

Determination of Optimal Stress Locations in 2D–9 Noded Element in Finite Element Technique

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Abstract—In Finite Element Technique nodal stresses are calculated through displacement as nodes. In this process, the displacement calculated at nodes is sufficiently good enough but stresses calculated at nodes are not sufficiently accurate. Therefore, the accuracy in the stress computation in FEM models based on the displacement technique is obviously matter of concern for computational time in shape optimization of engineering problems. In the present work same is focused to find out unique points within the element as well as the boundary of the element so, that good accuracy in stress computation can be achieved. Generally, major optimal stress points are located in domain of the element some points have been also located at boundary of the element where stresses are fairly accurate as compared to nodal values. Then, it is subsequently concluded that there is an existence of unique points within the element, where stresses have higher accuracy than other points in the elements. Therefore, it is main aim is to evolve a generalized procedure for the determination of the optimal stress location inside the element as well as at the boundaries of the element and verify the same with results from numerical experimentation. The results of quadratic 9 noded serendipity elements are presented and the location of distinct optimal stress points is determined inside the element, as well as at the boundaries. The theoretical results indicate various optimal stress locations are in local coordinates at origin and at a distance of 0.577 in both directions from origin. Also, at the boundaries optimal stress locations are at the midpoints of the element boundary and the locations are at a distance of 0.577 from the origin in both directions. The above findings were verified through experimentation and findings were authenticated. For numerical experimentation five engineering problems were identified and the numerical results of 9-noded element were compared to those obtained by using the same order of 25-noded quadratic Lagrangian elements, which are considered as standard. Then root mean square errors are plotted with respect to various locations within the elements as well as the boundaries and conclusions were drawn. After numerical verification it is noted that in a 9-noded element, origin and locations at a distance of 0.577 from origin in both directions are the best sampling points for the stresses. It was also noted that stresses calculated within line at boundary enclosed by 0.577 midpoints are also very good and the error found is very less. When sampling points move away from these points, then it causes line zone error to increase rapidly. Thus, it is established that there are unique points at boundary of element where stresses are accurate, which can be utilized in solving various engineering problems and are also useful in shape optimizations.

Keywords—Finite element, Lagrangian, optimal stress location, serendipity.

I. INTRODUCTION

IN this paper we have tried to determine the optimal locations for stresses in the elements as well as at the

boundaries by using generalized procedure. Barlow noticed this phenomenon, when using simple structural elements in the representation of air frame structure [1], [2]. Similar phenomenon was also noticed by in more advanced elements [15]. The wider implication is given by Iron and Strang [7], [8], [14]. This concept is used by Moan to determine the optimal locations [9]. Further Barlow rationalized the concept of optimal stress points and outlined the methodology by which the location of such points can be determined [3]. Hinton & Owen also explained the similar procedure [6]. This phenomenon of existence of certain points of higher accuracy than would generally be expected to occur is known as super convergence. Sehgal showed that apart from ± 0.577 points, the origin of local axes is also a very good sampling point for quadratic rectangular element [12]. Barlow tried to address the relationship between optimal sampling points, reduced integration and geometric distortion [3]. Budkowska and Fu used an analytical procedure for determination of optimal stress points [4]. It was also noticed that at optimal location the stress magnitude is insensitive to the increased value of poison ratio. It was also indicated that the optimal stress location for various stress components may not be the same. Further, many researchers worked on stress computation with different approaches and formulations as published in [5], [10], [11] and [13].

In most of the time engineers and designers are interested to calculate the stresses at the boundaries. The Barlow's procedure fails to give idea about the best sampling point at the boundary for direct determination of stresses. Thus, a generalized procedure for the determination of the optimal stress location inside the element as well as the boundaries of the element has been evolved and to verify the same with numerical experimentation.

II. DETERMINATION OF OPTIMAL LOCATION FOR NINE-NODED LAGRANGIAN RECTANGULAR ELEMENT

For nine noded rectangular element the basic approximate displacement function ϕ_{as} is assumed as

$$\phi_{as} = \alpha_1 + \alpha_2\xi + \alpha_4\xi\eta + \alpha_5\xi^2 + \alpha_6\eta^2 + \alpha_7\xi^2\eta + \alpha_8\xi\eta^2 + \alpha_9\xi^2\eta^2 \quad (1)$$

In the proposed generalised procedure, we assumed that the exact displacement function is of much higher order, containing 27 terms, i.e. with full quartic polynomial plus 2 extra quantic order terms as under:

$$\phi_{ac} = \beta_1 + \beta_2\xi + \beta_3\eta + \beta_4\xi\eta + \beta_5\xi^2 + \beta_6\eta^2 + \beta_7\xi^2\eta + \beta_8\xi\eta^2 +$$

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$$\begin{aligned} & \beta_9 \xi^2 \eta^2 + \beta_{10} \xi^3 + \beta_{11} \eta^3 + \beta_{12} \xi^3 \eta + \beta_{13} \xi \eta^3 + \beta_{14} \xi^3 \eta^2 + \\ & \beta_{15} \xi^2 \eta^3 + \beta_{16} \xi^3 \eta^3 + \beta_{17} \xi^4 + \beta_{18} \eta^4 + \beta_{19} \xi^4 \eta + \beta_{20} \xi \eta^4 + \\ & \beta_{21} \xi^4 \eta^2 + \beta_{22} \xi^2 \eta^4 + \beta_{23} \xi^4 \eta^3 + \beta_{24} \xi^3 \eta^4 + \beta_{25} \xi^4 \eta^4 + \beta_{26} \xi^5 + \\ & \beta_{27} \eta^5 \quad (2) \end{aligned}$$

Then it is assumed that at nodes

$$(\phi_{as})_N = (\phi_{ac})_N \quad (3)$$

where $N = 1, 2, 3, 4, 5, \dots, 9$

Now putting the values of the local coordinates (ξ, η) at node which are combination of -1.0, 0.0, 1.0 of all the nine nodes and solving them in terms of α we get:

$$\begin{aligned} \alpha_1 &= \beta_1 & \alpha_2 &= \beta_2 + \beta_{10} + \beta_{26} \\ \alpha_3 &= \beta_1 + \beta_{11} + \beta_{27} & \alpha_4 &= \beta_4 + \beta_{12} + \beta_{13} + \beta_{16} \\ \alpha_5 &= \beta_5 + \beta_{17} & \alpha_6 &= \beta_6 + \beta_{18} \\ \alpha_8 &= \beta_8 + \beta_{14} + \beta_{20} + \beta_{24} & \alpha_9 &= \beta_9 + \beta_{21} + \beta_{22} + \beta_{25} \quad (4) \end{aligned}$$

For equality of derivatives we have

$$\begin{aligned} \frac{\partial \phi_{as}}{\partial \xi} &= \frac{\partial \phi_{ac}}{\partial \xi} \\ \frac{\partial \phi_{as}}{\partial \eta} &= \frac{\partial \phi_{ac}}{\partial \eta} \\ \frac{\partial \phi_{as}}{\partial \xi} &= \alpha_2 + \alpha_4 \eta + 2\alpha_5 \xi + 2\alpha_7 \xi \eta + \alpha_8 \eta^2 + 2\alpha_9 \xi \eta^2 \quad (5) \end{aligned}$$

$$\begin{aligned} \frac{\partial \phi_{ac}}{\partial \xi} &= \beta_4 + \beta_4 \eta + 2\beta_5 \xi^2 + 2\beta_7 \xi \eta + \beta_8 \eta^2 + 2\beta_9 \xi \eta^2 + 3\beta_{10} \eta^2 + \\ & 3\beta_{12} \xi^2 \eta + \beta_{13} \eta^3 + 3\beta_{14} \xi^2 \eta^2 + 2\beta_{15} \xi \eta^3 + 3\beta_{16} \xi^2 \eta^3 + 4\beta_{17} \xi^3 + \\ & 4\beta_{19} \xi^3 \eta + \beta_{20} \eta^4 + 4\beta_{21} \xi^3 \eta^5 + 2\beta_{22} \xi \eta^4 + \beta_{23} \xi^3 \eta^3 + 3\beta_{24} \xi^2 \eta^4 + \\ & 4\beta_{25} \xi^3 \eta^4 + 5\beta_{26} \xi^5 \quad (6) \end{aligned}$$

Substituting the values of $\alpha_2, \dots, \alpha_9$ from (4) in (5) and for (6) rearranging the equations after eliminating similar terms, we get: -

$$\begin{aligned} & \beta_{10}(3\xi^2 - 1) + \beta_{12}\eta(3\xi^2 - 1) + \beta_{13}\eta(\eta^2 - 1) + \beta_{14}\eta^4(3\xi^2 - \\ & 1) + 2\beta_{15}\xi\eta(\eta^2 - 1) + \beta_{16}\eta(3\xi^2 - 1) + 2\beta_{17}\xi(2\xi^2 - 1) + \\ & 2\beta_{19}\xi\eta(2\xi^2 - 1) + \beta_{20}\eta^2(\eta^2 - 1) + 2\beta_{21}\xi\eta^2(2\xi^2 - 1) + \\ & 2\beta_{22}\xi\eta^2(\eta^2 - 1) + 2\beta_{23}\xi\eta(2\xi^2\eta^2 - 1) + \beta_{24}\eta^2(3\xi^2\eta^2 - 1) + \\ & 2\beta_{25}\xi\eta^2(2\xi^2\eta^2 - 1) + \beta_{26}(5\xi^4 - 1) = 0.0 \quad (7) \end{aligned}$$

The above equality is valid only when

$$\begin{aligned} (3\xi^2 - 1) &= 0.0 \Rightarrow \xi = \pm \frac{1}{\sqrt{3}} \\ \eta(3\xi^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi = \pm \frac{1}{\sqrt{3}} \\ \eta(\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \eta = \pm 1.0 \\ \eta^2(3\xi^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi = \pm \frac{1}{\sqrt{3}} \\ 2\xi\eta(\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi = 0.0 \text{ or } \eta = \pm 1.0 \\ \eta^2(3\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{3}} \\ 2\xi(2\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \xi = \pm \frac{1}{\sqrt{2}} \\ 2\xi\eta(2\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi = \pm \frac{1}{\sqrt{2}} \\ \eta^2(\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \eta = \pm 1.0 \\ 2\xi\eta^2(2\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi = \pm \frac{1}{\sqrt{2}} \\ \xi\eta^2(\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \eta = \pm 1.0 \end{aligned}$$

$$\begin{aligned} 2\xi\eta(2\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{2}} \\ \eta^2(3\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{3}} \\ 2\xi\eta(2\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{2}} \\ (5\xi^4 - 1) &= 0.0 \Rightarrow \xi = \pm \frac{1}{\sqrt{5}} \end{aligned}$$

Similarly, for equality of derivatives with respect to η we have

$$\frac{\partial \phi_{as}}{\partial \xi} = \frac{\partial \phi_{ac}}{\partial \xi}$$

$$\frac{\partial \phi_{as}}{\partial \xi} = \alpha_3 + \alpha_4 \xi + 2\alpha_6 \eta + 2\alpha_7 \xi^2 + \alpha_8 \xi \eta + 2\alpha_9 \xi^2 \eta \quad (8)$$

$$\begin{aligned} \frac{\partial \phi_{ac}}{\partial \eta} &= \beta_3 + \beta_4 \xi + 2\beta_6 \eta + \beta_7 \xi^2 + 2\beta_8 \xi \eta + 2\beta_9 \xi^2 \eta + 3\beta_{11} \eta^2 + \\ & 3\beta_{12} \xi^3 + 3\beta_{13} \xi \eta^2 + 2\beta_{11} \xi^3 \eta + 2\beta_{15} \xi^2 \eta^2 + 3\beta_{16} \xi^3 \eta^2 + 4\beta_{18} \eta^3 + \\ & \beta_{19} \xi^4 + 4\beta_{20} \xi \eta^3 + 4\beta_{22} + 4\beta_{24} \xi^3 \eta^3 + 4\beta_{25} \xi^4 \eta^3 + 5\beta_{27} \eta^4 \quad (9) \end{aligned}$$

Again, similarly putting the values $\alpha_2, \dots, \alpha_9$ from (4) in terms of β in (9) & (10) and for (6) rearranging the equation after eliminating similar terms, we get: -

$$\begin{aligned} & \beta_{11}(3\eta^2 - 1) + \beta_{12}\xi(\xi^2 - 1) + \beta_{13}\xi(3\eta^2 - 1) + \beta_{14}2\xi(\xi^2 - 1) + \\ & 2\beta_{15}\xi^2(3\eta^2 - 1) + \beta_{16}\xi(3\xi^2\eta^2 - 1) + 2\beta_{18}\eta(2\eta^2 - 1) + \\ & 2\beta_{19}\xi^2(\xi^2 - 1) + \beta_{20}2\xi\eta(2\eta^2 - 1) + 2\beta_{21}\xi^2\eta(\xi^2 - 1) + \\ & 2\beta_{22}\xi^2\eta(2\eta^2 - 1) + \beta_{23}\xi^2(3\xi^2\eta^2 - 1) + \beta_{24}2\xi\eta(2\xi^2\eta^2 - 1) + \\ & \beta_{25}2\xi^2\eta((2\xi^2\eta^2 - 1) + \beta_{27}(5\eta^4 - 1) = 0.0 \quad (10) \end{aligned}$$

This equality holds good only when

$$\begin{aligned} (3\eta^2 - 1) &= 0.0 \Rightarrow \eta = \pm \frac{1}{\sqrt{3}} \\ \xi(3\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = \pm \frac{1}{\sqrt{3}} \\ \xi(\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \xi = \pm 1.0 \\ \xi^2(3\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = \pm \frac{1}{\sqrt{3}} \\ 2\xi\eta(\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi = \pm 1.0 \\ \xi^2(3\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{3}} \\ 2\eta(2\eta^2 - 1) &= 0.0 \Rightarrow \eta = 0.0 \text{ or } \eta = \pm \frac{1}{\sqrt{2}} \\ 2\xi\eta(2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \eta = \pm \frac{1}{\sqrt{2}} \\ \xi^2(\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \xi = \pm 1.0 \\ 2\xi^2\eta(2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \eta = \pm \frac{1}{\sqrt{2}} \\ \xi^2\eta(\xi^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi = \pm 1.0 \\ 2\xi\eta(2\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{2}} \\ \xi^2(3\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{3}} \\ \xi^2\eta(2\xi^2\eta^2 - 1) &= 0.0 \Rightarrow \xi = 0.0 \text{ or } \eta = 0.0 \text{ or } \xi\eta = \pm \frac{1}{\sqrt{2}} \\ (5\eta^4 - 1) &= 0.0 \Rightarrow \eta = \pm \frac{1}{\sqrt{5}} \end{aligned}$$

The derived results have been given in Table I which are optimal stress locations for derivatives with respect to ξ and η . Now theoretically we have obtained optimal stress locations for derivatives with respect to ξ and η in local co-ordinates.

III. DETAILS OF OPTIMAL STRESS LOCATIONS

From Table I optimal stress location whichever occurs

frequently with derivatives ξ and η are tabulated below in local co-ordinates shown in Table II.

TABLE I
OPTIMAL STRESS LOCATIONS FOR RESPECTIVE DERIVATIVES

For terms Having the Following β coefficients	For derivatives with respect to ξ	For derivatives with respect to η
β_9	$\xi = 0.0$ or $\eta = 0.0$	$\xi = 0.0$ or $\eta = 0.0$
β_{10}	$\xi = \pm 1/\sqrt{3}$	
β_{11}		$\eta = \pm 1/\sqrt{3}$
β_{12}	$\xi = \pm 1/\sqrt{3}$ or $\eta = 0.0$	$\xi = 0.0$ or $\xi = \pm 1$
β_{13}	$\eta = 0.0$ or $\eta = \pm 1$	$\xi = 0.0$ or $\eta = \pm 1/\sqrt{3}$
β_{14}	$\eta = 0.0$ or $\xi = \pm 1/\sqrt{3}$	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1$
β_{15}	$\eta = 0.0$ or $\xi = 0.0$ or $\eta = \pm 1$	$\xi = 0.0$ or $\eta = \pm 1/\sqrt{3}$
β_{16}	$\eta = 0.0$ or $\xi = \pm 1/\sqrt{3}$	$\xi = 0.0$ or $\xi = \pm 1/\sqrt{3}$
β_{17}	$\xi = 0.0$ or $\xi = \pm 1/\sqrt{2}$	
β_{18}		$\eta = 0.0$ or $\eta = \pm 1/\sqrt{2}$
β_{19}	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$	$\xi = 0.0$ or $\xi = \pm 1$
β_{20}	$\eta = 0.0$ or $\eta = \pm 1$	$\eta = 0.0$ or $\xi = 0.0$ or $\eta = \pm 1/\sqrt{2}$
β_{21}	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1$
β_{22}	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1$	$\eta = 0.0$ or $\xi = 0.0$ or $\eta = \pm 1/\sqrt{2}$
β_{23}	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$	$\xi = 0.0$ or $\xi = \pm 1/\sqrt{3}$
β_{24}	$\eta = 0.0$ or $\xi = \pm 1/\sqrt{3}$	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$
β_{25}	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$	$\xi = 0.0$ or $\eta = 0.0$ or $\xi = \pm 1/\sqrt{2}$
β_{26}	$\xi = \pm \frac{1}{\sqrt{5}}$	
β_{26}		$\xi = \pm \frac{1}{\sqrt{5}}$

TABLE II
OPTIMAL STRESS LOCATIONS

$x(\xi)$	$y(\eta)$
0.0	0.0
± 1.0	± 1.0
$\pm 1/\sqrt{3}$	$\pm 1/\sqrt{3}$
$0.0 \& \pm 1.0$	$0.0 \& \pm 1.0$
$0.0 \& \pm 1/\sqrt{3}$	$0.0 \& \pm 1/\sqrt{3}$
$\pm 1.0 \& \pm 1/\sqrt{3}$	$\pm 1.0 \& \pm 1/\sqrt{3}$
$\pm 1/\sqrt{2}$	$\pm 1/\sqrt{2}$
$\pm 1/\sqrt{5}$	$\pm 1/\sqrt{5}$

IV. DISCUSSION ABOUT THE RESULTS

Using Barlow's criteria, the locations which satisfy fully the ξ^2 and η^3 terms are $\pm 1/\sqrt{3}$. However, 0.0 location fails to satisfy ξ^2 and η^3 terms but repeats many times. Other very good locations are $(0.0 \& \pm 1/\sqrt{3})$ within the elements and on the boundary $(0.0 \& \pm 1.0)$ and $(\pm 1.0 \& \pm 1/\sqrt{3})$.

V. CONCLUSION

From above theoretical calculations it is quite clear that assuming very high order polynomials, we get number of optimal locations at our disposal for selecting the best ones. For this purpose, numerical experimentation proposed to be performed on number of structural engineering problems to verify the same and arrive at final conclusion.

VI. NUMERICAL VERIFICATION

For numerical verification purpose, one engineering stress analysis problem chosen is thick beam. The thick beam was solved for Lagrangian 9 noded elements and the stresses were found at number of selected locations, further to verify the same stresses are also calculated at number of same locations (Sets in Figs. 1-3) by using quartic order 25 noded Lagrangian elements (Fig. 4) for the same mesh configuration. Here assumption has been made that higher order element as standard and will yield accurate results being a higher order element.

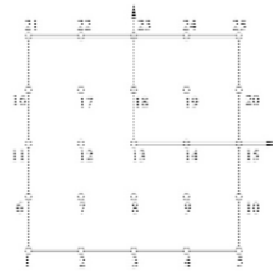


Fig. 1 25- Noded Element

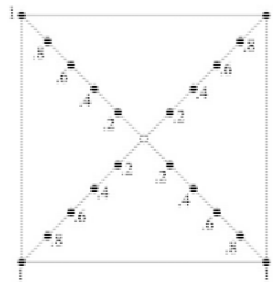


Fig. 2 Set 1

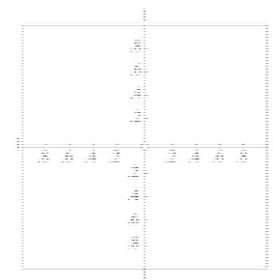


Fig. 3 Set 2

A thick cantilever beam of length 48 cm height 12 cm and thickness 1.0 cm is shown in Fig. 5 (a). The elastic modulus E taken as 2000000 kg/m² for beam is assumed to be homogeneous and isotropic and same is discretised in to 16 elements as shown in Fig. 5 (b). Then root mean square (RMS) stress errors are found and results are plotted with respect to error and distance of location in element in local co-ordinates.

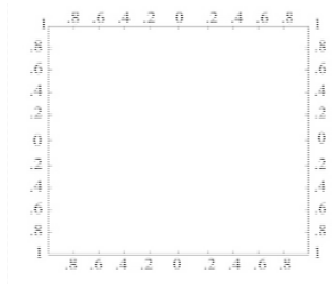


Fig. 4 Set 3

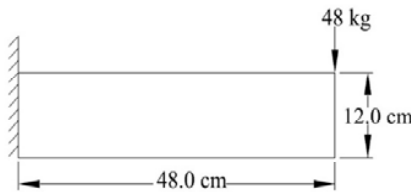


Fig. 5 (a) A Thick cantilever Beam

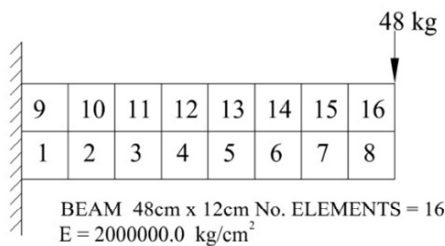


Fig. 5 (b) Discretised Thick Cantilever Beam

VII. NUMERICAL RESULTS OF THICK BEAM

For Lagrangian elements, plot of errors as shown in Fig. 6 where along the set 1 points the minimum error is 0.12 at location $\pm 1/\sqrt{3}$ (0.57) from origin along diagonal axis. At origin error is 0.44 and the maximum error is 1.05 at the corner nodes. Along set 2, the minimum error is 0.19 at ± 0.745 locations, at origin error is 0.44 and maximum error is 0.45 at mid side points. Further along set 3, the minimum error is 0.45 at mid side points and maximum error is 1.05 at corner nodes.

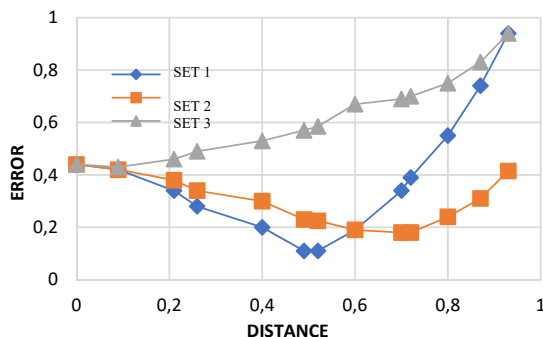


Fig. 6 Error plot for thick beam (9-noded)

VIII. FINAL CONCLUSION

It is clear from above numerical case study that $\xi = \eta = \pm 1/\sqrt{3}$ i.e. ± 0.577 are the best optimal location within the element apart from other various locations derived above. If we wish to calculate stresses at the boundary then mid-side nodes (0.0, ± 1), are the best locations and along the local coordinate axes, locations at (0.0, ± 0.577) are the best one. Thus, we can very well draw the conclusion that theoretically arrived optimal stress locations exist in the element as well as at the boundary where researchers can calculate stress in their optimisation problems. Also, location can be verified in laboratory with various standard problems using stress sensors or strain gauges which can also be another research area.

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