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A Case Study on the Numerical-Probability Approach for Deep Excavation Analysis

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***Abstract*—**Urban advances and the growing need for developing infrastructures has increased the importance of deep excavations. In this study, after the introducing probability analysis as an important issue, an attempt has been made to apply it for the deep excavation project of Bangkok's Metro as a case study. For this, the numerical probability model has been developed based on the Finite Difference Method and Monte Carlo sampling approach. The results indicate that disregarding the issue of probability in this project will result in an inappropriate design of the retaining structure. Therefore, probabilistic redesign of the support is proposed and carried out as one of the applications of probability analysis. A 50% reduction in the flexural strength of the structure increases the failure probability just by 8% in the allowable range and helps improve economic conditions, while maintaining mechanical efficiency. With regard to the lack of efficient design in most deep excavations, by considering geometrical and geotechnical variability, an attempt was made to develop an optimum practical design standard for deep excavations based on failure probability. On this basis, a practical relationship is presented for estimating the maximum allowable horizontal displacement, which can help improve design conditions without developing the probability analysis.

***Keywords*—**Numerical probability modeling, deep excavation, allowable maximum displacement, finite difference method, FDM.

I. INTRODUCTION

PROBABILITY analysis has numerous applications as an efficient tool in analyzing experimental and laboratory data for theoretical and practical issues [1]. The spatial variety of geotechnical soil properties as one of the most important natural issues with uncertainty has resulted in the growing significance of probability analysis in geotechnical engineering [2]. Probability analysis in geotechnical engineering includes the use of probability in the analysis of soil related issues in order to provide accurate insight of its behavior [3]. An accurate probability analysis will only result when there is adequate statistical knowledge of parameters with uncertainty [3], [4]. Such knowledge is usually obtained through various field samplings and developing different experiments. This is while existing limitations in geotechnical projects often result in limited statistical information and thus, the development of probability analysis in geotechnics has always been considered a challenging issue [4]. This issue has often led geotechnical engineers to evaluate related issues by disregarding these uncertainties and considering them as definite [5].

Probability analysis has extensive applications in various branches of geotechnics. Among these applications is the development of probability analysis in the realm of slope stability. Studies carried out in this realm frequently indicate that safety factors of two slopes with similar conditions that have been calculated as similar based on certainty analysis can be different, for which the reason can be the spatial and randomized variability of soil properties [6]-[10]. Based on the study by Kitch et al., which was carried out in two forms of certainty and probability with the limit equilibrium method on a reinforced slope with geosynthetic, it is shown that conditions with the greatest failure probability in probabilistic conditions with similar collapse conditions with the least safety factor in certainty condition are not necessarily similar and thus carrying out probability analysis in regard to slopes is of great significance [11]. Based on this analysis pattern, Luo et al. [12] indicate that an increase in the number of geosynthetic layers in slopes effectively reduces their probability of failure. In another study, the probability of slope failure with cohesive materials by means of two probability techniques, including the classic analytical technique and Random Finite Element Method (RFEM) are evaluated. The results indicate that using the RFEM has numerous advantages compared to traditional methods in slope stability analysis. Among the most important advantages is evaluating the failure in the most critical mechanism and thus developing the analysis based on the conservative approach [8].

Among other practical realms of probability analysis, a number of researchers have set out to develop this analysis pattern in regards to dam projects. In one of these studies that were done dynamically, spatial uncertainty of inherent properties of the concrete dam and uncertainty of seismic wave parameters were given consideration. On this basis, while using the finite element method and Monte Carlo sampling method, the effect of considering this random process was evaluated. Carrying out this analysis in the form of probability analysis provided the means for evaluating different failure conditions and probabilities related to each of them. Based on the results obtained, the probability of sliding failure is greater than crushing failure and the probability of crushing is greater than cracking [13]. The development of dynamic probability analysis in other structures was also followed by a number of researchers [14], [15].

Spatial variability of inherent soil properties in the realm of building tunnels also results in the significance of probability analysis in this field. Based on the studies carried out by Gong et al. [16], in which the Monte Carlo randomized sampling method was used, it is proven that considering uncertainty results in unpredictable behavior along the tunnel's longitudinal direction. Therefore, in this study, probability

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analysis is considered a necessary framework put forth in evaluating the performance of tunnels longitudinal direction.

Accurate analysis of deep excavations with regard to the growing increase in urban activities and reduction in construction space has gained great importance. Similar to other geotechnical projects introduced, probability analysis in these structures can also have numerous advantages [17]-[19]. In this realm, Castaldo et al. [20] have presented a simple probability method in the form of a case study in order to evaluate the damages to adjacent buildings as a result of deep excavation. In this study, the probability of different forms of damage is evaluated as a function of input parameters. Hsiung et al. [21] studied the deformation of Jakarta's metro excavation in Indonesia in the form of numerical analysis. In this study, in order to analyze the deep excavation, a two-dimensional finite element model, with an up-down construction method and a concrete slab supportive system was considered and the variability of the soil modules s was evaluated in this case. Based on the results obtained, the softening soil model and measured soil modules from the project provide an adequate estimate of a logical failure probability of the wall deformation. In another study, an attempt has been made to evaluate the vertical spatial variability of protected excavation in sand by considering the different structural and geotechnical bending and shear failure using the RFEM, and thus the performance of the maintenance system was assessed. This study emphasizes the importance of spatial variability in soil properties by considering multiple modes of failure for complex problems of soil-structure interaction effects [22]. Among these studies, the role of probability analysis in improving the results of geotechnical project analyses is quite clear. For this reason, the probability analysis in deep excavation is greatly needed. On the other hand, with a review of the literature, it is observed that the existing ideas in estimating vertical and horizontal displacement resulting from deep excavation, which are generally divided into two groups of empirical and numerical, are not quite effective [23]-[29]. Thus, lack of an appropriate analytical standard in estimating the allowable displacement of deep excavations has always been considered a challenge. In this study, the role of probability analysis was evaluated in improving the results of deep excavation outcomes, while selecting Bangkok's metro deep excavation project and the Finite Difference method in two-dimensional form. The validity of the mentioned numerical method after the numerical modeling of this deep excavation was evaluated and confirmed after making a comparison of results with field measured values. Then, in order to evaluate the effect of geotechnical parameter variability, probability modeling was carried out by selecting the Monte Carlo method and the results were obtained. While applying some of the practical aspects of probability analysis, an attempt has been made to improve the results of these analyses. Finally, probability analysis is used for providing an appropriate standard in order to estimate allowable displacement of deep excavations.

II. PROBABILITY ANALYSIS TECHNIQUES

In the development of a probability analysis, randomized spatial parameter sampling techniques are often used. Until now, various techniques have been presented in this realm. Some of these techniques include the following: 1) Latin Hypercube Method [30], 2) Point probability and overall probability method [31], 3) Monte Carlo Method [32], 4) Bivariate probability Budyko analysis [33], and 5) Probability soft alert method [34].

Randomized methods encompass an extensive domain of quasi-mathematical analyses for evaluating the behavior of a system. Therefore, one of the primary advantages of using the above methods is that they are much simpler in comparison to deterministic methods, and there is no need for complicated mathematical equations. Among the various randomized probability methods, the Monte Carlo method, as an easier to use method, is more applicable than other methods presented [35].

The Monte Carlo method is a computational algorithm that uses randomized sampling as input for probability analysis. This method is often used to carry out simulations of mathematics or physics systems. Tendency to use the Monte Carlo method is greater when calculating the exact respond is not possible or unjustified by definite algorithms [36]. In the current study, while selecting this model and integrating it with the Finite Difference Numerical model, an attempt is made to use it for developing the probability-numerical model. The Finite Difference Method (FDM) is a numerical method for approximate estimation of differential equations. In this method, the derivative functions are estimated with their equivalent differences. The basis of this method for solving equations is using the approximation function with Taylor's method [37]. Due to the productivity of computation time in carrying out numerical analysis, it is best to use two-dimensional numerical simulation [38]-[42].

III. NUMERICAL SIMULATION AND VALIDATION OF BANGKOK'S METRO PROJECT

Bangkok's metro project, which is designed in the proximity of the crowded urban city, includes approximately 20 kilometers of tunnels that have been built by a Tunnel Boring Machine (TBM) (Fig. 1). This tunnel includes 18 underground stations that have been built by the cut and cover method. The length, width and depth of boring in stations reach up to 230 m, 25 m and 16-32 m, respectively. In this study, deep excavation related to Sukhumvit station is evaluated. Soil layers in the studied station bounds are generally in the form of alternate sand and clay layers. The exact sequence of these layers is shown in Fig. 2. In addition, the geotechnical properties of these layers are reported in Table I. It is noted that the soil profile was adopted based on a site investigation consisting of a series of four boreholes.

The studied station was built by the top-down construction method with a configuration of the central platform type. Deep excavation related to this station has a width of 23 m, length of 200 m, and depth of 21 m. In this excavation, reinforced concrete diaphragm walls with thickness of 1 m and depth of 27.9 m are used. In addition, concrete slabs with various thicknesses in three different levels are used as the primary support system for diaphragm walls. Fig. 3 gives a view of the retaining structures related to the Sukhumvit station excavation. The construction process adopted at this station is summarized in Table II. Also, structural properties related to this structure are shown in Table III.

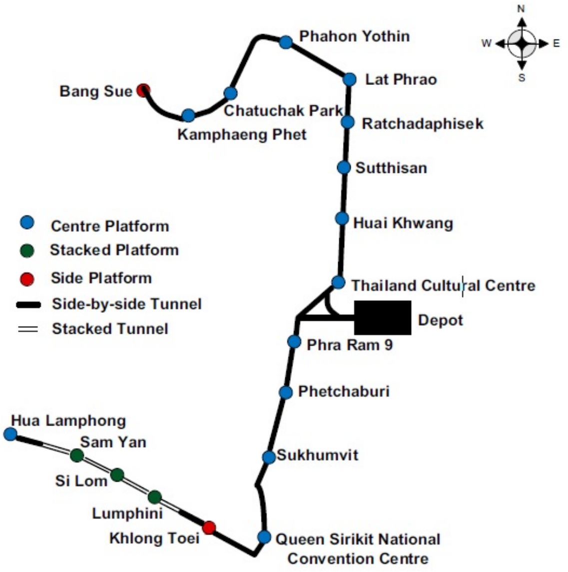

Fig. 1 Bangkok's MRT project

With regard to the fact that the ratio of length (L) and width (B) of the Sukhumvit station is equal to 8.7, the project can be evaluated in two-dimensional form and the effect of three-dimensional behavior is disregarded [43]. On this basis, two-dimensional numerical modeling using the plain strain approach was used. In addition, due to the geometrical symmetry, while adopting the lateral symmetry modeling approach, half of the model was analyzed. The length and width of the numerical model is equal to 91.5 m and 45 m, respectively, and the number of meshes is equal to 1200. With regard to the presence of different layers of soil in the region, the sequence of seven layers of soil was considered in the geometry of the numerical model, for which, the Mohr Coulomb numerical model was used for all layers. The geotechnical properties of these soil layers are applied based on Table II. With regard to the necessity of creating an equilibrium state in the numerical model prior to project construction, the primary geometry is designed without considering the project. The project after mesh geometry of soil layers of the model is presented in Fig. 4.

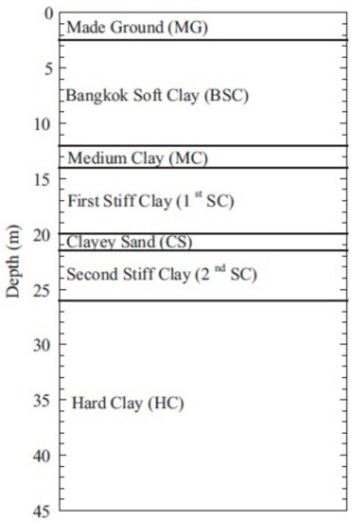

Fig. 2 Soil properties at Sukhumvit station

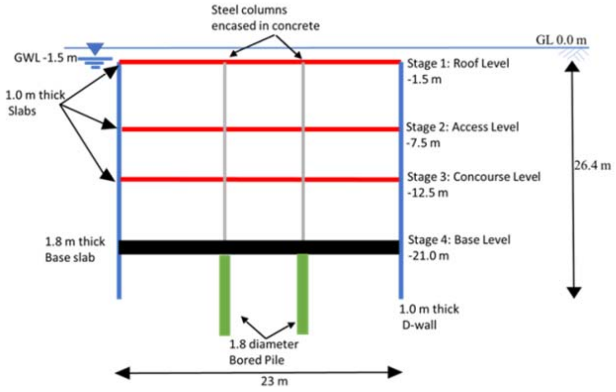

Fig. 3 Geometry of retaining structures used in the Sukhumvit station excavation

TABLE I
Soil Parameters for Mohr Coulomb Model (MCM)

Layer	Soil type	Depth (m)	γ_b (kN/m3)	s_u (kPa)	c′ (kPa)	ϕ′ (o)	ψ (o)	E_u (MPa)	E′ (MPa)	ν	Analysis type
1	MG	0–2.5	18	-	1	25	0	-	8	0.3	Drained
2a	BSC 1	2.5–7.5	16.5	20	-	-	0	10	-	0.495	Undrained
2b	BSC 2	7.5–12	16.5	39	-	-	0	20.5	-	0.495	Undrained
3	MC	12–14	17.5	55	-	-	0	27.5	-	0.495	Undrained
4	1st SC	14–20	19.5	80	-	-	0	40	-	0.495	Undrained
5	CS	20–21.5	19	-	1	27	0	-	53	0.25	Drained
6	2nd SC	21.5–26	20	120	-	-	0	72	-	0.495	Undrained
7	HC	26–45	20	240	-	-	0	240	-	0.495	Undrained

TABLE II
CONSTRUCTION PROCESS OF SUKHUMVIT MRT STATION

Stages	Construction activities
1	
1.1	Construction of diaphragm walls
1.2	Construction of bored piles
1.3	Installation of steel columns
1.4	Excavation to the depth of 1.5 m
2	
2.1	Completion of roof floor concrete casting
2.2	Excavation to the depth of −7.5 m
3	
3.1	Completion of second floor concrete casting
3.2	Excavation to the depth of −12.5 m
4	
4.1	Completion of third floor concrete casting
4.2	Excavation to the depth of −21 m

In this modeling, the phreatic level is considered in natural form at a depth of 1.5 m. In order to create equilibrium state at the boundaries, roller support is used at both horizontal and right boundaries at the bottom of the model. In such conditions, primary analysis of the model is provided in order to create equilibrium state. The model's maximum unbalance forces change and gravitational stress conditions after obtaining equilibrium state are presented in Figs. 5 and 6, respectively.

Project implementation in numerical models is carried out step by step based on Table III. In each phase of numerical modeling, after excavation and installing the structures in that phase, the model is allowed to equilibrate. Fig. 7 shows the ultimate geometry of soil and structure elements. Fig. 8 shows the project deformation after carrying out the final phase.

TABLE III
INPUT PARAMETERS FOR STRUCTURAL COMPONENTS

Parameter	Diaphragm wall (1 m thick)	Platform slab (1 m thick)	Base slab (1.8 m thick)	Column (0.8 m dia. at 11.4 spacing)	Pile (1.8 m dia. at 11.4 m spacing)
Axial stiffness, EA (MN/m)	28000	28000	50400	1712	3852
Flexural rigidity, EI(MN/m^2/m)	2333	2333	13608	91.3	1040
Weight, w (KN/m2)	16.5	25	45	25	25
Poisson's ratio, ν	0.15	0.15	0.15	0.15	0.15

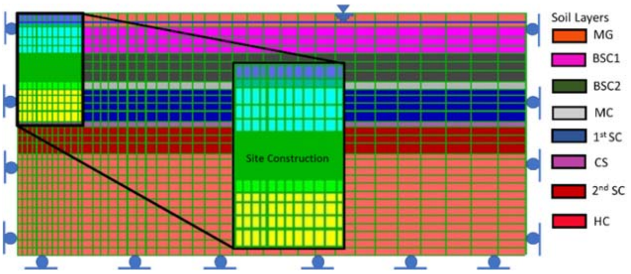

Fig. 4 Finite Difference Model, mesh and soil groups

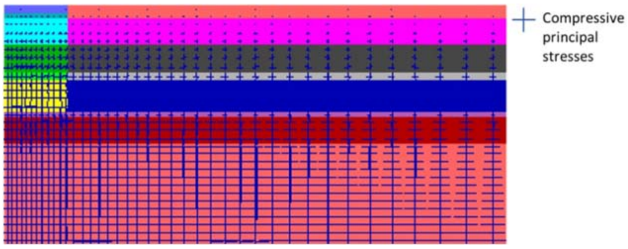

Fig. 6 Principle stress components

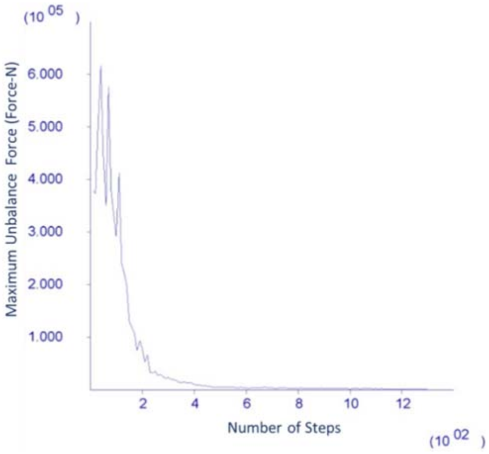

Fig. 5 Change of model maximum unbalanced force

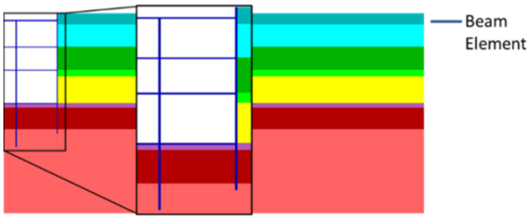

Fig. 7 Final geometry of soil and structure elements

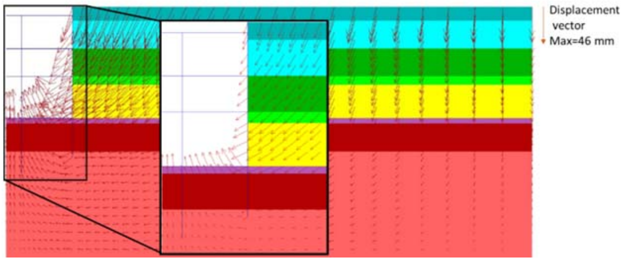

Fig. 8 Excavation displacement vectors after final analyses

Prior to carrying out probability analyses, it is necessary to evaluate the accuracy of the numerical model results. For this purpose, the results of this model were compared with the results obtained from field instruments. Fig. 9 shows a comparison of lateral wall displacement measured on the field and predicted by the finite difference model for the above metro stations carried out with the Mohr Coulomb model. As it is shown in the figure, displacement measured by the FDM has significant agreement with displacement measured on the site. It is necessary to note that error in the results is obtained by the root mean square error (RMSE). The error obtained in the 4th phase is less than 3% and less than 10% for phase 1 to phase 3.

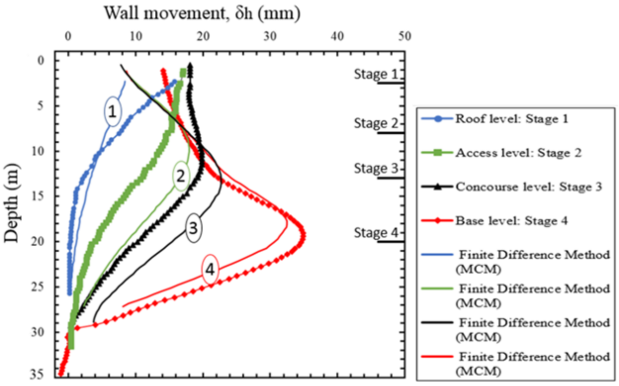

Fig. 9 Measured and numerical predicted lateral wall displacements

IV. IMPLEMENTING AN INTEGRATED NUMERICAL PROBABILITY MODEL

In order to implement an integrated numerical probability model, it is necessary that statistical properties of urban sediments are initially evaluated. Therefore, after geometrical modeling, project analysis and a comparison of validation results, statistical analysis was done regarding cohesion (C) properties and the internal friction angle of soil components. The histogram related to these properties is shown in Figs. 10 and 11. Based on the results, the lognormal distribution is determined as the best statistical distribution of the cohesion parameter and normal distribution as the best statistical distribution for the internal friction angle. Statistical moments of these distribution functions are reported in Table IV.

TABLE IV
STATISTICAL DISTRIBUTION AND RELATED MOMENTS ESTIMATED FOR SHEAR STRENGTH PARAMETERS

	Distribution	Mean	Deviation	Location	Scale	Threshold
Friction (°)	Normal	26.96	9.365	-	-	
Cohesion (kPa)	Log Normal	-	-	2.508	1.027	0

In this study, the Monte Carlo statistical method was used for developing the numerical probability model. In this method, according to the flowchart presented in Fig. 12, first a series of random numbers are produced coinciding with statistical distribution functions of each parameter (normal distribution for internal friction angle parameter and lognormal distribution for the cohesion parameter) and based on their statistical information. Then, these statistical properties are used in developing the finite difference numerical model. Afterwards, the considered results are extracted and finally in the last stage, the model is prepared to carry out the next computational cycle. The remodeling of this process continues until the number of cycles reaches the number of considered attempts and finally the results are prepared for ultimate probability analyses. It is noteworthy that in this integrated model, while writing the functions, producing random and illogical data out of the range has been prevented. In addition, functions are written in order to specify equilibrium conditions and refrain from saving results in attempts that are unbalanced due to illogical properties.

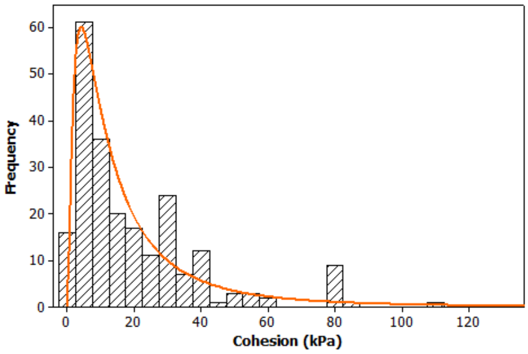

Fig. 10 Compliance of lognormal statistical distribution for the cohesion parameter

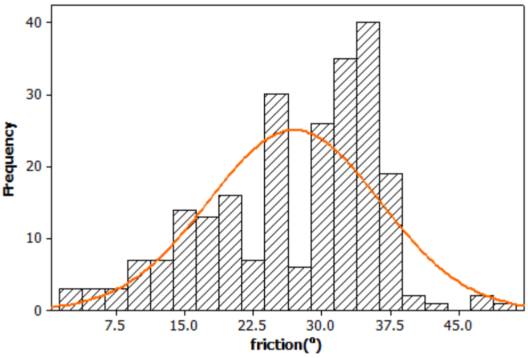

Fig. 11 Compliance of normal statistical distribution for internal friction angle

With the development of the integrated numerical probability model, it is possible to come to probabilistic conclusions. In this study, with 500 estimations of the Monte Carlo method, the mentioned model was used in order to assess the reliability of Bangkok's metro project. The histogram of the safety factor and the maximum horizontal displacement is shown in Fig. 13. Based on the statistical analyses, the best distribution adaptable to the safety factors and maximum displacement histogram is identified as the gamma statistical distribution. Coefficients of these statistical distributions are presented in Table V.

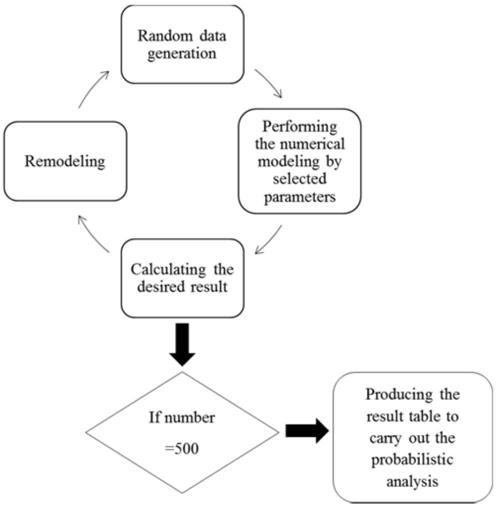

Fig. 12 Flowchart of probability analysis by considering uncertainty parameters of soil

TABLE V
THE BEST PREDICTED STATISTICAL DISTRIBUTION AND RELATED MOMENTS OF SAFETY FACTOR AND MAXIMUM ALLOWABLE HORIZONTAL DISPLACEMENT

Parameters	Distribution	Shape	Scale	Threshold
Safety factor	Gamma	8.643	0.9082	1.497
Horizontal Max Displacement (m)	Gamma	1.211	0.0002913	0.003553

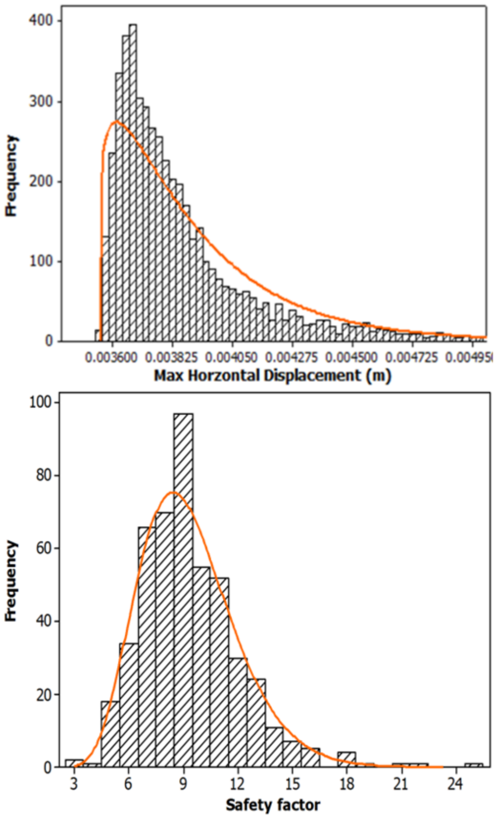

Fig. 13 Histograms of maximum horizontal displacement and safety factor

As it is concluded from Fig. 13, statistical variability of the internal friction angle parameters and cohesion based on normal and lognormal distribution, respectively, result in variability of the safety factor and horizontal displacement of excavation based on gamma distribution. Accordingly, the distributions can be used in order to assess the confidence terms of definitive analysis results of the Bangkok excavation project. For this means, by selecting a benchmark value for the safety factor, it is possible to calculate the failure probability based on the integrating of the fitted distribution curve at amounts less than this value. For example, Fig. 14 shows the process of calculating failure probability for a safety factor of 5. Failure probability for certain safety factors of the above-mentioned study is put forth in Table VI.

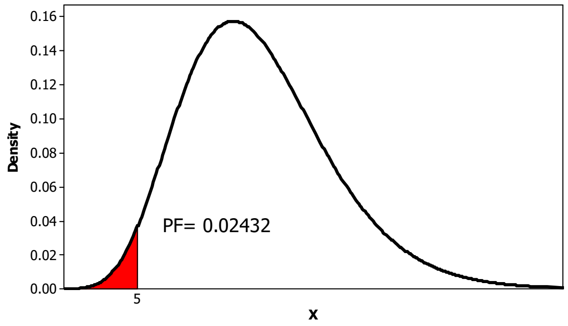

Fig. 14 Evaluation of failure probability with regard to Gamma distribution for safety factors and benchmark of 5

TABLE VI
CALCULATING FAILURE PROBABILITY FOR CERTAIN SAFETY FACTORS

Safety Factor	Probability Factor (%)
1.5	0.00
3.5	0.08
5	2.43
7	19.24

By evaluating the results obtained from the probability analysis and histogram of its obtained safety factor, it is clear that stability conditions in the Bangkok metro excavation project are provided by considering the very low probability of failure (about 0%). In other words, in this project, even for very high safety factors, failure probability is in an acceptable range. This means that the support system designed in this project has very high efficiency. Regardless of its relatively high efficiency, this support system cannot be economical. Based on this probability analysis, the support system considered for this excavation was redesigned such that with an increase in failure probability in the allowable range, adequate economic conditions are met, while providing an acceptable safety factor.

V. PRESENTING A PROBABILITY STANDARD TO CALCULATE ALLOWABLE HORIZONTAL DEFORMATION OF EXCAVATION

One of the existing uncertainties in designing deep excavations is lack of adequate knowledge of the allowable displacement range of excavations in various structural and geometrical conditions. It seems that accurate knowledge of this parameter can be obtained from probability analysis. In this research, the probability of calculating this parameter is introduced by maintaining failure probability conditions based on safety factor design values. For this purpose, by calculating the failure probability for each specific safety factor value, this probability can be used to estimate the allowable displacement range.

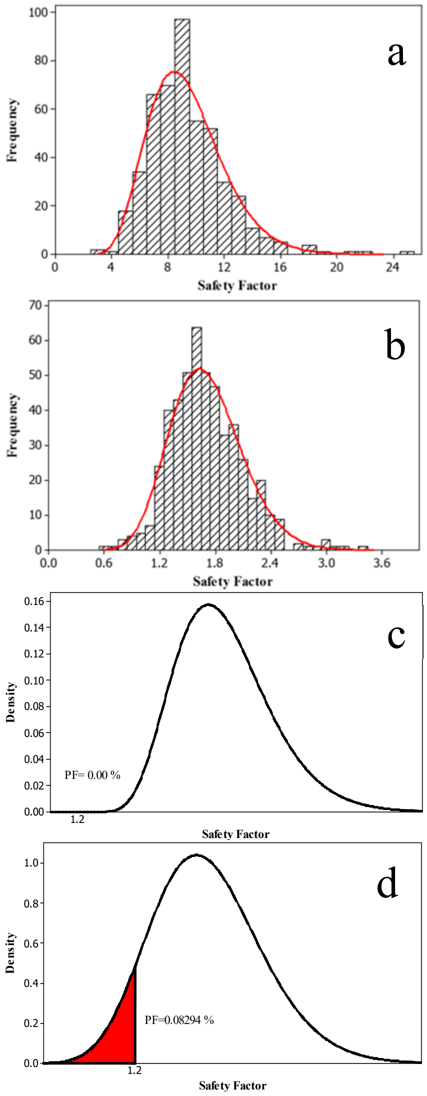

Fig. 15 Frequency distribution of safety factors before reduction in flexural strength of support system (a) and after (b) failure probability for safety factor of 1.2 before reduction in flexural strength of the support system (c) and after (d)

With regard to the generality of excavations with an anchorage support system in urban environments, for the evaluation of allowable displacement conditions of deep excavations, a single-layered probability model including a support system is utilized. It is noteworthy that with regard to the effect of excavation dimensions in the allowable horizontal displacement of the wall, the analyses of this section are developed based on various ratio of excavation height to width (h/w). Also, in order to create similar analytical conditions for a variety of proportions h/w, different boundary positions of the model are considered as dependent. The area of this analysis is assumed as dry. The geometry of this model is presented in Fig. 16.

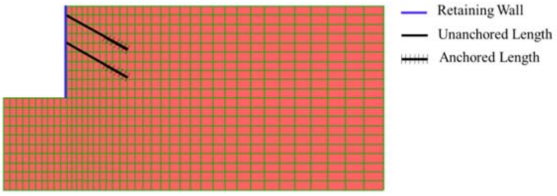

Fig. 16 Geometry of dimensionless excavation model analyzed in evaluating of allowable horizontal displacement

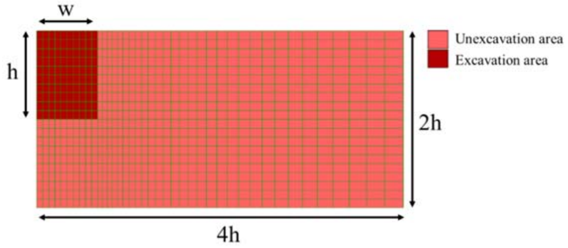

Fig. 17 Geometry of the support system considered for the general probability analysis model

The soil behavioral model was selected similar to the Mohr Coulomb model. In addition, in order to evaluate the probability behavior of the excavation, the normal distribution function was used for the internal friction angle parameter and the lognormal distribution function was used for the cohesion parameter. The structural system considered in this general model includes a retaining wall by two anchors of 30 degrees from the horizon. The fixed length of the anchors is dependent on the excavation height and equal to 0.25 h and the total length is assumed to be equal to 0.75 h. Soil properties and structures used in this model are reported in Table VII. In addition, the geometry of excavation and mentioned structures are presented in Fig. 17.

Probability analysis of the general deep excavation model is carried out by 500 attempts of the Monte Carlo method. In these attempts, three amounts of 1, 1.5 and 2 were considered for the h/w ratio and by considering the equal failure probability, displacement related to the safety factor is obtained. Fig. 18 shows the calculation process for the failure probability of 50.52% and ratio of 1.5 h/w. Based on this figure, for failure probability equal to 50.52 % and allowable safety factor of 1.5, the equivalent maximum allowable horizontal displacement of 0.008264 m was obtained. Adaptive Figures of maximum allowable horizontal displacement and allowable safety factor for different values of h/w are presented in Fig. 19.

Based on Fig. 19, and by making use of multivariate interpolation, it is possible to present (1) for predicting maximum allowable horizontal displacement of deep excavations based on the ratio of geometrical dimensions and allowable safety factor at an equal probability level, the correlation coefficient is 0.98.

$$\delta_{max} = 0.0011 \times \text{Exp}(3.5991\left(\frac{h}{w}\right) + (-2.162\left(\frac{h}{w}\right) + 0.8817) \times SF) \qquad (1)$$

In this equation, SF is the safety factor considered for the design, δ_{max} is maximum allowable horizontal displacement (m), and h/w is the ratio of height to width of the excavation. In this study, (1) is presented as the resulting indicator of one of the probability analysis applications, which can have the effective functional aspects.

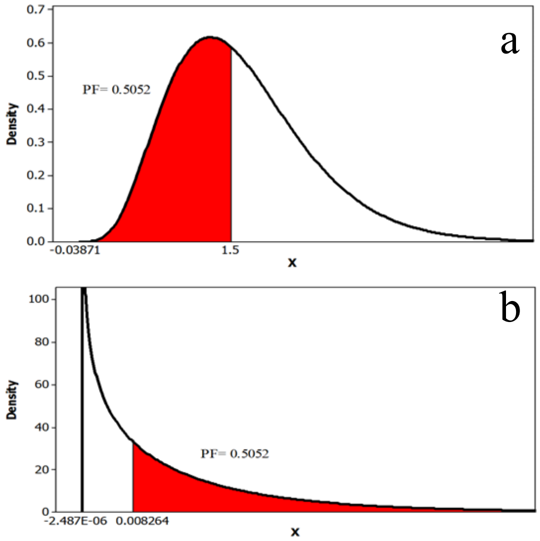

Fig. 18 Statistical distribution of Gamma fit for safety factor of 1.5 (a) horizontal allowable displacement value (m) (b)

VI. DISCUSSION AND CONCLUSIONS

By evaluating the results of probability analysis development in relation to the case study excavation, it was observed that disregard for variability of cohesion parameters and internal friction angle results in a very low failure probability. Even though at first glance the failure probability of near zero is very ideal; however, it is evident that spending excessively to further stabilize the created excavation has created this failure probability, which is a great challenge for the economic optimality of the project. The probability analysis carried out in this project with an estimate of failure probability, proposed redesign of the support system such that failure probability in the allowable range would increase. On this basis, the probabilistic redesign process of Bangkok's excavation support system was carried out and a 50% reduction in the flexural strength of the support system was observed, which resulted in an increase in failure probability from 0% to 8%. It is evident that a failure probability of 8% is still considered an optimum value, which does not cause any damage to excavation conditions. This is while this reduction in resistance can result in abundant economic savings. However, in the current study, considering that "probabilistic redesign of the excavation" is introduced as one of the practical purposes of probability analysis, and by considering the breadth of this issue, other strength and geometric conditions of structural elements have not been considered. Thus, the authors have proposed accurate implementation of its various aspects

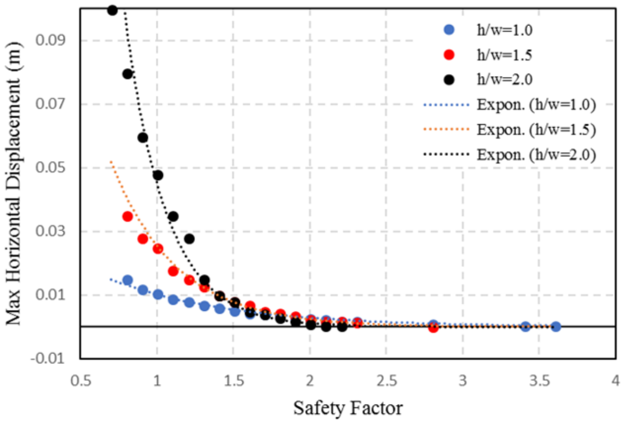

Fig. 19 Fit of the negative exponential function on corresponding data of maximum allowable horizontal displacement of the excavation for different values of the h/w ratio

Overall results of this study indicate that the role of probability analysis in deep excavation projects is of great significance and disregarding it can result in irreparable consequences, especially in conditions in which failure probability is high. Thus, it is necessary that probability analysis is used in the mechanical design and economic phase of these projects from different aspects in order to optimize the intended designs.

TABLE VII
SOIL AND STRUCTURAL PARAMETERS USED IN THE GENERAL ANALYZED DEEP EXCAVATIONS

	Density (Kg/m3)	Cohesion (kPa)				Friction (°)			Young Modulus (MPa)	Poisson Ratio	Area (m^2)	Second Moment, (m^4)	Spacing (m)	Yield strength (KN)	Normal Stiffness (MPa/m)	Shear Stiffness (MPa/m)	Perimeter (m)	Prestress anchor force (KN)
Soil	2000	Distribution	L	S	T	Distribution	M	Std	240	0.495	-	-	-	-	-	-	-	-
		Log Normal	9.33	1.01	0	Normal	26.96	9.36										
Retaining Wall	-	-				-			3055	0.15	1.24	0.87	-	-	-	-	-	-
Anchor	-	-				40			91e4	-	0.0015	-	0.5	1e17	1	1	0/62	750

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