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Engineering Optimization Using Two-Stage Differential Evolution

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***Abstract*—**This paper employs a heuristic algorithm to solve engineering problems including truss structure optimization and optimal chiller loading (OCL) problems. Two different type algorithms, real-valued differential evolution (DE) and modified binary differential evolution (MBDE), are successfully integrated and then can obtain better performance in solving engineering problems. In order to demonstrate the performance of the proposed algorithm, this study adopts each one testing case of truss structure optimization and OCL problems to compare the results of other heuristic optimization methods. The result indicates that the proposed algorithm can obtain similar or better solution in comparing with previous studies.

***Keywords*—**Differential evolution, truss structure optimization, optimal chiller loading, modified binary differential evolution.

I. Introduction

OPTIMIZATION approaches have been widely used in various engineering problems in recent years. Most of developed optimization methods are designed for dealing with continuous optimization problems. One of famous heuristic approaches, DE, proposed by Storn and Price [1] is used to deal with optimization problems in continuous space. It has been recognized as a powerful heuristic optimization approaches and has been applied in many fields pertaining to engineering problems [2]-[4]. However, DE is not easy to solve the binary-valued optimization problem due to the fact that searching exploration in binary-valued optimization problems is weak. Thus, new binary strategies for DE algorithms have been proposed to evolve solutions for binary-valued optimization problems. Due to the limitation of binary coding, the binary DE have good exploration ability but lack of exploitation in searching binary optimal solution due to the limitation of binary coding.

Truss optimization problem has been a standard optimization problem in the fields of structure optimization problems. The truss optimization problem is attractive due to its direct applicability in the design of structures. Many heuristic optimization algorithms have been developed to search the global optimum for truss structures optimization problems. Many heuristic algorithms are widely applied to solve the many different types of engineering applications [5]-[9].

OCL is a key issue for energy saving. In air condition systems of commercial buildings where multiple chillers are operated in parallel (multichiller systems), each chiller can operate independently; thus, chiller operation schedule adjusted by time schedule to obtain the requirement of refrigeration ton (RT) and a flexible maintenance schedule [10] can be easily arranged in the commercial building.

The key issue of saving energy in multichiller system is how to operate appropriate numbers of operating chillers and control points so that each chiller operates at optimal efficiency. Multichiller systems are composed of chillers of varying features or even of various types of chillers; the optimized control setting is not easy to find out by using traditional optimization approaches. Many researches have discussed the OCL problem [11]-[16].

This study proposed a two-stage DE algorithm; the integrated framework includes DE with binary-valued and real-valued types of variables, to solve minimum weight of truss structure and OCL problems. MBDE [17], [18] is involved to increase searching diversity in the first stage; the real-valued DE algorithm is used in the second stage for exploitation. The proposed two-stage algorithm, by the integration the advantages of MBDE and DE, can have good exploration and exploitation in solving optimization problems. The comparison results indicates that proposed method can obtain better solutions than those found in literatures [5], [6], [15], [16] and it is proven that two-stage DE is suitable for solving various engineering optimization problems.

II. Truss Structure Optimization and OCL Problems

A. Truss Structure Optimization

Truss optimization can be simply classified into three categories: size, shape, and topology. Size optimization of truss structures is to optimize the cross-sectional area of truss members while the coordinates of the nodes and existence of truss members are held constant. In shape optimization, the coordinates of the nodes become design variables, and others (cross-sectional area and existence of truss members) remain constant. The connectivity and existence of truss member have become the design variables in the topology optimization of truss structure. Because the optimization of size and shape simultaneously are nonlinear programming problems [5], consideration of all the three types of variables makes the problems very complex due to the different characteristics of the design variables, including both discrete and continuous variables. The formulation of the truss optimization problem can be described as follows:

$$\text{Minimize:}\quad \mathrm{W(A)} = \sum_{i}^{n} \rho_i L_i A_i i \tag{1}$$

The boundary constraints are subject from G1 to G6 listed

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below:

- G1: Truss structure is acceptable to the user.
- G2: Truss is kinetically stable.
- G3: $\sigma_i \leqq \sigma_{\text{allow}}$, $i=1,2\ldots\ldots m$
- G4: $\delta_j \leqq \delta_{allow}$, $j=1,2\ldots\ldots n$
- G5: $A_i^{min} \leqq A_i \leqq A_i^{max}$, $i=1,2\ldots\ldots m$
- G6: $\xi_j^{min} \leqq \xi_j \leqq \xi_j^{max}$, $j=1,2\ldots\ldots n$
- Constraint G1: The number of basic nodes for supports and loads must exist. The definition of basic nodes is that these nodes must have loading or be set as fixed position, and it must be existed during the search process.
- Constraint G2: Grubler's criterion [19] is used to check the degree-of-freedom (DOF) of the truss for kinetic stability. Once a truss is a non-mechanistic (non-positive DOF value), the stiffness matrix is then used to check whether it is positive definite.
- Constraints G3 and G4: All members of the truss must have stresses within the allowable strength of the material and all nodal displacements must not deflect more than allowable limits.
- Constraints G5 and G6: The design variables of the cross section area and positions of non-basic nodes should be bonded within the pre-specified value.

The design variables A_i are the cross-sectional areas of structure with m members; ξ_j stands for the n real-valued coordinates of all non-basic nodes present in the truss. ρ_i is the material density of truss member; L_i is the length of each truss member. The parameters σ_{allow} and δ_{allow} indicate the allowable strength of the member and the allowable deflection of the node defined by designer. The existence or void of a truss member in the ground structure is determined by comparing the cross-sectional area of the member with a pre-defined small critical cross-sectional area ε. If the cross-sectional area is small than critical area ε, the member is assumed to be removed from the truss structures. On the contrary, the truss member will be kept in the truss structure with cross-sectional area. The advantage of this representation can both represent size and topology of a truss member in a real-typed value. The setting of lower and upper bounds on the cross-sectional area must be in the range of [A_i^{min}, A_i^{max}], and the value of A_i^{min} is the same as the negative value of A_i^{max}. The reason of introducing negative area is to obtain an almost equal probability of any member being present or absent in a truss [1]. In this study, the definition of critical cross-sectional area ε and negative area are employed in the case considering topology optimization of truss structures. The penalty function of the constraint violations used in this study listed in (6) are applied as in [5].

B. OCL Problems

Multichiller system, composed of two or more chillers, provides many advantages in operating control, such as flexible operation, reserving capacity, and less frequent system shutdowns for maintenance. Each chiller can be operate independently in a multichiller system and provide various refrigeration capabilities; the chillers can provide a wide range of RT requirement according to operate on different or similar performance curves in HVAC system. The architecture of a multichiller system is as shown in Fig. 1.

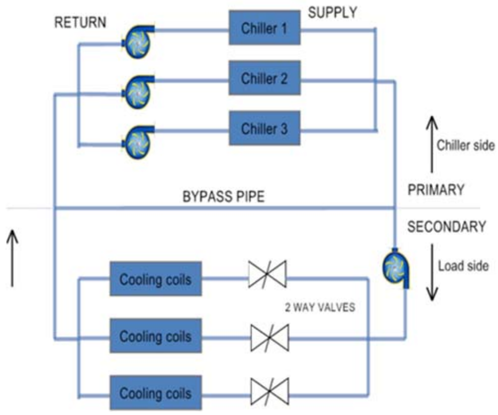

Fig. 1 Multichiller system architecture

In the design of a multichiller system, maximum peak load demand is defined as the maximum capacity of a chiller. Maximum peak load generally only occurs in summer because actual venue requirements and change of temperature, and the multichiller system operates low efficiency by controlled at low partial load mode during the remaining time. The partial load rate (PLR) of the chiller can be expressed as (2):

$$PLR = \frac{chiller\ load}{chiller\ rated\ load} \tag{2}$$

The power consumption of a single chiller, according to previous study [16], is defined as (3) by using PLR as a control parameters.

$$P_i = a_i + b_i \times PLR_i + c_i \times PLR_i^2 \tag{3}$$

The objective of OCL was to figure out the optimized partial loading rate for each chiller in the multichiller system to obtain minimized power consumption and also satisfied the RT requirement. The restriction of OCL is shown in (4), and the second restriction is that the partial loading of each chiller cannot be less than 30% [16].

$$\sum_{i=1}^{n} PLR_i \times Q_i = CL \tag{4}$$

III. Two-Stage DE

A. DE

DE [1], proposed by Storn and Price, was developed as a population-based global optimization algorithm for real-value numerical optimization problems. The upper and lower bounds for each design variable are used to generate initial population randomly. The objective function values for all individuals in the population are calculated and the best individual x_{best}, G of each generation is selected for mutation. The three main steps of DE, mutation, crossover, and selection are then iterated until the convergence state is satisfied.

The main target of the DE mutation operator is to calculate different mutated vectors. For each individual's vectors $X_{i,G}$ in the population, DE uses a mutation operation to generate a new mutated vector according to (5) and (6):

$$V_{i,G+1} = X_{i,G} + F \times (V_{r1,G} - X_{r2,G}) \quad (5)$$

$$V_{i,G+1} = X_{best,G} + F \times \left(V_{r1,G} - X_{r2,G}\right) \quad (6)$$

The index G is used as index of the current iteration generation. In (5) and (6), $r1$ and $r2$ are random numbers between 1 and NP. Two random vectors, $X_{r1,G}$ and $X_{r2,G}$, are selected from population of current iteration G to generate new mutation vector $V_{i,G+1}$ shown in (5). The vectors $X_{best,G}$ represents the best target vector of the current generation. F is scaling factor, a real value between zero and one, is introduced to increases or decreases the differential variation between the two vectors. After all individual entity generate a mutated vector by applying the mutation mechanism, the crossover operator is used to generate trial vectors in the next stage.

In the crossover operator, the trial vector $U_{i,G+1}$ is generated by a combination of parts of the mutated vector, $V_{i,G+1}$, and the target vector $X_{i,G}$. The crossover parameter is shown in (7) and (8), and C_r represents the crossover probability. If the generated random number R is smaller than the C_r value, the variable of mutation vector, $V_{i,G+1}$,will be chosen to be the variable of the trial vector. Otherwise, the variable of the target vector $X_{i,G}$ is selected as the variable of the trial vector. The mutation and crossover operators are used to diversify the search area of the optimization problems.

$$if\ R \le C_r\ , U_{i,G+1} = V_{i,G+1} \quad (7)$$

$$if\ R > C_r\ , U_{i,G+1} = X_{i,G} \quad (8)$$

All trial vectors $U_{i,G+1}$ have been selected to be candidates for selection operations. If the cost value of trial vectors $U_{i,G+1}$ is better than target vector $X_{i,G}$, the trial vector is chose as for new target vector $X_{i,G+1}$. Otherwise, the target vector $X_{i,G}$ is selected.

B. Binary DE

A MBDE used in this study proposed by [17], [18] was proposed by Wu and Tseng in 2010. MBDE uses bit string to represent the individuals in a population. The modified mutation mechanism is developed to solve discrete optimization problems. The evolutionary procedures of MBDE are the same as those of DE. Initial population of MBDE is randomly generated by uniform random number generator for value 0 or 1. The objective function value of all individuals of the population is calculated and a novel binary mutation mechanism is used to generate mutated individuals. The binary crossover mechanism is then applied to build trial solutions. If the objective function value of the trial solution is better than original solution, the trial solution will be selected for the next generation. Otherwise, the new solution for the next iteration is replaced by the original solution.

The main idea of modified binary mutation mechanism in this study is to find the common and difference feature pattern of two individuals and then apply a mutation operation to mutate bits in the different feature patterns using different mutation rates. The modified binary mutation mechanism is also based on the bit-string frameworks and logical operations. But the mutation is truly applied to two groups of bits which are decided by using an XOR logical operation. Bits with code “0” in the string after XOR operation represent common bits having identical codes of “1” or “0” in both strings. Bits with code “1” represent difference bits having different codes in two selected strings. The binary mutation equation is illustrated in (9) and (10). In (9), XOR operator is used to determine the difference bits and the common bits between the $X_{i,G}$ and $X_{r1,G}$ solutions. Each single bit belongs difference bits of $X_{i,G}$ are changed from 0 to 1 or 1 to 0 while random number R is higher than mutation rate F1. Similarly, the common bits of $X_{i,G}$ are mutated by using mutation rate F2. In general, the mutation rate F1 is higher than the mutation rate F2, because the common bits may have chance to become the feature pattern of a final optimum solution and the difference bits need higher probability to mutated. In the final stage, we combine these binary strings, including common and difference patterns, to represent trial solution $V_{i,G+1}$ mutated individuals. The flowchart of the new binary mutation mechanism is shown in Fig. 2. Though it uses logical operations with binary strings, the proposed algorithm tries to follow only the process of DE to perform optimum search without enhancement by other optimization algorithms.

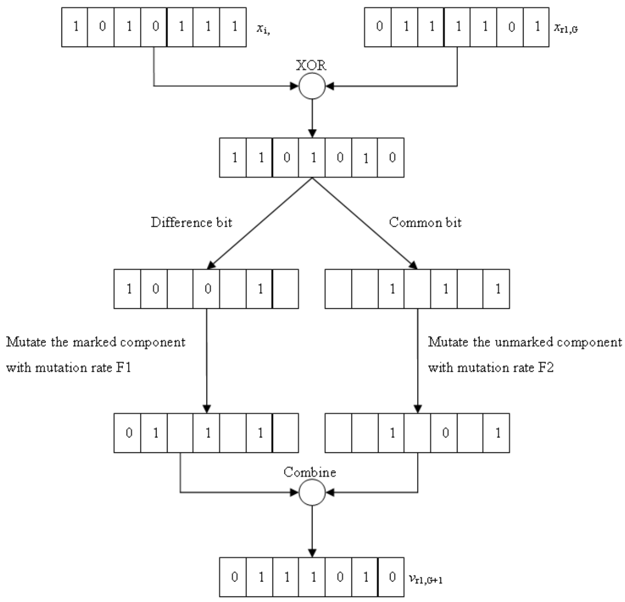

Fig. 2 Binary mutation mechanism in MBDE

$$V_{i,G+1} = F1\left(X_{i,G} \oplus X_{r1,G}\right) + F2!\left(X_{i,G} \oplus X_{r1,G}\right) \quad (9)$$

$$V_{i,G+1} = F1\left(X_{i,G} \oplus X_{best,G}\right) + F2!\left(X_{i,G} \oplus X_{best,G}\right) \quad (10)$$

C. Two-Stage DE

The proposed two-stage DE is to combine two kinds of one-stage approaches, MBDE and DE, in solving truss optimization and OCL problems. In stage 1, the solving of MBDE begins after the cross-sectional area of each truss member and PLR parameter of each chiller has been encoded to binary, restrictions have been considered, and the objective equation has been defined. The first stage can explore the possible solutions with coarse resolution. The candidates around the optimum solution obtained in the first stage are subject to real-coded DE in the second stage for rapidly converging to the optimal solution in continuous space.

IV. Result Study and Discussion

A. Truss Structure Optimization

The objective of this case is to minimize the weight through optimizing cross-section areas and topology of each truss member. The geometry model and loading conditions of a 15 member truss with six nodes are shown in Fig. 3. The material properties and design constraints are:

- Modulus of elasticity $E = 1\times10^4$ksi
- Density $\rho = 0.1$lb/in^3
- Maximum allowable stress $\sigma_{allow} = 25$ ksi
- Allowable displacement $\delta_{allow} = 2.0$ in.

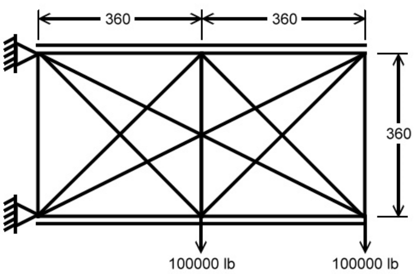

Fig. 3 15 member, 6 node truss structure

The area of the cross section following the setting of literature used [5] is limited between -35 in^2 and 35 in^2. The critical area of the truss member is set to 0.09 in^2. Case two will be analyzed using two-stage DE, to compare the performance of those two methods. The optimized truss with five truss members removed is illustrated in Fig. 4, and the cross-sectional area and weight employing two-stage DE, GA [5] and ant algorithm [6] are listed in Table I, respectively. The weight using two-stage DE is 4730.48 lbs, lighter than the weight found by GA of 4731.65, and the ant algorithm, 4730.824. The maximum stress and displacement using two-stage DE are all below the allowable constraints also shown in Table I. The convergence history of two-stage DE for this case is shown in Fig. 5, and the best solution employing two-stage DE has an overall weight of 4787.49 lbs at 168 iterations. The number of objective function evaluations of two-stage DE is 16,500 (2610 for stage one, 13,890 for stage two), much less than that found in the literature (GA = 80,850, ant algorithm = 41,000). It shows that two-stage DE obtains lower weight with fewer objective function evaluations than either GA or the ant algorithm.

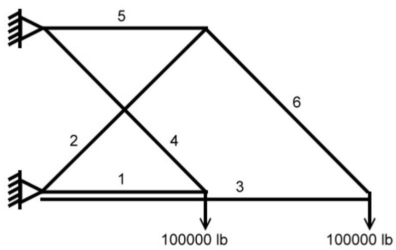

Fig. 4 Optimized truss structure found by two-stage DE

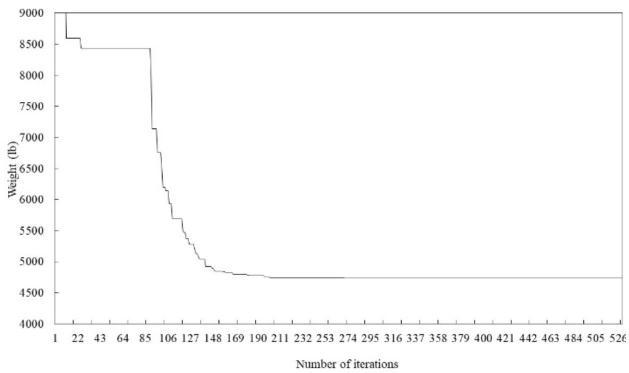

Fig. 5 The convergence history of two-stage DE for case one (15 members with 6 node truss structure)

TABLE I
MEMBER AREAS OF THE OPTIMIZED TRUSS

Truss Number	Area(in^2)		
	Two-stage DE	Deb and Gulati [5]	Luh and Lin [6]
1	5.386	5.219	5.428
2	20.433	20.310	14.308
3	14.310	14.593	20.265
4	7.646	7.772	7.617
5	28.881	28.187	20.549
6	20.366	20.650	28.876
Weight	4730.48	4731.65	4730.824
Max. Calculation	15,900	85,050	41,000
Max. Displacement (in)	1.999	2.000	1.999
Max. Stress (Ksi)	18.564	18.567	19.161

B. OCL

The objective of OCL in this study was to calculate the lowest power consumption combinations of all six chiller under and satisfied different RT requirements. The venue RT requirements were of five types: 6858 (90%), 6477 (85%), 6096 (80%), 5717 (75%), and 5334 (70%). The power consumption is defined in (3). The rated RT of each chiller and the power consumption parameters (a_i, b_i, c_i) are listed in Table II. In order to compare the result obtained in previous literature [15], [16], mean standard deviation (SD), maximum, minimum and average values were computed in this case through 30 runs. The population was 20 and the iteration number of each run is 20,000. In parameter setting of MBDE, the crossover rate was 0.5; the mutation rates F1 and F2 were 0.5 and 0.005, respectively. The mutation rate and crossover rate of DE were each 0.5. In order to obtain the same numbers of calculation with other study [15], [16], the total number of iterations was given 1,000.

TABLE II
POWER CONSUMPTION COEFFICIENT AND RATED RT INFORMATION

Chiller	a_i	b_i	c_i	Rated RT
1	399.345	-122.12	206.30	1280
2	287.116	80.04	700.48	1280
3	-120.505	1525.99	-502.14	1280
4	-19.121	898.76	-98.15	1280
5	-95.029	1202.39	-352.16	1280
6	191.75	224.86	524.04	1280

Table III summarizes the optimization results of the proposed method with DCSA [16]. The optimized results solved by two-stage DE were very close than those of DCSA in minimum values. In comparison with SD values, the proposed method has proven its stability than can obtain better result than those of DCSA in all CL conditions.

TABLE III
RESULT COMPARISON WITH DCSA

CL	Algorithm	MIN (kW)	Average (kW)	Max (kW)	SD
6868(90%)	Two-stage DE	4738.575	4733.575	4738.575	3.919E-06
	DCSA	4738.575	4738.575	4738.575	5.313E-7
6477(85%)	Two-stage DE	4421.649	4421.649	4421.650	6.355E-05
	DCSA	4421.649	4421.650	4421.650	2.301E-4
6096(80%)	Two-stage DE	4143.706	4143.709	4143.714	3.211E-04
	DCSA	4143.706	4143.710	4143.709	4.299E-4
5717(75%)	Two-stage DE	3838.208	3838.217	3838.225	6.702E-04
	DCSA	3840.055	3840.458	3843.766	9.428E-1
5334(70%)	Two-stage DE	3507.270	3507.278	3507.302	1.356E-03
	DCSA	3507.270	3507.715	3511.760	1.036

TABLE IV
RESULT COMPARISON WITH METHODS PROPOSED BY OTHER STUDIES

CL	Chiller No.	PSO [15]	Power (kW)	DCSA [16]	Power (kW)	This Study	Power (kW)
6898 (90%)	1	0.8026	4739.53	0.8127	4738.575	0.8127	4738.575
	2	0.7799		0.7496		0.7495	
	3	0.9996		1.0000		1.0000	
	4	0.9998		1.0000		1.0000	
	5	0.9999		1.0000		1.0000	
	6	0.8183		0.8385		0.8386	
6477 (85%)	1	0.7606	4423.04	0.727731	4421.649	0.7204	**4421.648**
	2	0.6555		0.656132		0.6342	
	3	1.0000		1.000000		1.0000	
	4	1.0000		1.000000		1.0000	
	5	1.0000		1.000000		1.0000	
	6	0.6835		0.716524		0.7463	
6096 (80%)	1	0.6591	4147.69	0.642735	4143.706	0.6423	4143.706
	2	0.5798		0.562645		0.5627	
	3	0.9991		1.000000		0.9999	
	4	0.9979		1.000000		0.9999	
	5	0.9921		1.000000		0.9999	
	6	0.5710		0.594490		0.5947	
5717 (75%)	1	0.7713	3921.07	0.843697	3840.055	0.8432	**3838.207**
	2	0.7177		0.783794		0.7832	
	3	0.3000		0.000001		0.0000	
	4	0.9991		1.000000		0.9999	
	5	1.0000		1.000000		0.9999	
	6	0.7187		0.883049		0.8824	
5334 (70%)	1	0.6265	3675.34	0.6418	3507.270	0.7499	3507.270
	2	0.7403		0.6621		0.6824	
	3	0.3093		0.3301		0.0000	
	4	0.9546		0.9906		1.0000	
	5	0.9511		0.9990		1.0000	
	6	0.6250		0.5806		0.7763	

The result comparison with PSO [15] and DCSA [16] was shown in Table IV. The result found by two-stage DE was better than those of PSO in all CL conditions (from 75% to 90%). When CL was 90%, 80% and 70%, a solution similar to that of DCSA was found. Furthermore, the solution obtained by two-stage DE was superior to that of DCSA under other CL conditions (85% and 75%), the results indicated that those of the proposed algorithm were superior to those of other studies [15], [16].

V. CONCLUSION

In this paper, a two-stage DE is successfully investigated for truss optimization and OCL problems. The MBDE proposed in this study is applied in the first stage to locate a diversity of solution candidates with coarse resolution, and then real-valued representation with DE in the second stage is used to obtain a better solution by exploiting the results obtained in the first stage. Truss optimization and OCL problems are used to illustrate the high viability of the proposed algorithm in comparison with the results found by previous studies [5], [6], [15], [16]. The results show that two-stage DE can obtain better or similar results than the approaches found in the literature in those two types of engineering optimization problems.

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