results show that the second policy outperforms the first policy, with the expected cost savings of 12.33%.

IV. Conclusions and Future Research

In this paper, we have proposed two condition-based maintenance policies for an N-unit system considering the economic dependency among units. Units of the system deteriorate independently, and deterioration process of each unit is modeled as a three-state continuous time homogeneous Markov chain with two working states and a failure state. Inspection takes place every Δ time units to observe the state of each unit. The average production rate of units varies in different working states and demand rate of the system is constant. Maintenance action is determined by the total number of failed units in the system at the time of inspection.

Our objective has been to determine the optimal number of failed units at an inspection time to initiate maintenance for both maintenance policies minimizing the long run expected average cost per unit time. The problem has been formulated and solved in the semi-Markov decision process (SMDP) framework. A numerical example has been provided to illustrate the proposed maintenance policies, and comparison of the two policies has been made. The results have shown that it is cost effective to consider opportunistic preventive maintenance of the units in the warning state when the failed units are replaced .

In future research, we will extend the model developed in this paper by considering a general N state deterioration process instead of a three state deterioration. It is also interesting to consider a joint spare part ordering and preventive maintenance policy for the multi-unit system studied in this paper.

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Nooshin Salari is a Ph.D. candidate in the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada. Her research interests are in stochastic modeling, condition-based maintenance (CBM), maintenance and production optimization with a special focus on multi-unit systems. Her research is driven by practical problems such as optimal joint production and maintenance control for energy systems. She is a student member of IIE and member of the Association of Mechanical and Industrial Engineering Graduate Students (AMIGAS).

Viliam Makis is a Professor in the Department of Mechanical and Industrial Engineering, University of Toronto. His research interests are in modeling and optimization of partially observable stochastic systems with a special focus on investigating structural properties of the optimal operating policies for such systems, off-line and on-line model parameter estimation and filtering. His research is driven by practical problems such as the development of fault detection and diagnostic schemes for conditionbased maintenance (CBM), optimal joint production and maintenance control, the optimal multivariate quality control for both short and long production runs, and the development of optimal sampling schemes for monitoring partially observable stochastic processes. He is a Fellow of the Int. Society of Engineering Asset Management, a Senior Member of the Institute of Industrial Engineers, of the American Society for Quality, and a member of INFORMS and CORS.