International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:1, 2013

A Comparative Study on Different Approaches to Evaluate Ship Equilibrium Point

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Abstract**—**The aim of this paper is to present a comparative study on two different methods for the evaluation of the equilibrium point of a ship, core issue for designing an On Board Stability System (OBSS) module that, starting from geometry information of a ship hull, described by a discrete model in a standard format, and the distribution of all weights onboard calculates the ship floating conditions (in draught, heel and trim).

Keywords**—**Algorithms, Computer applications, Equilibrium, Marine applications, Stability System.

I. INTRODUCTION

THE ships are still vital to the economy of many countries and they still carry some 95 per cent of world trade. In 1998 the world's cargo fleet totalled some 775 million tonnes deadweight and was increasing by 2 per cent a year [1]. The average deadweight was about 17000. Although aircraft have displaced the transatlantic liner, ships still carry large numbers of people on pleasure cruises and on the multiplicity of ferries in all areas of the globe. Ships, and other marine structures, are needed to exploit the riches of the deep. Although one of the oldest forms of transport, ships, their equipment and their function, are subject to constant evolution.

Changes are driven by changing patterns of world trade, by social pressures, by technological improvements in materials, construction techniques and control systems, and by pressure of economics. As an example, technology now provides the ability to build much larger, faster ships and these are adopted to gain the economic advantages those features can confer [2]. A crucial information for a naval operator is the current floating conditions (in draught, heel and trim) of the ship related to the actual loading condition but more interesting is the possibility to evaluate the new floating conditions corresponding to a different load distribution and its reserve of stability. An intuitive, qualitative understanding of stability and of the risks of insufficient stability must have existed for millennia. The foundations for scientific physical explanation and for a quantitative assessment of hydrostatic stability were laid by Archimedes in antiquity. Yet despite many important contributions and partially successful attempts by scientists in early modern era like Stevin, Huygens and Hoste among others it took until almost the mid-eighteenth century before a mature scientific theory of ship hydrostatic stability was formulated and published. Pierre Bouguer and Leonard Euler were the founders of modern ship stability theory, who quite independently and almost simultaneously arrived at their landmark result for hydrostatic stability criteria. The full implementation of computational methods for evaluating these criteria and their acceptance by practitioners took even several decades longer. A beautiful excursus of the history can be found in [3]. The availability of computer systems on board provides the possibility to simulate in real time the effect of a change in load condition and its influence in the new stability condition. In every case the core of all stability systems is the framework that evaluates the ship equilibrium point in terms of draft, heel and trim angle related to a particular load conditions. In this paper the authors focus on a comparative study of two different approaches to achieve this framework: sectional approach and mesh approach.

The skill to evaluate the equilibrium point by numerical procedure gives the ability of a well structured code to perform the righting arms calculations when the ship is "free to trim" as required by the newest regulations from 90 years (SOLAS '92).

This paper is organized as follows: in section 2 it is reported the mathematical approach to the problem and the simplifications needed; in section 3 and section 4 are presented the sectional approach and the mesh approach respectively; in section 5 two test cases obtained with the two approaches are presented and, in section 6, some conclusions and future work are inserted.

II. PROBLEM FORMULATION

The ship is a complex structure and it is, not in physical sense, a rigid body. However, for the purpose of studying its stability it is permissible so to regard it. Throughout this paper the ship will be regarded as a rigid body in calm water and not underway. For the ship in waves or underway there are hydrodynamic forces acting on it which may affect the buoyancy forces. This problem is well described in [4], [5].

Nevertheless, for stability purposes, it is usual to ignore hydrodynamic forces except for high-speed craft, including hydrofoils. For planning hull the hydrodynamic forces prevail in assessing stability [6]-[8].

From the statics of rigid bodies follows that necessary and sufficient condition so that a body is in equilibrium are

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(Cardinal equations of statics):

- Equilibrium of forces (the sum of the forces acting on the body is zero) (see Eq. (1));
- Equilibrium of moments (the sum of moments produced by such forces respect to a polo is zero) (see Eq. (2)).

$$\sum_{i=1}^{n} \vec{F}_i = 0 \qquad (1)$$

$$\sum_{i=1}^{n} M_{x_i} = 0\,; \sum_{i=1}^{n} M_{y_i} = 0\,; \sum_{i=1}^{n} M_{z_i} = 0 \qquad (2)$$

If we take into account a body hull in order to assess the equilibrium condition, for a rigid body, it is worth noting that the Eq.(1) and (2) are simplified because the only forces that acting on the system are the weight ($\vec{\Delta}$) acting in centre of gravity ($\vec{\mathrm{G}}$) and the buoyancy ($\vec{S}$) acting in its centre ($\vec{B}$), as shown in Fig. 1.

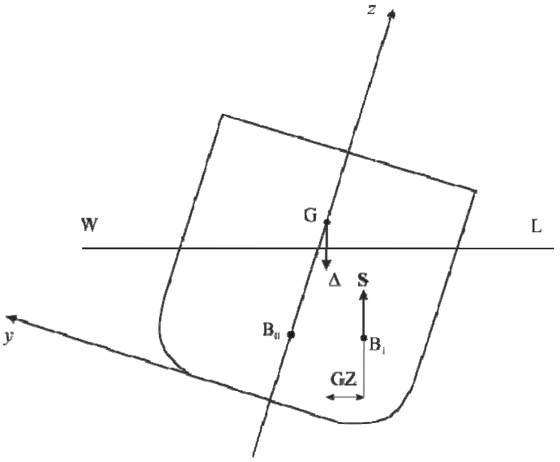


Fig. 1 Forces which act on any floating object.

As the weight and centre of gravity are both functions of the loading condition, they could be considered fixed for the equilibrium assessment. Otherwise the buoyancy and its point of application depend on the hull geometry and are non-linear functions of the position of the ship in the water. This position is function of the heel angle (ϑ_{T}), the trim angle (ϑ_{L}) and the draft (T). In order to evaluate the equilibrium condition of non-linear system described above, it is necessary to use adaptive convergence methods, where iterations on the draft, trim and heel angles are performed until both sum of the forces and sum of moments acting on the ship are zero.

This paper presents two different approaches to solve the problem of the evaluation of equilibrium condition, i.e. sectional approach and mesh approach. The sectional approach comes from the traditional description of the hull by means of the vertical lines plan while the mesh approach takes advantage of the three-dimensional description of the hull surface used in modern CAD applications.

III. SECTIONAL APPROACH

The Sectional Approach uses as input the description of the hull in terms of transversal sections. The ability of the developed in-house [4]-[5] software is to evaluate the stability at upright condition and at large angles for a ship in still water and in longitudinal waves. In the *Fig. 2* a sketch of the used body hull representation has been presented. It is worth noting that in *Fig. 2* the blue lines represent the submerged parts of the ship, while the black ones show the entire hull.

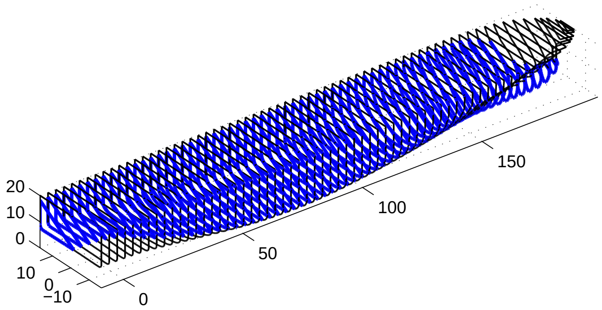


Fig. 2 Example of body hull input for Sectional Approach

The ship waterline is the result of geometrical intersection between the hull and undisturbed free surface. The computational tool is able to evaluate any geometrical property of the hull at any longitudinal or transversal angle of inclination: by a numerical integration over the sections all the geometrical characteristics are evaluated, for example the sectional submerged area and the breadth of the waterline. Then by a longitudinal integration all global characteristics are computed, e.g. the centre of buoyancy and the hull volume.

The second feature of the code focuses on the calculation of the righting arm GZ, given a fixed volume and a transversal angle. Calculations are carried out considering the ship free to trim, and superimposing the transversal angle. Therefore this routine is able to find the righting arm (GZ) for any load condition, any imposed roll angle.

In Fig. 3 the flow chart used inside the numerical code is shown. In particular, it is possible to note that the procedure uses a geometric core in order to evaluate the floating characteristics ($\vec{\mathrm{S}}, \vec{\mathrm{B}}$) starting from the position of the ship in the water ($\vartheta_{\mathrm{L}}, \vartheta_{\mathrm{T}}, \mathrm{T}$), and an iterative method that assure to find the equilibrium condition, solving the non-linear system, under any (feasible) load condition. The iterative method is based on two one-dimensional Newton-Raphson method, where the derivatives are evaluated using an explicit approximation of them.

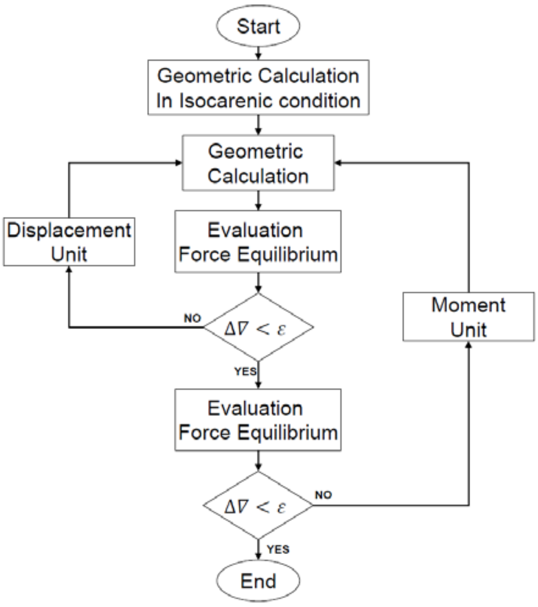


Fig. 3 Sectional approach flow chart.

IV. MESH APPROACH

The Mesh Approach assumes as input the 3D hull model and the distribution of all weights onboard. The 3D hull model input file describes the ship hull geometry. It is a mesh file described by a collection of facets, each of them composed by three vertices in the geometric reference system.

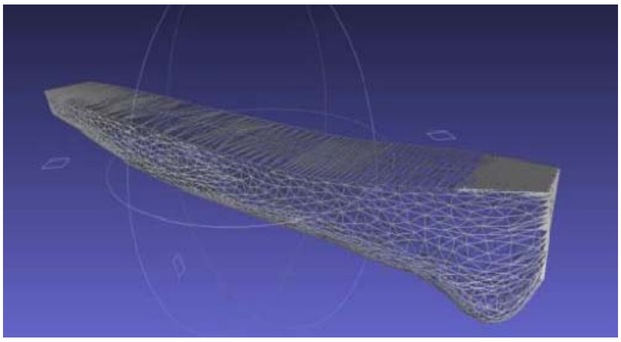


Fig. 4 An example of mesh used for modelling the hull.

Consider the ship in a particular floating condition established by the draft, heel and trim parameters. The calculation of the submerged volume of the ship is obtained as the summation of the volume of all tetrahedrons having a common vertex on cutting plane and bases equal to the submerged part of each facet of the hull model, while the centre of buoyancy, representing the geometric centre of the submerged part of the ship, is calculated referring always to the submerged part of the facet using geometric algorithm.

In the calculation, the exact contribution of the facets is determined. In *Fig. 5* an example of partially immersed triangular facet is showed. Such facet will contribute in the calculation only for the polygon represented by the vertex (N1, N2, V2, V3) as shown in *Fig. 5* (b) and can be decomposed as set of triangular facets (N1, N2, V2) and (V3, N1, V2) as shown in *Fig. 5* (c):

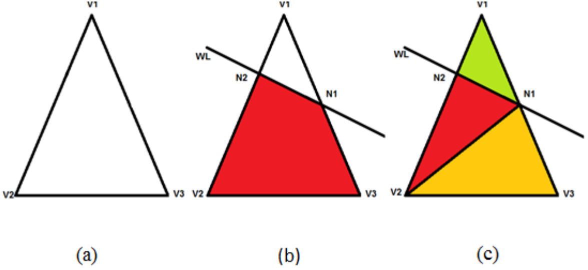


Fig. 5 Triangular facet (a), submerged (b), rebuilt as sum of triangular facets

The algorithm for this calculus comes from gaming programming [9]. In fact, rigid body simulation brings many new capabilities and challenges. In many action games, for example, the player can jump on a body in the water and expects it to float in a believable manner. For this to happen, the game must simulate buoyancy realistically. The Mesh Approach implements an efficient method to compute buoyancy on rigid bodies (see [10]-[12], through algorithms based on a FEA-like method (Finite Elements Analysis).

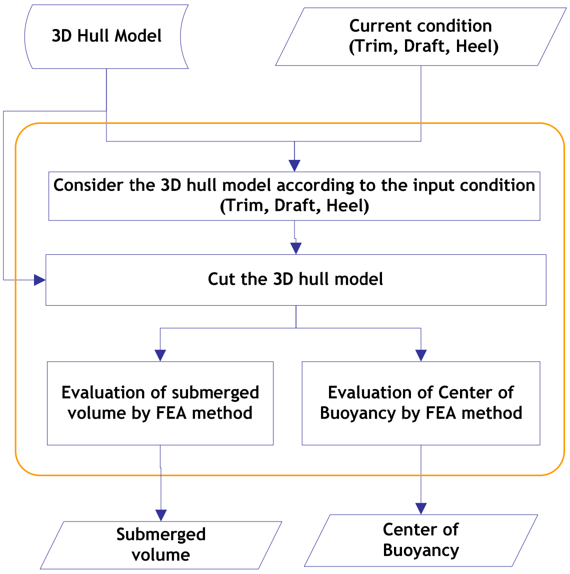


Fig. 6 Mesh Approach: Flow chart for the calculation of submerged volume and Centre of Buoyancy.

Starting from the loading conditions given as input, the displacement and the location of the center of gravity are calculated. The output of the calculus is the triple of parameters that defines the floating condition, i.e. the draft (T), the heel (ϑ_T) and the trim (ϑ_L) angles. The calculations take into account the following constraints: the nonnegative property of the mean draft, and that the heel angle and trim angle lie in the interval of $[-\pi/2, \pi/2]$.

Hydrostatic analysis generally involves iterations; then attention has been paid to the optimization of the iteration numbers trying to perform one shot analysis to determine the exact contribution of total weight force, buoyancy force, waterline area, inertia, etc.

For obtaining the floating condition while the ship freely floats in the still water (i.e. the draft, heel angle and trim angle to balance the displacement and the total weight so that the perpendicular distance between the acting lines of the buoyancy and weight, i.e. the global righting lever, to be zero) a Newton-like iteration method has been used. The diagram flow of the Mesh Approach is reported in *Fig. 6.* A detailed explanation can be found on [13].

V. CASE STUDY

To compare the two previous approaches, the authors present two different loading conditions on the same ship. Among the possible systems of coordinates the paper follows the DIN 81209-1 standard as shown in *Fig. 7*.

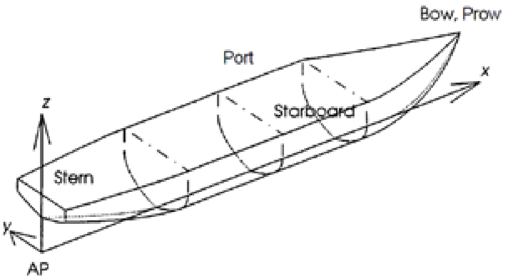
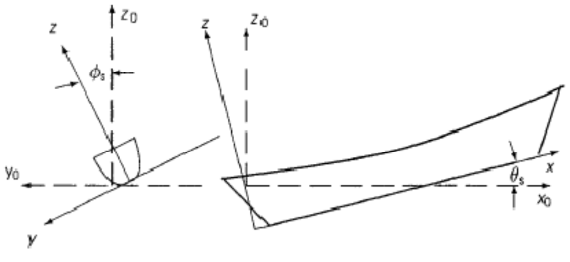


Fig. 7 System of coordinates according to DIN 81209-1 standard

The *x-axis* runs along the ship and is positive forwards, the *y-axis* is transversal and positive to port, and the *z-axis* is vertical and positive upwards. The origin of coordinates lies at the intersection of the centreline plane with the transversal plane that contains the aft perpendicular. The system of coordinates used for the hull surface is also employed for the location of weights. By its very nature, the system in which the hull is defined is fixed in the ship and moves with it. To define the various floating conditions related to the position assumed by the ship (as reported in [14]), it is necessary to use another system, fixed in space defined in ISO 7463 as x_0, y_0, z_0.

Let this system initially coincident to the system x, y, z. A vertical translation of the system x, y, z with respect to the space-fixed system x0, y0, z0 produces a draught change. If the ship-fixed z-axis is vertical, it say that the ship floats in an upright condition. A rotation of the ship-fixed system around an axis parallel to the x-axis is called heel (Fig. 8) if it is temporary, and list if it is permanent. The heel can be produced by lateral wind, by the centrifugal force developed in turning, or by the temporary, transverse displacement of weights. The list can result from incorrect loading or from flooding.



(a) heel (b) trim

Fig. 8 Heel and Trim

If the transverse inclination is the result of ship motions, it is time-varying and it call it roll. When the ship-fixed x-axis is parallel to the space-fixed x0-axis, it says that the ship floats on even keel. A static inclination of the ship-fixed system around an axis parallel to the ship-fixed y-axis is called trim. If the inclination is dynamic, that is a function of time resulting from ship motions, it is called pitch. A graphic explanation of the term trim is given in Fig. 8. The trim is measured as the difference between the forward and the aft draught, so it is measured in metres. Trim is positive if the ship is trimmed by the head.

The main dimensions of the hull used into the two test are reported in Table.

TABLE I
MAIN DIMENSIONS OF THE HULL USED IN TEST CASES.

Length Between Perpendiculars (LBP)	126.5 m
Breadth (B)	21.6 m
Depth (D)	10.23 m

To verify the correctness of given results of the two approaches, the same loading condition are inserted on a COTS software named AVEVA Marine [15].

A. First Test Case: Lightweight

The first test case considers the ship completely empty. No other information is required in addition to the lightweight as reported in TABLE .

In

TABLE the comparison results are reported, where the trim is evaluate by the stern, and the positive heel is in port side direction. It’s very nice to see the goodness of the two different approaches.

TABLE II
FIRST TEST CASE: LOADING CONDITION.

Title	Weight	LCG[1]	TCG[1]	VCG[1]
Lightweight	5000.0 t	60.52 m	0.05 m	7.92 m

[1] (LCG, TCG, VCG) = Longitudinal, Transversal and Vertical component of Centre of gravity.

TABLE III
FIRST TEST CASE: COMPARISON RESULTS

Title	AVEVA Marine	Sectional Approach	Mesh Approach
Mean draft at midship	4.411 m	4,4222 m	4.418 m
Heel	2.68 °	2.6901°	2.621°
Trim	2.551 m	2.5265 m	2.559 m

B. Second Test case: Deep Loading

The second test case considers the ship in deep condition. All loads are reported in TABLE and the comparison results in *TABLE* . It’s very nice to see the goodness of the two different approaches.

TABLE IV
SECOND TEST CASE: LOADING CONDITION.

Title	Weight	LCG	TCG	VCG
Lightweight	5000 t	60.52 m	0.05 m	7.92 m
Fuel Oil	1000 t	62.50 m	0.01 m	1.50 m
Fresh Water	120 t	61.50 m	0.00 m	3.00 m
Crew and Effects	60 t	66 m	0.00 m	7.00 m
Other Effects	1200 t	68 m	-0.02 m	8.20 m

TABLE V
SECOND TEST CASE: COMPARISON RESULTS.

Title	AVEVA Marine	Sectional Approach	Mesh Approach
Mean draft at midship	5.796 m	5,801 m	5.802 m
Heel	1.23 °	1.2103 °	1.205 °
Trim	1.493 m	1.4840 m	1.501 m

VI. CONCLUSION

In this paper, we have presented the logical and functional architecture of the equilibrium condition component of the OBSS module. Two different approaches are described showing their equivalence. The increase in computer performance over time continues to be exponential growth in not only the performance of supercomputers but also that of personal micro-computers. What is also interesting is the application of Graphics Processing Units (GPUs) rather than CPUs to solving CFD flows ([16]). In fact, using the GPUs, through the NVIDIA's CUDA (Compute Unified Device Architecture) programming model it’s possible to harness the massive parallelism of the GPU based systems optimised for floating-point calculations and matrix inversion much more than CPUs (which are required to perform a much broader range of operations), reaching elevate speedup (over 10×). This observation give sense to next research that will be addressed to realize the equilibrium condition component of the OBSS via GPU to reduce the elaboration time. An example of the speedup that can be reach by CUDA in very different fields than gaming or graphics can be found in [17].

ACKNOWLEDGMENT

The authors are grateful to other collaborators involved into design and implementation activities. A particular acknowledgment goes to Luisa Mancarella (Apphia s.r.l) for her support in development of mesh approach.

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