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Effect of Two Radial Fins on Heat Transfer and Flow Structure in a Horizontal Annulus

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Abstract**—**Laminar natural convection in a cylindrical annular cavity filled with air and provided with two fins is studied numerically using the discretization of the governing equations with the Centered Finite Difference method based on the Alternating Direction Implicit (ADI) scheme. The fins are attached to the inner cylinder of radius r_i (hot wall of temperature T_i). The outer cylinder of radius r_o is maintained at a temperature T_o ($T_o < T_i$). Two values of the dimensionless thickness of the fins are considered: 0.015 and 0.203. We consider a low fin height equal to 0.078 and medium fin heights equal to 0.093 and 0.203. The position of the fin is 0.82π and the radius ratio is equal to 2. The effect of Rayleigh number, *Ra*, on the flow structure and heat transfer is analyzed for a range of *Ra* from 10^3 to 10^4. The results for established flow structures and heat transfer at low height indicate that the flow regime that occurs is unicellular for all *Ra* and fin thickness; in addition, the heat transfer rate increases with increasing Rayleigh number and is the same for both thicknesses. At median fin heights 0.093 and 0.203, the increase of Rayleigh number leads to transitions of flow structure which correspond to significant variations of the heat transfer. The critical Rayleigh numbers, $Ra_{c.app}$ and $Ra_{c.disp}$ corresponding to the appearance of the bicellular flow regime and its disappearance, are determined and their influence on the change of heat transfer rate is analyzed.

Keywords**—**Natural convection, fins, critical Rayleigh number, heat transfer, fluid flow regime, horizontal annulus.

I. Introduction

THE analysis of heat transfer in an annular cavity with fins has been studied in depth in recent decades due to its importance in many applications which can be simplified in such a model. This type of research can be applied to the cooling of nuclear reactors, electronic equipment [1] and solar collectors [2]. A very large number of researches [3]-[6] have been carried out to study the flow structure and the heat transfer by natural convections in different geometries including square and annular enclosures.

Nag et al. [3] studied the effect of a fin attached to one of the active walls of a differentially heated square cavity on heat transfer. The effect of fins with different conductivity values (insulating to fully conductive) was analyzed and the results showed that as the fin thickness is reduced, heat transfer across the cavity decreases at first, until a critical thickness of the fin is reached. Beyond this value, heat transfer increases as the fin thickness is further reduced. It was also shown that the conductivity of the fin has a negligible effect on the rate of heat transfer. Elatar et al. [4] studied the effect of the fin thickness with different thermal conductivities on the mean Nusselt number in a square enclosure with active vertical walls. This study showed that for the fins with thermal conductivity ratios between the fin and the air equal to 10 and 100, the thickness of the fins has a minimal impact on heat transfer, for the fins with a thermal conductivity ratio equal to 1000, this impact is nonexistent. It has also been mentioned that the rate of heat transfer increases with the increase in the height of the fins and the thermal conductivity. The study of the influence of the fins of height 0.2 on heat transfer in an annular cylindrical cavity filled with air by [5] has shown that the heat exchanges generated in an annular cylindrical cavity are favored in a significant way in the case of the developed convection regime, by placing the fins in the upper part of the annular space with the appearance of a bicellular regime. After that, [6] studied the effect of fins with moderate heights ($0.14 \le h \le 0.25$) on heat transfer and flow structure in a cylindrical annular cavity. The thickness of the fins considered is equal to 0.109. The results show that the multicellular regime is characterized by a heat transfer rate significantly higher than that corresponding to unicellular regime.

The aim of this work is to analyze the effect of the variation of the Rayleigh number in a cylindrical annular cavity equipped with two fins having three heights and two thicknesses through the flow structure, the intensity of the fluid as well as the heat transfer obtained.

II. Methodology

The schematic of the geometry analyzed is consisting of an annular space filled with air (Pr = 0.7), bounded by two isothermal cylinders, coaxial, horizontal, very long, which allows us to consider the problem as two-dimensional. Temperatures remain uniform along the cylinder walls and the fins. The isothermal fins are arranged on the hot wall of temperature T_i (inner cylinder of radius r_i) symmetrically with respect to the vertical plane containing the axis of the cylinders. The cold wall (the outer cylinder of radius r_o) is at a temperature T_o ($T_o < T_i$). The radius ratio of the cylinders is $R = r_o/r_i = 2$. The mathematical modeling of the problem is based on the principles of conservation of momentum, energy and mass. The system

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obtained is completed by introducing the boundary conditions on the hot and cold walls and on the boundaries of the fins assumed to be very conductive.

The fluid occupying the annular space is air (Pr = 0.7) assumed to be incompressible viscous, obeying the Boussinesq approximation. The problem considered is two-dimensional, steady, and laminar. The non-dimensional governing equations written in vorticity-stream function formulation are:

$$\Delta\psi + \omega = 0 \tag{1}$$

$$\frac{\partial\omega}{\partial t} + U\frac{\partial\omega}{\partial r} + \frac{V}{r}\frac{\partial\omega}{\partial\varphi} = Pr\Delta\omega + Ra\,Pr\left(\sin\varphi\frac{\partial T}{\partial r} + \frac{\cos\varphi}{r}\frac{\partial T}{\partial\varphi}\right) \tag{2}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial r} + \frac{V}{r}\frac{\partial T}{\partial\varphi} = \Delta T \tag{3}$$

The stream function is related to the radial and tangential components of the dimensionless velocity by the following relations:

$$U = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}, V = -\frac{\partial\psi}{\partial r} \tag{4}$$

Ra and Pr are respectively, the Rayleigh and Prandtl numbers defined by:

$$Ra = \frac{g\beta(T_i - T_o)r_i^3}{\nu\alpha}, Pr = \frac{\nu}{\alpha} \tag{5}$$

α, ν and β are respectively, the thermal diffusivity, the kinematic viscosity and the coefficient of thermal expansion.

The boundary conditions write:

- On the inner wall:

$$r = 1: \psi = 0, \frac{\partial\psi}{\partial r} = 0, \frac{\partial^2\psi}{\partial r^2} + \omega = 0, T = 1 \tag{6}$$

- On the outer wall:

$$r = R: \psi = 0, \frac{\partial\psi}{\partial r} = 0, \frac{\partial^2\psi}{\partial r^2} + \omega = 0, T = 0 \tag{7}$$

- On the border of the isothermal fins:

$$\psi = 0, \frac{\partial\psi}{\partial\varphi} = 0, \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2} + \omega = 0, T = 1 \text{ for } 1 \le r \le 1 + h \tag{8}$$

$$\psi = 0, \frac{\partial\psi}{\partial r} = 0, \frac{\partial^2\psi}{\partial r^2} + \omega = 0, T = 1 \text{ at } r = 1 + h \tag{9}$$

It is furthermore considered that the problem is symmetrical. The symmetry conditions write:

$$\varphi = 0 \quad \text{and} \quad \varphi = \pi : \psi = 0, \omega = 0 \quad \text{and} \quad \frac{\partial T}{\partial\varphi} = 0 \quad \forall r$$

The integration of the system (1)-(3) is based on the discretization of these equations by the Centered Finite Difference method with an ADI scheme. This leads to solve tridiagonal matrix systems using Thomas algorithm. The initialization of the calculations is carried out by the introduction of zero fields of the stream function and vorticity, and a pure conduction temperature field ($T = 1 - \ln r/\ln R$). The fluid flows considered in this study are steady-state and laminar. The fins held at the angular position $\varphi_m = 0.82\pi$ from downward, have a dimensionless thickness $w = (\varphi_{max} - \varphi_{min})/\pi$.

The average Nusselt number is defined by:

$$\overline{Nu} = -\frac{R}{\pi}\ln R\int_0^{\pi}\left.\frac{\partial T}{\partial r}\right|_{r=R} d\varphi \tag{10}$$

III. Results

The present work investigates the effect of Rayleigh number Ra on the flow structure and the heat transfer rate in a cylindrical annular cavity equipped with fins of different heights for two thicknesses 0.015 and 0.203. The Rayleigh number varies from 1000 to 10 000, and we consider three fins heights 0.078, 0.093 and 0.203.

At the low height h = 0.078, a unicellular regime, similar to the case without fins is established for all Rayleigh numbers and fin thicknesses. Fig. 1 shows the streamlines and the isotherms for the thickness w = 0.023 at $Ra = 10^3$ and $Ra = 10^4$. A distortion of the isothermal lines is noticed at the large Rayleigh number.

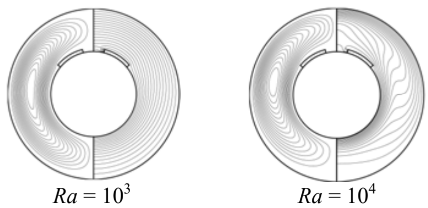

Fig. 1 Streamlines and isotherms at h = 0.078 for w = 0.203

The variation of the intensity of the main cell when Ra increases from 10^3 to 10^4 at this low height 0.078 for the two thicknesses indicates that the rise in the Rayleigh number leads to a continuous growth of ψ_{max} which remains the same for the two thicknesses especially for low Rayleigh numbers (Fig. 2). For $Ra = 10^3$ at w = 0.015 and w = 0.203, ψ_{max} is equal to 2.424. For $Ra = 10^4$ at w = 0.015 ψ_{max} is equal to 15.07 and at w = 0.203 ψ_{max} is equal to 14.90.

The Nusselt number reflecting the importance of the heat transfer rate indicates that the heat transfer increases gradually as the Rayleigh number varies from 10^3 to 10^4 (Fig. 3). The variation curves of $\overline{Nu}$ as function of Ra for both thicknesses are almost juxtaposed for this low height. The heat transfer remains practically the same for both thicknesses 0.015 and

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0.203 at a given Rayleigh number. At $Ra = 10^3$, $\overline{Nu} = 1.044$ for $w = 0.015$ and $\overline{Nu} = 1.066$ for $w = 0.203$. The relative difference is 2%. A correlation between Nusselt number and Rayleigh number can be proposed: $\overline{Nu} = 0.345 \times ln(Ra) - 1.313$ with the coefficient $R^2 > 96\%$.

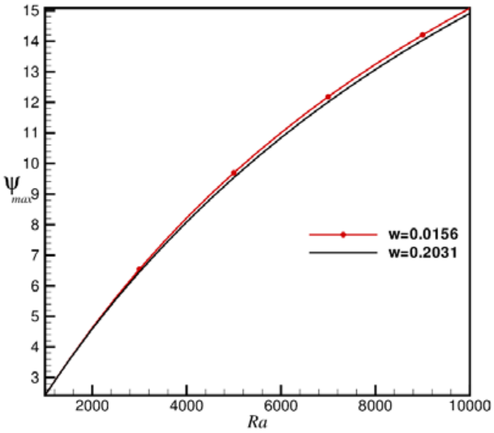


Fig. 2 Variation of the intensity of the main cell as function of Rayleigh number for $h = 0.078$

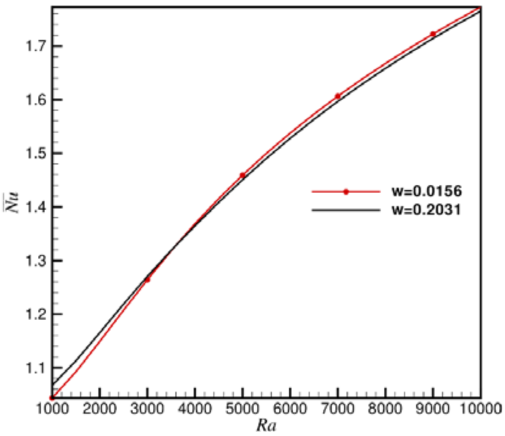


Fig. 3 Variation of $\overline{Nu}$ as function of Rayleigh number for $h = 0.078$

At greater heights, 0.093 and 0.203, a secondary cell appears in the upper part of the cavity at a critical Rayleigh number of appearance $Ra_{c,app}$.

For the thickness $w = 0.015$ at $h = 0.093$, only the unicellular regime is established for the different Ra (Fig. 4 (a)). At $h = 0.203$, the secondary cell appears with a small size at $Ra_{c.app} = 2740$, grows in size until reaching its maximum at the Rayleigh number equal to 3050 and disappears after at the critical Rayleigh number of disappearance equal to 3060 (Fig. 4 (b)).

For the thickness $w = 0.203$, at $h = 0.093$ and $h = 0.203$, the critical Rayleigh number of appearance is equal to 1000 (Fig. 5). When we keep increasing the Rayleigh number, the secondary cell increases in size until Ra = 10 000 for h = 0.203, or disappears at the critical Rayleigh number of disappearance of the bicellular regime $Ra_{c,dis}$, which is equal to 6500 at $h = 0.093$ (Fig. 5 (a)).

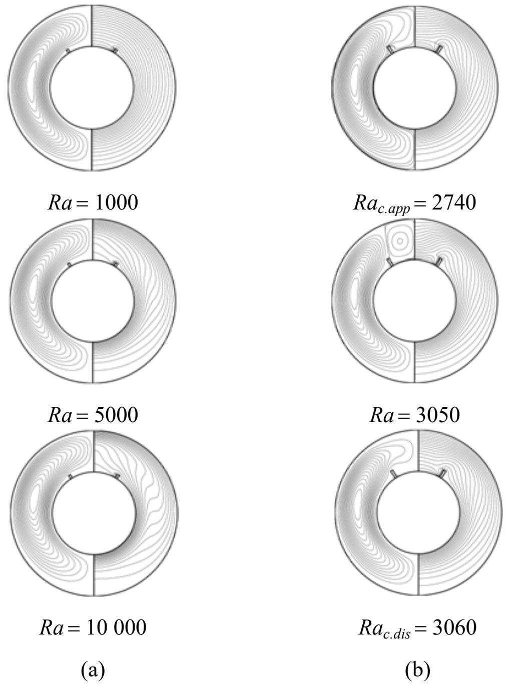


Fig. 4 Streamlines and isotherms for $w = 0.015$ at (a) $h = 0.093$, and (b) $h = 0.203$

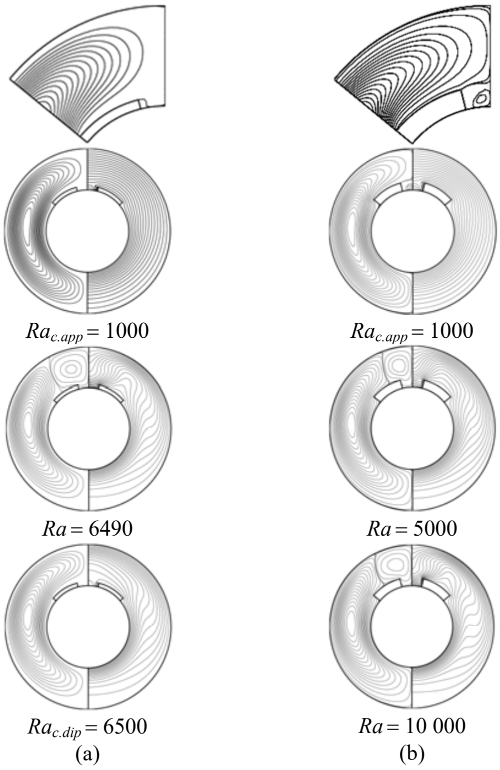
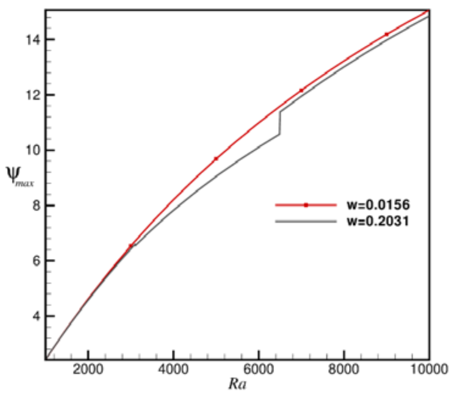


Fig. 5 Streamlines and isotherms for $w = 0.203$ at (a) $h = 0.093$ and (b) $h = 0.203$

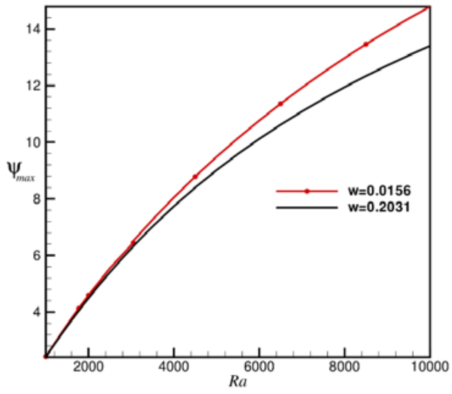
Fig. 6 indicates the variation in the main cell intensity as a function of Rayleigh number. For $h = 0.093$ and $h = 0.203$, the appearance of the secondary cell at $Ra_{c.app}$ does not indicate a significant change in the intensity of the main cell, despite the fact that this cell no longer fills the entire annular space in the presence of the secondary cell. Only the disappearance of the bicellular regime at $Ra_{c.dis}$ is marked by an accentuated growth of ψ_{max} for $w = 0.203$ at $h = 0.093$ (Fig. 6 (a). For $w = 0.203$ at $h = 0.093$, the intensity varies by 0.3% between $Ra = 2720$ and $Ra_{c.app} = 2730$ and then at the disappearance of the secondary cell at $Ra_{c.dis} = 6500$, ψ_{max} increases by 7.56% when the Rayleigh number varies between $Ra = 6490$ and $Ra_{c.dis} = 6500$.

Fig. 7 shows that for the heights 0.093 and 0.203, the average Nusselt number varies almost linearly for Rayleigh numbers less than 2500 and then undergoes an increase related to the increase of the secondary cell size. The increase in the Rayleigh number thus leads to a growth in $\overline{Nu}$ until finding an abrupt diminution when there is a change in flow regime at $Ra_{c.dis}$, related to the disappearance of the secondary cell. According to this, the heat transfer in the cavity differs depending to the Rayleigh number. In another study of laminar natural convection inside a cylindrical annular cavity equipped with two blocks of thickness $w = 0.203$. El Amraoui et al. [7] distinguished between the appearance of the secondary cell at $Ra_{c.app}$ and its increase at $Ra_{c.inc}$. They concluded that, for given heights the increase in the size and the intensity of the secondary cell at $Ra_{c,inc} > Ra_{c,app}$ causes a significant growth on $\overline{Nu}$, which is much greater than that noticed at the appearance of the secondary cell at $Ra_{c,app}$.

At $h = 0.093$ (Fig. 7(a)) for $1000 \leq Ra \leq 3100$ and $6500 \leq Ra \leq 10\,000$, the heat transfer is almost the same for $w = 0.015$ and $w = 0.203$, which means that the variation in the fins thickness has no significant effect on the heat transfer rate for these intervals of Ra. Beyond this interval the greatest thickness $w = 0.203$ offers the highest heat transfer. The maximum difference is 16% at $Ra = 6490$. At the height $h = 0.203$ (Fig. 7(b)), $\overline{Nu}$ for $w = 0.203$ is the largest except for $2740 \leq Ra \leq 3050$ where the smallest thickness $w = 0.015$ causes a heat transfer greater than that obtained by $w = 0.203$. Beyond this range, the thickness 0.203 provides the highest heat transfer.

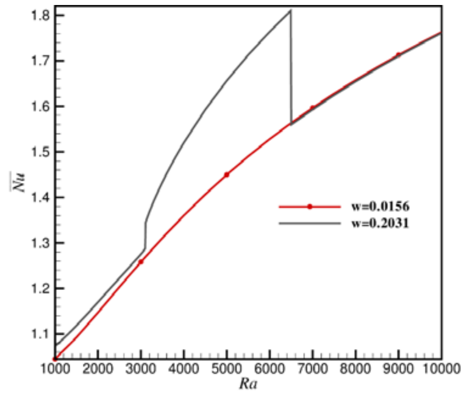


(a)

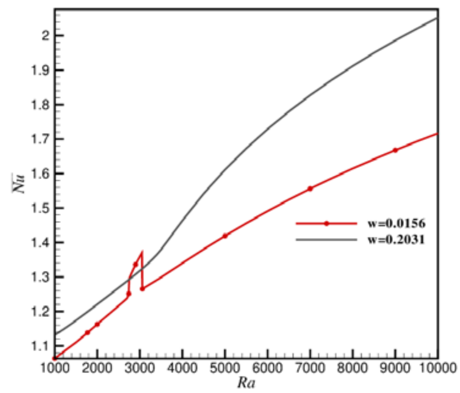


(b)

Fig. 6 Variation of the intensity of the main cell as function of Rayleigh number for (a) $h = 0.093$ and (b) $h = 0.203$



(a)



(b)

Fig. 7 Variation of $\overline{Nu}$ as function of Rayleigh number for (a) $h = 0.093$ and (b) $h = 0.203$

IV. Conclusion

A numerical analysis of the thermo-convection in a cylindrical annular cavity filled with air was carried out for two fins thicknesses $w = 0.015$ and $w = 0.203$ and three fin heights 0.078, 0.093 and 0.203. The variation of the Rayleigh number Ra, from 10^3 up to 10^4 allowed us to determine the critical Rayleigh numbers of transition between the unicellular regime and the bicellular regime. The main points found according to the fin heights are:

1. For the low height $h = 0.078$, the flow regime established is unicellular for all values of *Ra* number and for both values of *w*. The increase in *Ra* has the effect of increasing the intensity of the main cell as well as the heat transfer rate. The heat transfer is practically the same for the two thicknesses. A correlation between Nusselt number and Rayleigh number is determined.
2. For median heights $h = 0.093$ and $h = 0.203$, the bicellular regime appears and increases abruptly for *Ra* values approaching 3000, and the critical Rayleigh numbers which define the development of the secondary cell, as well as the critical Rayleigh numbers which characterize the disappearance of the bicellular regime are determined. The variation in heat transfer rate as a function of *Ra* has shown that the increase in *Ra* leads to an intensification in the heat transfer rate followed by an abrupt decrease corresponding respectively to the growth of the secondary cell and its disappearance. For certain *Ra* values, the thickness $w = 0.015$ can offer a heat transfer greater than that obtained for the greater thickness $w = 0.203$.

References

[1] S. Acharya, Sukanta K. Dash, *Natural convection heat transfer from a hollow horizontal cylinder with external longitudinal fins: A numerical approach*. Numerical Heat Transfer, Part A: Applications, 2018.

[2] T Bhattacharyya, R Anandalakshmi, K. Srinivasan, *Heat Transfer Analysis on Finned Plate Air Heating Solar Collector for its Application in Paddy Drying*. Energy Procedia, 2017.

[3] A. Nag, A. Sarkar, V. M. K. Sastri, *Effect of Thick Horizontal Partial Partition Attached to One of the Active Walls of a Differentially Heated Square Cavity*. Numerical Heat Transfer, Part A: Applications: An International Journal of Computation and Methodology, 1994.

[4] A. Elatar, M. A. Teamah, M. A. Hassab, *Numerical study of laminar natural convection inside square enclosure with single horizontal fin*. International Journal of Thermal Sciences, 2016.

[5] Y. Taher, A. Cheddadi, M. T. Ouazzani*, Heat transfer in an annular space provided with fins: numerical simulation of the effect of the fins width.* Phys. Chem. News, 2006.

[6] A. Idrissi, A. Cheddadi, M.T. Ouazzani*, Heat transfer in annular space containing heating blocks with moderate heights multicellular flows,* Phys. Chem. News, 2015.

[7] A. El Amraoui, A. Cheddadi, M.T. Ouazzani, *Effect of Flow Regimes on The Heat Transfer in An Annular Cavity with Two Internal Blocks*, Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, 2020.