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Non-parametric Linear Technique for Measuring the Efficiency of Winter Road Maintenance in the Arctic Area

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***Abstract*—**Improving the performance of Winter Road Maintenance (WRM) can increase the traffic safety and reduce the cost as well as environmental impacts. This study evaluates the efficiency of WRM technique, named salting, in the Arctic area by using Data Envelopment Analysis (DEA), which is a non-parametric linear method to measure the efficiencies of decision-making units (DMUs) based on handling multiple inputs and multiple outputs at the same time that their associated weights are not known. Here, roads are considered as DMUs for which the efficiency must be determined. The three input variables considered are traffic flow, road area and WRM cost. In addition, the two output variables included are level of safety in the roads and environment impacts resulted from WRM, which is also considered as an uncontrollable factor in the second scenario. The results show the performance of DMUs from the most efficient WRM to the inefficient/least efficient one and this information provides decision makers with technical support and the required suggested improvements for inefficient WRM, in order to achieve a cost-effective WRM and a safe road transportation during wintertime in the Arctic areas.

***Keywords*—**DEA, environmental impacts, risk and safety, WRM.

I. Introduction

CLIMATE change has become more obvious in the last few years. The result of climate change can be hotter weather in summer, as well as, lower temperature in wintertime, which can cause severe problems in road weather conditions and WRM service [1]. Low temperature and snowfall can cause low quality of transport network due to delays in time of trips, poor visibility, slippery road, accumulated snow on the road surface and high probability of accidents [2]. WRM can increase the friction between tire & pavement that controls the vehicle and ultimately leads to safe mobility. Friction affects braking distance, acceleration as well as directional control [3]. In order to achieve friction control, anti-icing and de-icing methods (e.g. salting) need to be applied on the road surface especially for heavily-trafficked roads [3].

WRM is about keeping roads safe i.e. clear roads with an acceptable safe driving condition. In addition, WRM has increased linearly over past decades as the mobility demand has increased [4]. For instance, in Norway, it has been estimated that personal transport and goods transport are going to increase by 28% and 70% respectively, by 2050 [5]. Therefore, this growth in demand results in increasing of emissions and increasing use of chemicals to control friction in WRM.

It is significant to note that in Arctic region (e.g. north of Norway), the winter period is longer than the typical three months period, it mostly starts in mid-November and finishes in mid-April, Therefore, to keep the desirable road conditions it is required more resources that the one used on places with the typical winter period. Due to this reason, comparison between winter data corresponding to different areas must consider the time duration of the winter period to avoid misinterpretations, as for example, registered number of accidents in the Arctic area is higher compared to other regions. In Norway, during 2005-2012 16% of fatalities registered in road traffic during winter were due to adverse weather conditions and difficult driving situations [6]. In 2013, Norwegian Transport Agency has set a twelve-year traffic policy and the main goal is to achieve the zero-casualties vision by 2025 [7]. This represents a clear statement about the need of improving road conditions during the whole year, but specially, during winter, where the driving conditions are more challenging to the drivers.

Sodium chloride (NaCl) is the effective material used during the salting road activity, as anti-icing and de-icing techniques at low temperature [8]. This method has been proved effective under specific conditions; however, the use of NaCl can have adverse effects on local environment, including local vegetation and animal species. The quantification of these adverse effects are proportional to the amount of salt use, therefore excessive use of NaCl must be avoided.

Current efforts in WRM strategies and actions aim the maximum possible traffic safety level in the roads during the winter, while minimizing the costs and negative environmental impacts. The main idea is to establish a balance between traffic safety, environmental impacts and economic resources used in different roads during wintertime [10]. To address this issue, this research implements a framework to measure the efficiency of WRM applying the method called DEA.

Even though DEA was introduced and discussed in details for first time in 1978 [9], it still currently represents a promising optimization technique for scenarios not covered yet, as for example the WRM case studied in this paper, nevertheless some maintenance scenarios have been

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previously covered [10]-[13].

DEA is a non-parametric mathematical method, which is based on fundamentals of linear programming. It measures the efficiency of a company, organization or DMUs using weighted multi-inputs and weighted multi-outputs. These weights factors are determined by DEA model based on the characteristics of the system introduced in the model by the selected input and output variables [10]. One of the objectives of DEA is to construct an efficiency frontier in accordance to efficient DMUs and these efficient DMUs are used as the benchmark for inefficient DMUs [11].

A DMU is considered efficient when its efficiency is on the efficiency frontier or inefficient when its performance not on this frontier [14], [11].

There are two formulations for DEA models; i) CCR formulation which is introduced by Charnes, Cooper and Rhodes (constant returns to scale) ii) BCC formulation which is explored by Banker, Charnes and Cooper (variable returns to scale) and it is also important to select orientation of the model, i.e. input-oriented or output-oriented characteristics [12].

Constant returns to scale means that either reduction or increase in the value of the input variables results in similar proportional change in outputs variables whereas, variable returns to scale means that either reduction or increase in the values of the input variables does not result in similar proportional change in output variables [10]. Furthermore, under the input-oriented model, the decision makers are able to implement modifications in the input variables for inefficient DMUs to promote changes in the output of the studied process or activity. However, for output-oriented model, decision makers are able to modify the values or characteristics of the output variables for inefficient DMUs [15]. Both of these models try either to minimize value of the input variables or to maximize values of the output variables that lead to increase in the efficiency of DMUs. To illustrate the mentioned concept, Fig. 1 displays an example of the study of a process over three DMUs (A, B and C), which includes one input variable and one output variable for the mentioned DMUs. After the method is applied, it is found that DMU_A and DMU_B are efficient and they construct an efficiency frontier. However, DMU_C is not efficient and it locates under the efficiency frontier. Therefore, DMU_C needs either a reduction in an input or an increase in an output in order to be considered as an efficient DMU.

Generally, input-oriented model is closely associated with operation and management issues while output-oriented model focuses on planning and strategy issues [16].

For the present study, it has been decided to use DEA-CCR model for evaluation of WRM efficiency. The decision of using DEA-CCR model was based on the fact that this model is the strictest model between the DEA models. In this way, it is warranty that if the DMU is efficient in DEA-CCR model, it will be efficient in DEA-BCC model. The major demand of the DEA-CCR model on the DMUs is introduced in the model by one extra free variable in the mathematical modelling. Furthermore, it is also significant to make decision for the orientation (either input-oriented or output-oriented) of the model. This decision is mostly based on the decision maker's choice, which means that decision makers are either flexible in reducing the inputs or increasing the outputs [12]. In this study, although the model orientation is not according to choice of decision makers, reduction in the inputs is considered to achieve a desired output such as reduction in WRM cost.

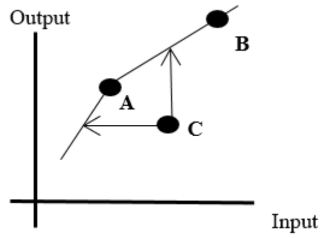

Fig. 1 An example of DEA approach

II. DEA-CCR MODEL

The original model for the calculation of the maximum efficiency for an individual DMU_p, Z_p, under CCR model is shown in (1) [9]:

$$Max\ Z_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \quad (1)$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1$$
$$u_r \geq 0\ ,\ v_i \geq 0$$
$$j = 1, \ldots, n \quad r = 1, \ldots, s \quad i = 1, \ldots, m$$

where i is the number of input variables, j is the number of DMUs, r is number of output variables, and x_{ij} and y_{rj} are values of the input and output variables of DMU_j considered in the modelling. Therefore, there are $\boldsymbol{n}$ DMUs, and every DMU has $\boldsymbol{s}$ output variables and $\boldsymbol{m}$ input variables. One restriction of the model is that the values of the input and output variables must be positive (non-negative and non-zero). In addition, u_r and v_i are decision variables in the model i.e. they are weights of inputs and weights of outputs respectively.

It is a common practice to solve the model described after it has been transformed into Linear Programming (LP) model, also called primary CCR model [9], illustrated by (2):

$$Max\ Z_p = \sum_{r=1}^{s} u_r y_{rp} \quad (2)$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{ip} = 1$$
$$\sum_{r=1}^{s} u_r y_{rj} \leq \sum_{i=1}^{m} v_i x_{ij}$$
$$u_r \geq 0\ ,\ v_i \geq 0$$
$$j = 1, \ldots, n \ \ r = 1, \ldots, s \ \ i = 1, \ldots, m$$

Decision variables u_r and $v_i$are then found after solving the linear system generated for each DMU_p, along with the maximum efficiency.

III. Physical Case

One of the main aspects of DEA models is that they can consider any variable (input variables, output variables and uncontrollable factors), without restriction in number or type of variables in the model without specifying parametric relationships and all variables have an equal influence on efficiency scores [11].

This research proposed the evaluation of efficiency for the WRM activity, known as salting, for a group of different roads. The three input variables considered are road area, material cost and Average Daily Traffic (ADT). These variables interact to define a road condition that is classified using the safety levels. It is accepted that to reach those safety levels some impacts on the surrounding environment can be also produced. The environment impact is classified as low, medium and high.

The graphical illustration of the WRM problem and the variables considered in this study can be seen in Fig. 2. We notice that the input variables, in practice, will determine the time consumed, quality and equipment involved during the WRM activity, named salting while, selected output variables are the mean to evaluate the performance of the mentioned WRM activity. However, the input of the mathematical modelling refers to the actual quantitative parameter called efficiency. The efficiency of final values reflects the influences of the input variables on the output variables of the process.

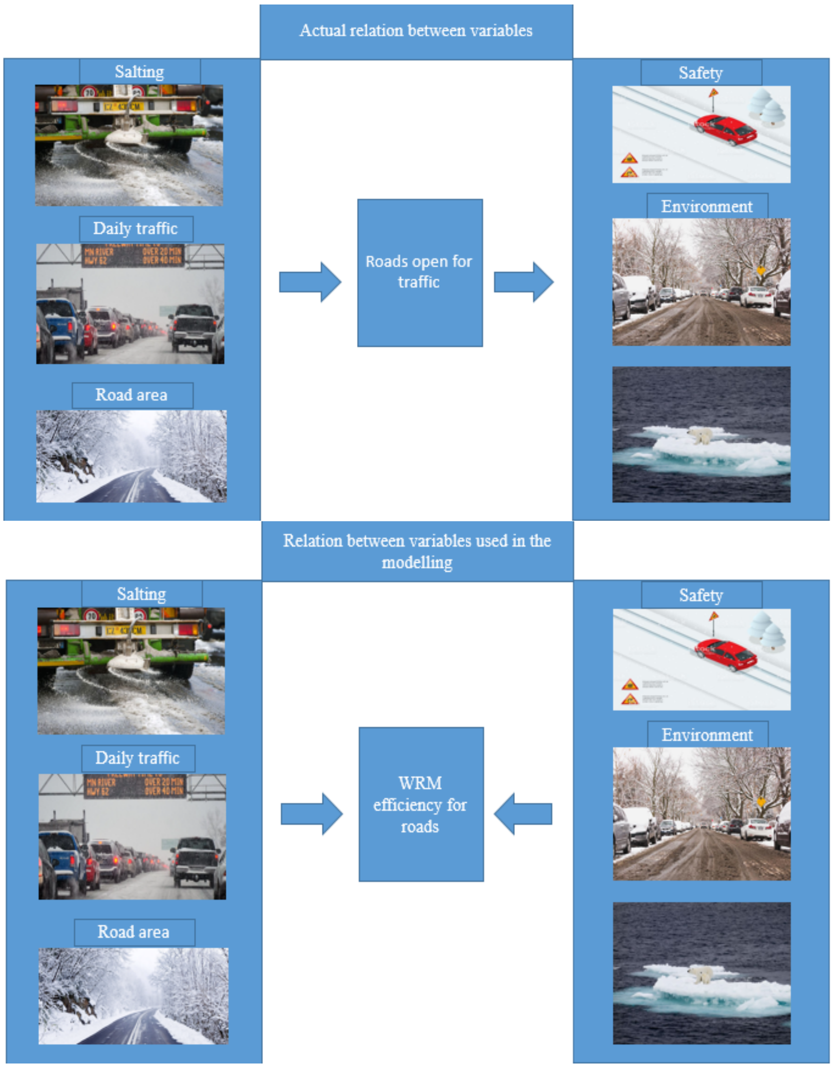

Fig. 2 Schematic view of the WRM analysis addressed in this study

This paper concentrates only on DEA-CCR input oriented model, which considers three inputs, one output and one uncontrollable factor.

After practical considerations were applied, some variables are aggregated to one variable [10]. For example, labor cost, material cost and equipment cost are aggregated to one variable i.e. WRM cost. Hence, the number of variables is reduced to ADT, area of the roads, maintenance cost as input variables.

In terms of the output variables, two scenarios have been selected. In the first scenario, safety level and environmental impacts will be selected both as outputs. The second scenario considers the safety level as the only output variable, because environmental impacts are considered as an uncontrollable factor. Uncontrollable factors cannot be controlled by decision makers [10]; however, they must be addressed. DEA models allow considering uncontrollable factors as uncontrollable variables inside the model [11].

The data used for this work correspond to a fictitious case, nevertheless, it reflects typical values of WRM of one lane road having different length at traffic conditions (for example light or heavy traffic road).

IV. Preparing Data Sets for the Selected Test Case

In order to use DEA model for measuring the efficiency of DMUs, the data set must fulfil a series of constrains or considerations imposed by the theory behind the model. Hereafter, there are six main considerations that have to be attended when preparing data:

1. Homogeneity
2. Input and Output Selection
3. Number of DMUs
4. Transformation of Qualitative Data
5. Mean Normalization
6. Isotonicity Relationship

The first three considerations guide the gathering of the row data for the modelling and the last three considerations guide the required treatment of the data before it can be used in the modelling.

A. Homogeneity among the DMUs

It is required to have homogeneity among the DMUs considered in the study case.

When a study case includes several DMUs, they must be in a way similar to each other. The similarity criterion must be established. It must perform a specific task or at least a similar task to the specified one in the study case [17], [18], [12]. For example, if the problem modelling corresponds to the evaluation of winter maintenance in roads, a DMU representing a storage facility for the salt cannot be included along with other roads in the DMU group.

In the study case of this paper the DMUs selected correspond to different roads on which slating is performed as WRM activity.

B. Input and Output Selection

Input and output variables selection will be based on the identification of relationships between them and the objective of the study case. The chosen input variables of this study are *ADT* of the roads in whole winter, total *area* of the road (length × width), and *WRM cost,* which is a summation of material cost (salt), labor cost and equipment cost (e.g. trucks) [11] whereas output selected variables are the *level of safety* in the road and *environmental impacts* associated to the WRM activities performed.

The classification of the level of safety in the roads includes three levels classifications defined as: High, Medium and Low. High safety level implies that no accident happens in that road during a specific period. Medium safety level implies that accident can happen in the same period, but they do not have any severe consequences. And Low safety level implies that during same period accident with casualties and severe injuries can happen.

Environmental impacts (uncontrollable factor) are classified as: High, Medium and Low. An interpretation of this type of undesirable impacts can be defined based on proximity of the road to fragile ecosystems. For example: High environmental impact is expected when salting takes place in roads close to the water bodies or vegetation. Medium environmental impact is expected when the road is close to either water or vegetation and low environmental impact is expected when the road is close to neither water nor vegetation.

C. Number of DMUs

Number of data sets plays a crucial role to specify the efficient frontier. A reasonable discrimination level can be achieved by an established rule of thumb, which states that the minimum number of DMUs can be $2 \times s \times m$, where s is number of outputs and m is number of inputs [18], which for this paper corresponds to a group of 12 DMUs.

Table I shows the raw data set after following the first three guidelines (considerations) described before. EIs is the abbreviation of Environmental Impacts. H, M and L are abbreviation of High, Medium and Low respectively.

TABLE I
Raw Data Set of Input And Output Variables

DMUs	ADT	Area(m2)	WRM.cost	EIs	Safety
A	10530	5202.57	€253000	H	H
B	15260	3994.83	€91000	L	H
C	16705	3716.12	€144000	L	L
D	15277	3205.15	€97000	L	M
E	16303	6781.90	€157000	H	H
F	14609	2684.90	€101000	M	M
G	13818	1458.60	€122000	H	L
H	15900	4500.00	€136000	L	L
I	10000	5300.00	€161000	H	M
J	12000	6720.00	€172000	M	M
K	17000	8900.00	€100000	M	M
L	10000	4000.00	€180000	H	L

D. Transformation of Qualitative Data

Qualitative values (i.e. safety and environmental impacts) must be quantified due to using them in the DEA-CCR mathematical formulation. So, it requires finding numerical surrogate variables and thus it has been decided to assign number “1” to low safety, number “2” to medium safety and number “3” to high safety and the same action is done for environmental impacts [12]. These transformations are named modified environmental impacts (M-EIs) and modified safety (M-S), both shown in Table II.

TABLE II
DATA SETS AFTER CONVERTING QUALITATIVE DATA TO QUANTITATIVE DATA

DMUs	ADT	Area (m^2)	WRM.cost	M-EIs	M-S
A	10530	5202.57	€253000	3	3
B	15260	3994.83	€91000	1	3
C	16705	3716.12	€144000	1	1
D	15277	3205.15	€97000	1	2
E	16303	6781.90	€157000	3	3
F	14609	2684.90	€101000	2	2
G	13818	1458.60	€122000	3	1
H	15900	4500.00	€136000	1	1
I	10000	5300.00	€161000	3	2
J	12000	6720.00	€172000	2	2
K	17000	8900.00	€100000	2	2
L	10000	4000.00	€180000	3	1

E. Mean Normalization

It is clear that data magnitude is not balanced in data sets. It is necessary to make a refinement in the data. A solution to balance data magnitude is to mean normalize the data which includes two main steps: i) calculating the average of the data set for every input and output ii) dividing every single input and output for each DMU by its associated mean value [19]. For instance, WRM cost for DMU_A is €25300, and the average WRM cost for all DMUs is €14283.33. So, the mean normalize of WRM cost for DMU_A is 25300/14283.33 = 1.77. Table III presents the mean values of data sets and Table IV presents the mean normalized of variables.

TABLE III
MEAN VALUES OF VARIABLES

	ADT	Area (m^2)	WRM.cost	M-EIs	M-S
Mean	13950.17	4705.34	€142833.3	2.08	1.92

TABLE IV
MEAN NORMALIZED OF VARIABLES

DMUs	ADT	Area	WRM.cost	M-EIs	M-S
A	0.75	1.11	1.77	1.44	1.57
B	1.09	0.85	0.64	0.48	1.57
C	1.20	0.79	1.01	0.48	0.52
D	1.10	0.68	0.68	0.48	1.04
E	1.17	1.44	1.10	1.44	1.57
F	1.05	0.57	0.71	0.96	1.04
G	0.99	0.31	0.85	1.44	0.52
H	1.14	0.96	0.95	0.48	0.52
I	0.72	1.13	1.13	1.44	1.04
J	0.86	1.43	1.20	0.96	1.04
K	1.22	1.89	0.70	0.96	1.04
L	0.72	0.85	1.26	1.44	0.52

F. Isotonicity Relationship

Isotonicity principle between input variables and output variables means that an increase in the value of any input should not produce a reduction in the values of output variables [20]. If the correlation between selected input variables and output variables is positive, they have isotonic relationship and they can be considered in the model [21].

In this study, the correlations between area and outputs are negative. To solve this issue, an approach called multiplicative inverse has been applied for each road (DMU). After this conversion, the correlations between this input variable and output variables are positive (i.e. an increase in this input results in an increase in output variables) and it can be included in the model [12]. The transformed variable is called "$Area_T$" in Table V. For instance, for DMU_A: $Area_T = 1/0.75 = 1.32$

TABLE V
DATA SETS APPLIED IN THE DEA MODEL

DMUs	ADT	$Area_T$	WRM.cost	M-EIs	M-S
A	1.32	1.11	1.77	1.44	1.57
B	0.91	0.85	0.64	0.48	1.57
C	0.84	0.79	1.01	0.48	0.52
D	0.91	0.68	0.68	0.48	1.04
E	0.86	1.44	1.10	1.44	1.57
F	0.95	0.57	0.71	0.96	1.04
G	1.01	0.31	0.85	1.44	0.52
H	0.88	0.96	0.95	0.48	0.52
I	1.40	1.13	1.13	1.44	1.04
J	1.16	1.43	1.20	0.96	1.04
K	0.82	1.89	0.70	0.96	1.04
L	1.40	0.85	1.26	1.44	0.52

V. RESULTS

Table VI presents the results including efficiency score and target WRM cost (T-WRM.cost) for each DMU. The results are calculated by running the DEA-CCR input oriented model, which is solved by MATLAB software. Target WRM cost for each DMU is obtained by multiplying its actual WRM cost (A-WRM.cost) with its associated efficiency score [10]. In addition, it is possible to measure the amount of overspending for inefficient DMUs in the data set [12]. For instance, the efficiency score for road C (DMU_C) is 47% and the target WRM cost is €144000 × 47% = €67277 which means that DMU_C is overspending by 53% (100% - 47% = 53%) compared to other efficient DMUs. Hence, it needs to spend €672767 without any changing in WRM quality in order to be considered as an efficient DMU.

TABLE VI
EFFICIENCY SCORE & TARGET WRM COST RESULTS FROM DEA MODEL

DMUs	Score of efficiency	A-WRM.cost	T-WRM.cost
A	0.94	€253000	€238731
B	1.00	€91000	€91000
C	0.47	€144000	€67277
D	0.83	€97000	€80568
E	1.00	€157000	€157000
F	1.00	€101000	€101000
G	1.00	€122000	€122000
H	0.41	€136000	€56331
I	0.84	€161000	€136013
G	0.60	€172000	€103768
K	1.00	€100000	€100000
L	0.71	€180000	€126900

To sum up the results from DEA model, out of 12 DMUs, 5 DMUs (DMU_B, DMU_E, DMU_F, DMU_G, DMU_K) are efficient (100% efficient). After these efficient DMUs, the most efficient DMU is DMU_A with 94% efficiency and the least efficient one is DMU_H with 41% efficiency, which can be definitely a warning for decision makers because resources are allocated to this DMU but overall obtained level of safety is not acceptable in a mentioned period.

Table VII illustrates the overall score of efficiency for DMUs calculated by DEA model with and without considering uncontrollable factor (i.e. environmental impacts). In fact, the differences between the efficiencies of these two models can simply show the impact of uncontrollable factor on efficiency scores of DMUs [12]. As it can be seen, if the DEA model runs without uncontrollable factor, the calculated efficiencies are lower for Road A, Road C, Road F, Road G, Road H, Road I, Road G, Road K and Road L. However, the efficiency scores stay the same for Road B, Road D and Road E, which shows that these DMUs are not influenced by uncontrollable factor.

TABLE VII
OVERALL SCORES OF EFFICIENCY OBTAINED BY TWO DEA MODELS

DMUs	Efficiency score with considering uncontrollable factors in the DEA model	Efficiency score without considering uncontrollable factors in the DEA model
A	0.94	0.77
B	1.00	1.00
C	0.47	0.36
D	0.83	0.83
E	1.00	1.00
F	1.00	0.99
G	1.00	0.91
H	0.41	0.34
I	0.84	0.50
G	0.60	0.51
K	1.00	0.72
L	0.71	0.33

VI. CONCLUSIONS

This paper presents non-parametric linear technique (DEA-CCR input oriented model) for measuring the efficiency of WRM in the Arctic region.

Although, target cost has been considered in this study, it is definitely possible to consider area of the road, which means that changes in the road area can influence efficiency score; for instance, dividing the entire road into different parts to increase the WRM quality in a shorter time. In addition, sometimes ADT can be controlled by traffic agency to avoid presence of special vehicles in critical roads when the weather condition is harsh or it is also feasible to impose some traffic limitations to control the number of cars in the roads.

The results in this study provide decision makers how to develop the efficiency scores for inefficient DMUs. It is important to highlight that the observations from the DEA models are not in detail to pinpoint the specific reasons for inefficient DMUs. Nevertheless, it warns about necessary actions that need to be taken to reinforce the WRM service for inefficient DMUs. Additionally, policy makers and managers can use these observations as guidelines (i.e. target WRM cost) which demonstrate potential performance to improve the efficiency for inefficient DMUs [22].

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