Effect of Density on the Shear Modulus and Damping Ratio of Saturated Sand in Small Strain

M. Kakavand, S. A. Naeini

Abstract—Dynamic properties of soil in small strains, especially for geotechnical engineers, are important for describing the behavior of soil and estimation of the earth structure deformations and structures, especially significant structures. This paper presents the effect of density on the shear modulus and damping ratio of saturated clean sand at various isotropic confining pressures. For this purpose, the specimens were compared with two different relative densities, loose Dr = 30% and dense Dr = 70%. Dynamic parameters were attained from a series of consolidated undrained fixed – free type torsional resonant column tests in small strain. Sand No. 161 is selected for this paper. The experiments show that by increasing sand density and confining pressure, the shear modulus increases and the damping ratio decreases.

Keywords—Dynamic properties, shear modulus, damping ratio, clean sand, density, confining pressure, resonant column/torsional simple shear.

I. INTRODUCTION

THE shear modulus (G) and damping ratio (D) are two important dynamic parameters of soil. In order to describe soil behavior, measurement of dynamic soil parameters, especially in small strains, is very important to preventing the damage to constructional structures in seismic problems. Therefore, conception of soil dynamic properties in geotechnical engineering is considerable special in seismic analysis. Some field experiments and geophysical tests in situ can obtain dynamic parameters via wave velocities. Maximum shear modulus, $G_{max}=\rho v_s^2$, that ρ is the soil density and Vs is shear wave velocity. Shear wave velocity (Vs) is acquired by site in-situ tests such as bore-hole, down-hole and cross-hole methods. However, for instance in high depth of soil, it could not be possible to perform these tests and sometimes for small projects it has no economic justification.

The first study on the dynamic response of soil in low strain range can be attributed to Iida [1]. Some researchers carried out resonant column tests in order to estimate relationships between the initial maximum shear modulus of sand and confining pressure and void ratio [2]-[8]. The studies of Hardin et al., Menq et al., Payan et al., SAxENA et al., Senetakis et al. and Wichtmann et al. show that the shear modulus of sand in small-strain are independent of some effective factors such as void ratio, confining pressure and the grain size characteristics and particle shape [2], [9]-[13].

Wichtmann et al. performed 163 free-free type resonant column tests on 25 different grain-size distribution curves quartz sand and with different coefficient of uniformity Cu. In constant void ratio, initial shear modulus were not affected by variations in the mean grain-size d50, but initial shear modulus significantly decreases by increasing coefficient of uniformity $Cu=d_{60}/d_{10}$ of the grain-size distribution curve. Some formulas are provided for the initial shear modulus G_{max} and the shear modulus coefficient K2 with the void ratio, relative density and uniformity coefficient of Cu [13].

Payan et al. used resonant column and bender elements tests to investigate the effect of non-plastic fine (0%, 5%, 10%, 15%, 20%, 25% and 30% by weight) and grain shape on dynamic properties of dry sand with isotropic stress path. The results showed that shear modulus decreases by increasing fine contents and by increasing the percentage of fine content, the contact between soil particles increases, as a result, these soils dampen the energy of the wave. In soil with fewer fines content, the grain size characteristics and particle shape are important to describe soil behavior and achieve the dynamic properties of soil. The results showed by increasing the percentage of non-plastic fine contents, the sand-dominant converted to silt-dominant [14].

Morsy et al. carried out laboratory experiments on dried and saturated Dabaa and Agami sands of Egypt with different void ratio and confining stress in small and medium shear strain ranges to determine dynamic properties, Poisson ratio and initial Yang modulus (Ei). The effect of void ratio on the shear modulus and damping ratio in lower void ratio is pronounced. Therefore, by decreasing void ratio, shear modulus increases and damping ratio decreases, and in dry sand with irregular and rough particles lower shear modulus and higher stress exponents compared to saturated sands is seen [15].

Payan et al. studied the effect of grain size characteristics and particle shape of sand via bender element apparatus in isotropic and anisotropic stress state along variable stress paths in very small shear strains. Several tests showed that participation in the shear modulus is different. In well-graded sand, there was great difference between the contributions of the principal stresses along and perpendicular to the direction of wave propagation, and were provided a mathematical relation for the initial shear modulus in isotropic and anisotropic stress [16].

Sadeghzadegan et al. studied the influence of clay content and saturation condition on the small and mid to large with bender element and cyclic triaxial apparatus on small to mid-shear strain. A series of saturated and unsaturated (70, 80, 90, 95 and 100% saturated rate) cyclic triaxial loading and bender

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element tests on clean sand and mixed with a range of Kaolinite including 0%, 10%, 20% and 30% by weight were carried out. The results showed that shear modulus increases by decreasing the degree of saturation, and shear modulus decreases by increasing plastic fine contents in all the ranges of degrees of saturation [17].

Most studies about clean sand focused on particle shape, grain size characteristics and confining pressure, but in this study, the density of specimens and confining pressure on saturated clean sand in small strain is evaluated. For this purpose, a series of undrained fixed-free torsional resonant column tests are conducted on clean Firouzkouh sand specimens with 30% and 70% relative density.

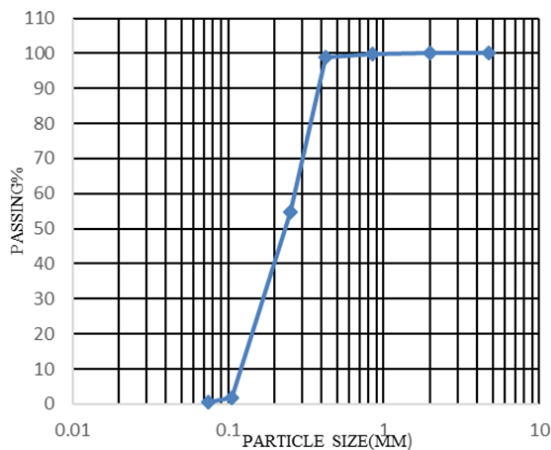Fig. 1 Optical microscope image of the Firouzkouh sand

Fig. 2 Particle size distribution curve of Firouzkouh sand

II. MATERIALS AND EXPERIMENTAL PROCEDURE

The sand No. 161 used in this article was obtained from Firouzkouh located in Iran. Firouzkouh sand No.161 is uniform quartz sand with sub-angular to sub-rounded grains. An optical microscope image of the Firouzkouh sand soil is given in Fig. 1. Table I shows the properties of the sand, and the grain-size distribution curves are depicted in Fig. 2 (ASTMD422 [18]). Indicator tests, grain size analysis, specific

gravity (ASTM D854 [19]), minimum specific gravity (maximum void ratio) (ASTM D4254 [20]) and maximum specific gravity (minimum void ratio) (ASTM D4252 [21]) were performed according to ASTM standards.

TABLE I
PROPERTIES OF FIROUZKOUH SANDS

	G_S	e_{max}	e_{min}	Cu	Cc	Fc
Firouzkouh sand	2.67	0.88	0.61	2.66	0.38	<0.35

Fig. 3 Resonant column device

The resonant column (RC) device used for this study was manufactured by Wykeham Farrance as shown in Fig. 3. This is a free-fixed type, which is a rotating sample head. The cubical top mass is equipped with one electrodynamics exciters, which accelerate a small mass. This acceleration and the resulting acceleration of the top mass are measured with acceleration transducers. Testing was conducted in accordance to (ASTM D4015 [22]).

The tested specimens are prepared in 50 mm diameter and 100 mm in height. The specimens were selected with two different relative densities (Dr), loose Dr = 30% and dense Dr = 70%. The initial void ratio for loose sand e0 = 0.796 and dense sand e0 = 0.684. The soil specimens were prepared by wet tamping method [23]. The soil specimens were prepared with moisture content of 5% then compacted in five layers on the porous stone of the apparatus in a plastic split mold. The first step is the saturation of the samples through carbon dioxide (CO_2), then water was passed through the specimens for two hours. The specimens were saturated with a Skempton's pore-water pressure parameters B value in excess of 95%. Then, to reach the target (B = 0.95%), the confining pressure and the back pressure increased step by step with a difference of 20 Kpa; then, the specimens consolidated isotropically in three effective confining pressures (100, 150, 200).

III. RESULTS

Generally, according to previous studies [2], [9], [13]-[15], shear modulus (G) and damping ratio (D) depend on the effective confining pressure, void ratio, fine content, grain size characteristics and particle shape. Experimental studies on different soils have shown that with increasing pressure, the

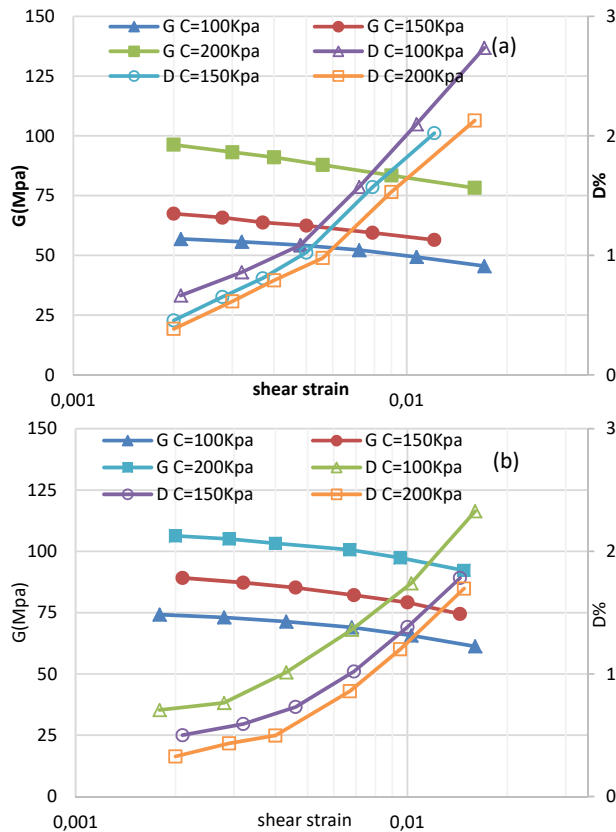shear modulus increases and damping ratio decreases.

Fig. 4 Variation of shear modulus and damping ratio against shear strain: (a) Dr = 30%; (b) Dr = 70%

The variation of shear modulus and damping ratio are plotted against shear strain at different confining pressures, as shown in Fig. 4. In loose specimen (Dr = 30%) with 100 Kpa, confining pressure at (ϒ=shear strain) ϒ = 0.002%, the value G is equal to 56.84 Mpa and the next shear strain is ϒ = 0.0032% and G is equal to 55.68 Mpa. With increasing confining pressure by 50 Kpa at the same condition and shear strain, shear modulus is 67.43 Mpa and 65.80 Mpa, respectively. On confining pressure 200 Kpa, at the same shear strains, shear modulus is 96.30 Mpa and 93.14 Mpa. The results showed that by increasing confining pressure in loose specimens from 100 Kpa to 150 Kpa and 200 Kpa, shear modulus increased by approximately 20% and 70%, respectively. In dense specimen (Dr = 70%) on confining pressure 100 Kpa at the least shear strain, G = 74.20 Mpa, and by increasing confining pressure from 100 Kpa to 150 Kpa at the same shear strain, G = 89.16 Mpa, at a confining pressure 200 Kpa, shear modulus is equal to 106.3 Mpa. It is observed that by increasing confining pressure from 100 Kpa to 150 Kpa and 200 Kpa in dense specimen, shear modulus increased by 20% and 40%, respectively. The results of the experiments show that by increasing confining pressure, the specimen stiffness increases so the shear modulus increases. It should be mentioned that by increasing confining pressure from 100 Kpa to 150 Kpa on loose and dense specimens, the shear modulus increased by 20%, and by increasing confining pressure from 100 Kpa to 200 Kpa in dense specimen the shear modulus increased by 40% but the loose specimen had a 70% increase, hence the effect of confining pressure in loose specimens is more than dense specimens.

The variation of shear modulus and damping ratio are plotted against shear strain at different density and same confining pressure as shown in Fig. 5. In loose specimens on ϒ = 0.002% at confining pressures 100 Kpa, 150 Kpa and 200 Kpa, the damping ratio is about 0.66%, 0.45% and 0.38%, respectively, and in dense specimens on ϒ=0.002% at confining pressures 100 Kpa, 150 Kpa and 200 Kpa, the damping ratio is about 0.70%, 0.50% and 0.32%, respectively. The results show that by increasing the confining pressure from 100 Kpa to 200 Kpa in loose specimen, the damping ratio is reduced by 68%.

According to Figs. 4 (a) and (b), the loose specimen on the first shear strain (ϒ = 0.002%) at 100 Kpa confining pressure, G = 56.84 Mpa and the dense specimen at this condition, the shear modulus is 74.20 Mpa and these specimens at the highest shear strain (ϒ = 0.017%), G = 45.5 and 61.25 Mpa, respectively. In confining pressure 150 Kpa, the loose and dense specimens on the first shear strain (ϒ = 0.002%) G = 67.42 and 89.15 Mpa, respectively, these specimens at the highest shear strain (ϒ = 0.013%) G = 56.46 and 74.46 Mpa, respectively. As density increased from 30% to 70%, the shear modulus increased by 30%. However, at confining pressure 200 Kpa, only a 15% increase in shear modulus was observed. According to the figures, the dense sand has a higher shear modulus than loose sand, because they have a lower void ratio

Damping ratio, in the smaller shear strains, is not appreciable, but by increasing shear strains, the difference in the damping ratio is obvious and the loose sand has a higher damping ratio in the same strain than the dense sand. For example in loose and dense specimens in the least shear strain, D = 0.66% and 0.70% at confining pressure 100 Kpa, and at highest shear strain D = 2.12% and 1.69%. The result of tests showed that the stiffness of specimens increases by increasing relative density and confining pressure.

IV. CONCLUSIONS

Several of consolidated undrained fixed – free type torsional resonant column tests in small strain were performed on loose and dense sand (Dr = 30 & Dr = 70) with three confining pressure (100 Kpa, 150 Kpa and 200 Kpa) to evaluate the dynamics properties of Firouzkouh sand. The results of this experimental study are summarized as follows:

− In loose specimen from 100 Kpa to 150 Kpa and 200 Kpa, shear modulus increased by approximately 20% and 70%, and in dense specimen, shear modulus increased by 20% and 40%, respectively; hence, the shear modulus increases with increasing confining pressure.
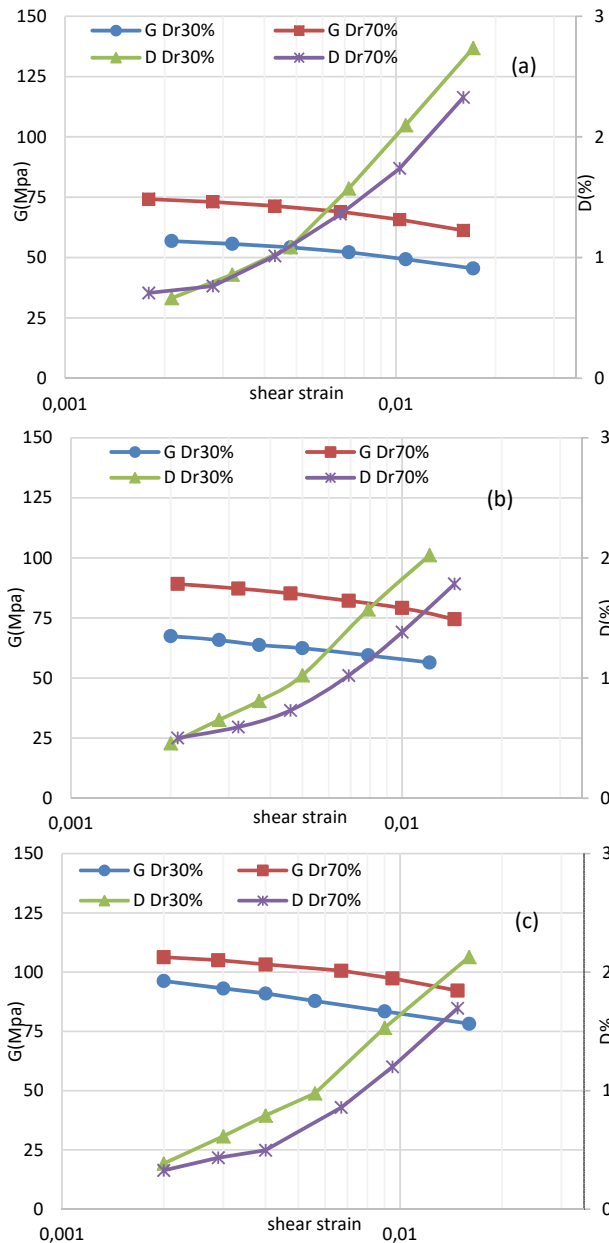
Fig. 5 Variation of G and D against shear strain: (a) confining pressure 100 Kpa; (b) confining pressure 150 Kpa;(c) confining pressure 200 Kpa

– The density increased from 30% to 70% in confining pressure 150 Kpa and 200 Kpa in the loose specimen, the shear modulus increased by 30% and 15%; therefore, the dense sand has a higher shear modulus than loose sand.
– The damping ratio decreases with increasing confining pressure, by increasing the confining pressure from 100 Kpa to 200 Kpa in loose specimens, the damping ratio is reduced by 68%.
– The damping ratio of dense and loose sand has no obvious change in very small shear strain, but in higher shear strains, the dense specimens have lower damping ratio.

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